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MATH

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CLASS 11&12th

CLASS 12th

Relations & Functions

01. Types of Relations

(A) **VOID, UNIVERSAL AND IDENTITY RELATIONS**

Void Relation-

Let A be a set. Then, $\phi \subseteq A \times A$ and so it is a relation on A. This relation is called the void or empty relation on set A.

Universal Relation-

Let A be a set. Then, $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.

NOTE ☞ It is to note here that the void relation and the universal relation on a set A are respectively the smallest and the largest relations on set A.
Both the empty (or void) relation and the universal relation are sometimes. They are called trivial relations.

Identity Relation-

Let A be a set. Then, the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

In other words, a relation I_A on A is called the identity relation i.e., if every element of A is related to itself only.

(B) **REFLEXIVE, SYMMETRIC, TRANSITIVE, ANTISYMMETRIC RELATIONS**

Reflexive Relation-

A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R is reflexive $\Leftrightarrow (a, a) \in R$ for all $a \in A$.

A relation R on a set A is not reflexive if there exists an element $a \in A$ such that $(a, a) \notin R$.

Symmetric Relation-

A relation R on a set A is said to be a symmetric relation iff

$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

i.e. $aRb \Rightarrow bRa$ for all $a, b \in A$.

Transitive Relation-

Let A be any set. A relation R on A is said to be a transitive relation iff

$(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $(a, c) \in R$ for all $a, b, c \in A$.

i.e. aRb and bRc

$\Rightarrow$ aRc for all $a, b, c \in A$.

Antisymmetric Relation-

Let A be any set. A relation R on set A is said to be an antisymmetric relation iff

$(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$

NOTE ☞ It follows from this definition that if $(a, b) \in R$ and $(b, a) \not\in R$, then also R is an antisymmetric relation.

(C) **Equivalence Relation**

A relation R on a set A is said to be an equivalence relation on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

02. Some Results on Relations

RESULT 1

If R and S are two equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on A.

OR

The intersection of two equivalence relations on a set is an equivalence relation on the set.

RESULT 2

The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.

RESULT 3

If R is an equivalence relation on a set A, then R^{-1} is also an equivalence relation on A.

OR

The inverse of an equivalence relation is an equivalence relation.

03. Kinds of Functions

ONE-ONE FUNCTION (INJECTION)

A function $f : A \to B$ is said to be a one-one function or an injection if different elements of A have different images in B.

Thus, $f : A \to B$ is one-one

$\Leftrightarrow$ $a \neq b \Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow$ $f(a) = f(b) \Rightarrow a = b$ for all $a, b \in A$

Algorithm

(i) Take two arbitrary elements *x, y* (say) in the domain *of f.*

(ii) Put *f(x)* = *f(y)*

(iii) Solve *f(x)* = *f(y)*. If *f(x)* = *f(y* gives x = y only, them $f : A \to B$ is a one-one function (or an injection). Otherwise not.

NOTE ☞ (i) Let $f : A \to B$ and let $x, y \in A$. Then, $x = y \Rightarrow f(x) = f(y)$ is always true from the definition. But, $f(x) = f(y) \Rightarrow x = y$ is true only when f is one-one.
(ii) If A and B are two sets having m and n elements respectively such that $m \leq n$, then total number of one-one functions from A to B is ${}^{n}C_{m} \times m!$.

MANY-ONE FUNCTION

A function $f : A \to B$ is said to be a many-one function if two or more elements of set A have the same image in B.

Thus, $f : A \to B$ is a many-one function if there exist $x, y \in A$ such that $x \neq y$ but *f(x)* = *f(y)*.

NOTE ☞ In other words, $f : A \to B$ is many-one function if it is not a one-one function.

ONTO FUNCTION (SURJECTION)

A function $f : A \to B$ is said to be an onto function or a surjection if every element of B is the f-image of some element of A i.e., if f(A) = B of range of f is the co-domain of f.

Thus, $f : A \to B$ is a surjection iff for each $b \in B$, there exists $a \in A$ such that f(a) = b.

INTO FUNCTION. A function $f : A \to B$ is an into function if there exists an element in B having no pre-image in A.

In other words, $f : A \to B$ is an into function if it is not an onto function.

Algorithm

Let $f : A \to B$ be the given function.

(i) Choose an arbitrary element y in B.

(ii) Put f(x) = y

(iii) Solve the equation f(x) = y for x and obtain x in terms of y. Let x = g(y).

(iv) If for all values of $y \in B$, the values of x obtained from x = g(y) are in A, then f is onto. If there are some $y \in B$ for which x, given by x = g(y), is not in A. Then, f is not onto.

BIJECTION (ONE-ONE ONTO FUNCTION)

A function $f : A \to B$ is a bijection if it is one-one as well as onto. In other words, a function $f : A \to B$ is a bijection, if

(i) it is one-one i.e. $f(x) = f(y) \Rightarrow x = y$ for all x, $y \in A$.

(ii) it is onto i.e. for all $y \in B$, there exists $x \in A$ such that f(x) = y.

REMARK If A and B are two finite sets and $f : A \to B$ is a function, then

(i) $f \text{ is an injection} \Rightarrow n(A) \leq n(B)$

(ii) $f \text{ is an surjection} \Rightarrow n(B) \leq n(A)$

(iii) $f \text{ is an bijection} \Rightarrow n(A) = n(B)$.