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Complete MATH

IIT-JEE · CBSE eBOOKS

CLASS 11&12th

CLASS 11th

RELATIONS & FUNCTIONS

01. Definition

(i) ORDERED PAIR

An ordered pair consists of two objects or elements in a given fixed order.
For example, if A and B are any two sets, then by an ordered pair of elements we mean pair (a, b) in that order, where $a \in A, b \in B$.

NOTE ☞ An ordered pair is not a set consisting of two elements. The ordering of the two elements in an ordered pair is important and the two elements need not be distinct.

(ii) EQUALITY OF ORDERED PAIRS

Two ordered pairs (a_1, b_1) and (a_2, b_2) are equal iff

$$a_1 = a_2 \text{ and } b_2 = b_2$$

i.e., $(a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2$ and $b_2 = b_2$

(iii) CARTESIAN PRODUCT OF SETS

Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$

(iv) CARTESIAN PRODUCT THREE SETS

Let A, B and C be three sets. Then, $A \times B \times C$ is the set of all ordered triplets having first element form A, second element form B and third element from C.

i.e., $A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}$

02. Number of Elements in the Cartesian Product of Two Sets

RESULT

If A and B are two finite sets, then $n(A \times B) = n(A) \times n(B)$.

PROOF

Let $A = \{a_1, a_2, a_3, \ldots, a_m\}$ and $B = \{b_1, b_2, b_3, \ldots, b_n\}$ be two sets having m and n elements respectively. Then,

$$\begin{aligned} A \times B = \{&(a_1, b_1), (a_1, b_2), (a_1, b_3), \ldots, (a_1, b_n) \\ &(a_2, b_1), (a_2, b_2), (a_2, b_3) \ldots, (a_2, b_n) \\ &\vdots \quad \vdots \quad \vdots \quad \vdots \\ &(a_m, b_1), (a_m, b_2), (a_m, b_3) \ldots, (a_m, b_n)\} \end{aligned}$$

Clearly, in the tabular representation of $A \times B$ there are m rows of ordered pairs and each row has n distinct ordered pairs.
So, $A \times B$ has mn elements.
Hence, $n(A \times B) = mn = n(A) \times n(B)$

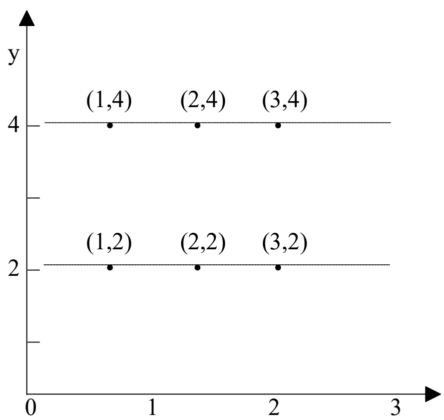
REMARK (i) If either A or B is an infinite set, then $A \times B$ is an infinite set.
(ii) If A, B, C are finitsets, then $n(A \times B \times C) = n(A) \times n(B) \times n(C)$

03. Graphical Representation of Cartesian Product of Sets

Let A and B be any two non-empty sets. To represent $A \times B$ graphically, we draw tow mutually perpendicular lines, one horizontal and other vertical. On the horizontal line, we represent the elements of set A and on the vertical line, the elements of B. If $a \in A, b \in B$, be draw a vertical line through a and a horizontal line through b. These two lines will meet in a point which will denote the ordered pair (a, b). In this manner we mark points corresponding to each ordered pair in $A \times B$. The set of points so obtained represents $A \times B$ graphically as illustrated in the following example.

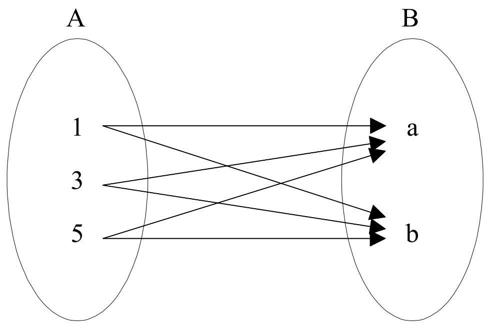
Example If $A = \{1, 2, 3\}$ and $B = \{2, 4\}$, find $A \times B$ and show it graphically.

Solution: Clearly, $A \times B = \{(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)\}$.
To represent $A \times B$ graphically, we draw two perpendicular lines OX AND OY as shown in Figure. Now we represent the set A by three points on OX and the set B by two points on OY. The set $A \times B$ is represented by the six points as shown in Figure.



04. Diagramatic Representation of Cartesian Product of Two Sets

In order to represent $A \times B$ by an arrow diagram, we first draw Venn diagrams representing sets A and B one opposite to the other as shown in Figure. Now, we draw line segments starting from each element of A and terminating to each element of set B.
If $A = \{1, 2, 3\}$ and $B = \{a, b\}$, then following figure gives the arrow diagram of $A \times B$.



05. Some Useful Results

RESULT 1
For any three sets A, B, C, prove that:
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

RESULT 2
For any three sets A, B, C, prove that:
$$A \times (B - C) = (A \times B) - (A \times C)$$

RESULT 3
If A and B are any two non-empty sets, then prove that:
$$A \times B = B \times A \Leftrightarrow A = B$$

RESULT 4
If $A \subseteq B$, show that $A \times A \subseteq (A \times B) \cap (B \times A)$.

RESULT 5
If $A \subseteq B$, prove that $A \times C \subseteq B \times C$ for any set C.

RESULT 6
If $A \subseteq B$, and $C \subseteq D$, prove that $A \times C \subseteq B \times D$.

RESULT 7
For any sets A, B, C, D prove that:
$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

COROLLARY
For any sets A and B, prove that $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$.