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Complete MATH

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CLASS 11&12th

CLASS 11th

Solutions of Triangle

01. Properties and Solution of Triangle

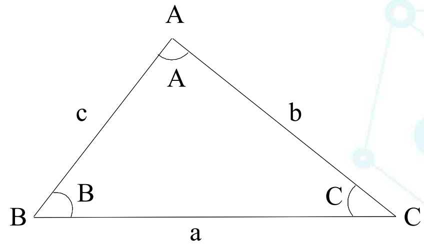
For a $\triangle ABC$, sides opposite to angles A, B and C i.e., BC, CA and AB are represented by a, b and c respectively. We denote half of the perimeter of the triangle by s, i.e., $2s = a+b+c$.

GEOMETRICAL PROPERTIES OF *A, B, C* and *a, b, c*.

1. $A+B+C=180^\circ$

2. $a+b>c, b+c>a, c+a>b$

3. $a>0, b>0, c>0$



For solving a Δ, we need some basic tools such as-

(1) SINE FORMULA

In any triangle ABC, the ratios of the sides to sine of the opposite angles are equal. i.e.,

$\frac{a}{\operatorname{Sin} A}=\frac{b}{\operatorname{Sin} B}=\frac{c}{\operatorname{Sin} C}=2R$, where R is circumradius of $\triangle ABC$.

(2) COSINE FORMULA

Can we express and angle of any triangle in terms of the sides of the triangle?
The formula which does that is known as cosine rule.

$$\operatorname{Cos} A=\frac{b^2+c^2-a^2}{2bc}, \operatorname{Cos} B=\frac{a^2+c^2-b^2}{2ac}, \operatorname{Cos} C=\frac{a^2+b^2-c^2}{2ab}$$

(3) PROJECTION FORMULA

$a=c\operatorname{Cos} B+b\operatorname{Cos} C$. Similarly, $b=a\operatorname{Cos} C+c\operatorname{Cos} A$ and $c=a\operatorname{Cos} B+b\operatorname{Cos} A$.

(4) NAPIER'S ANALOGY (TANGENT RULE)

Napier's Analogy states that

in any triangle ABC, $\operatorname{Tan}\frac{(A-B)}{2}=\frac{a-b}{a+b}\operatorname{Cot}\frac{C}{2}$.

Similarly,

$\operatorname{Tan}\frac{B-C}{2}=\frac{b-c}{b+c}\operatorname{Cot}\frac{A}{2}$; $\operatorname{Tan}\frac{C-A}{2}=\frac{c-a}{c+a}\operatorname{Cot}\frac{B}{2}$

(5) TO FIND THE SINE, COSINE AND TANGENT OF THE HALF-ANGLES IN TERMS OF THE SIDES

$\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$, $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$, $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ where b and c are sides opposite to angle A.