Complete MATH

IIT-JEE · CBSE eBOOKS

CLASS 11&12th

CLASS 11th

Binomial Expansion

01. Binomial Theorem

Theorem *If x and a are real numbers , then for all $n \epsilon N$,*

$$(x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + ...$$

$$+ ... + {}^nC_r x^{n-r} a^r + ... + {}^nC_{n-1} x^1 a^{n-1} + {}^nC_n x^0 a^n$$

$$i.e. \quad (x+a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r$$

Remark *The above expansion is also valid when x and a are complex numbers.*

Properties of Binomial Expansion

Property I *We have,*

$$(x+a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r$$

Since r can have values from 0 to n, the total number of terms in the expansion is (n + 1).

Property II *The sum of indices of x and a in each term is n.*

Property III *We have,*

$${}^nC_r = {}^nC_{n-r'} \qquad r = 0, 1, 2, ..., n$$

$$\Rightarrow {}^nC_0 = {}^nC_n, \; {}^nC_1 = {}^nC_{n-1}, {}^nC_2 = {}^nC_{n-2} = ...$$

So, the coefficients of terms equidistant from the beginning and the end are equal. These coefficients are known as the binomial coeffi-cients.

Property IV *Replacing a by−a, in expansion of $(x+a)^n$, we get*

$$(x-a)^n = {}^nC_0 x^n a^0 - {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 - {}^nC_3 x^{n-3} a^3$$

$$..... + ... + (-1)^r \, {}^nC_r x^{n-r} a^r + ... + (-1)^n \, {}^nC_n x^0 a^n.$$

i.e., $$(x-a)^n = \sum_{r=0}^{n} (-1)^r \, {}^nC_r x^{n-r} a^r$$

Property V Putting $x = 1$ and $a = x$ in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ... + {}^nC_r x^r + ... + {}^nC_n x^n$$

i.e., $$(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$$

This is the expansion of $(1+x)^n$in ascending powers of x.

Property VI Putting $a = 1$ in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0 x^n + {}^nC_1 \, x^{n-1} + {}^nC_2 x^{n-2} + ...$$

$$+ {}^nC_r x^{n-r} + ... + {}^nC_{n-1} x + {}^nC_n$$

i.e., $$(1+x)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r}$$

This is the expansion of $1+x)^n$ in descending powers of x.

General Term In a Binomial Expansion

We have,

$$(x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + ...$$

$$+ .... + {}^nC_r x^{n-r} a^r + ... + {}^nC_n x^0 a^n$$