

Complete MATH

IIT-JEE · CBSE eBOOKS CLASS 11&12th



CLASS 11th Sequences and Series

01. Arithmetic Progression

Definition A sequence $a_1, a_2, a_3, ..., a_n, ...$ is called an arithmetic progression (A. P.), if the difference of any term and the previous term is always same.

i.e., $a_{n+1} - a_n = \text{Constant} (= d)$ for all $n \in N$

or, $a_{n+1} - a_n$ is independent of n.

The constant difference 'd' is called the common difference.

(i) Properties of Terms of an A.P.

- **Property 1** A sequence is an A.P. iff its n^{th} term is a linear expression in n i.e. $a_n = An + B$. In such a case, the coefficient of n i.e. A is the common difference of the A.P.
- **Property 2** In a finite A.P. a_1 , a_2 , a_3 , ..., a_n the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of its first and last term.
- **Property 3** Three numbers a, b, c are in A.P. iff 2b = a + c.
- **Property 4** If a_n , a_{n+1} , a_{n+2} are three consecutive terms of an A.P., then $2a_{n+1} = a_n + a_{n+2}$
- **Property 5** Twice of any term of an A.P. is equal to the sum of any two terms equidistant from it.

i.e., $2a_n = a_{n+k} + a_{n-k}$ for all k = 1, 2, 3, ...

- **Property 6** If an constant if added to or subtracted from each term of an A.P. then the resulting sequence is also an A.P. with the same common difference.
- **Property 7** If each term of an A.P. is multiplied or divided by a non-zero constant k, then the resulting sequence is also an A.P. with common difference kd or d/k, where d is the common difference of the given A.P.

(ii) Selection of terms of An A.P.

Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient :

Number of terms	Terms	Common Difference
3	a - d, a , $a + d$	d
4	a - 3d, a - d, a + d, a + 3d	2d
5	a - 2d, a - d, a, a + d, a + 2d	d
6	a - 5d, $a - 3d$, $a - d$, $a + d$, $a + 3d$, $a + 5d$	2d

It should be noted that in case of an odd number of terms, the middle term is a and the common difference is d while in case of an even number of terms the middle terms are a - d, a + d and the common difference is 2d.

(iii) Sum to *n* terms of An A.P.

The sum S_n of *n* terms of an A.P. with first term '*a*' and common difference '*d*' is given by

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

Also, $S_n = \frac{n}{2} \{a_1 + a_n\}$, where $a_1 = a$.

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