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Complete PHYSICS

IIT-JEE · NEET · CBSE eBOOKS

CLASS 11&12th

CLASS 11th

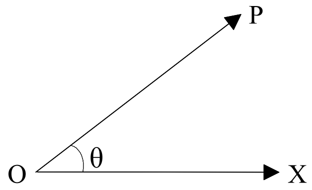
Basic Mathematics

Mathematics is the supporting tool of Physics. The elementary knowledge of basic mathematics is useful in problem solving in Physics. In the chapter we study Elementary Algebra, Trigonometry, Coordinate Geometry and Calculus (differentiation and integration).

01. Trigonometry

Angle

Consider a revolving line OP.
Suppose that it revolves in anticlockwise direction starting from it s intial position OX.
The angle is defined as the amount of revolution that the revolving line makes with its initial position.
From figure the angle covered by the revolving line OP is $\theta = \angle POX$



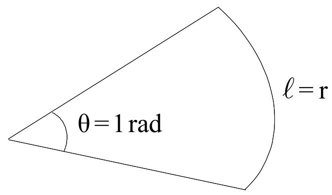
The angle
is taken **positive** if it is traced by the revolving line in anticlockwise direction and
is taken **negative** if it is covered in clockwise direction.

$$1° = 60' \text{ (minute)}$$
$$1' = 60'' \text{ (second)}$$

1 right angle = 90° (degrees) also 1 right angle = $\frac{\pi}{2}$ rad (radian)

One radian is the angle subtended at the centre of a circle by an arc of the circle whose length is equal to the radius of the circle.

$$1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$$



To convert an angle from degree to radian multiply it by $\frac{\pi}{180^\circ}$

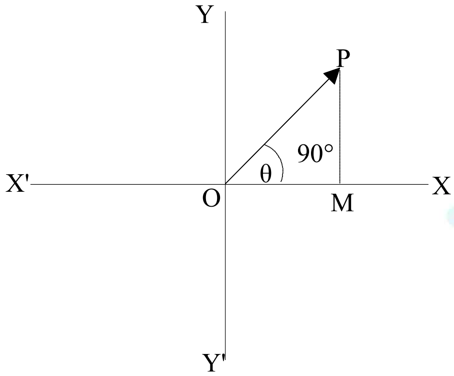
To convert an angle from radian to degree multiply it by $\frac{180^\circ}{\pi}$

Trigonometrical Ratios (Or T Ratios)

Let two fixed line XOX' and YOY' intersecting at right angles to each other at point O. Then,
(i) Point O is called origin.
(ii) XOX' known as X-axis and YOY' are Y-axis.

(iii) Point O is called origin.
(iv) XOX' known as X-axis and YOY' are Y-axis.
(v) Portions XOY, YOX', XOY' and YOX are called I, II, III and IV quadrant respectively.

Consider that the revolving line OP has traced out angle θ (in I quadrant) in anticlockwise direction. Form P, draw perpendicular PM on OX. Then, side OP (in front of right angle) is called hypotenuse, side MP (in front of angle θ) is called **opposite side or perpendicular** and side OM (making angle θ with hypotenuse) is called **adjacent side or base**.



The three sides of a right angled triangle are connected to each other through six different rations, called trigonometric ratios or simply T-ratios :

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{MP}{OP} \qquad \cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{OM}{OP}$$

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{MP}{OM} \qquad \cot\theta = \frac{\text{base}}{\text{perpendicular}} = \frac{OM}{MP}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{OP}{OM} \qquad \operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OP}{MP}$$

It can be easily proved that :

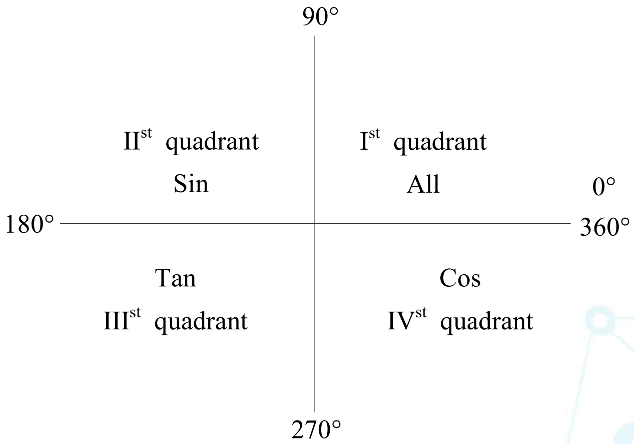
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

The T-ratios of a few standard angles ranging from 0° to 180°

Angle (θ)	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan\theta$	0.	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Four Quadrants and ASTC Rule*



In first quadrant, all trigonometric ratios are positive.
In second quadrant, only sinθ and cosecθ are positive.
In third quadrant, only tanθ and cotθ are positive.
In fourth quadrant, only cosθ and secθ are positive.

* Remember as Add Sugar To Coffee or After School To College.

Trigonometrical Ratios of General Angles (Reduction formula)

(i) Trigonometric function of an angle $2n\pi + \theta$ where n=0, 1, 2, 3,..... will be remain same.

$$\sin(2n\pi+\theta) = \sin\theta \qquad \cos(2n\pi+\theta) = \cos\theta \qquad \tan(2n\pi+\theta) = \tan\theta$$

(ii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will remain same if n is even and sign of trigonometric function will be according to value of that function in quadrant.

$$\sin(\pi-\theta) = +\sin\theta \qquad \cos(\pi-\theta) = -\cos\theta \qquad \tan(\pi-\theta) = -\tan\theta$$
$$\sin(\pi+\theta) = -\sin\theta \qquad \cos(\pi+\theta) = -\cos\theta \qquad \tan(\pi+\theta) = +\tan\theta$$
$$\sin(2\pi-\theta) = -\sin\theta \qquad \cos(2\pi-\theta) = +\cos\theta \qquad \tan(2\pi-\theta) = -\tan\theta$$

(iii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will be changed into co-function if n is odd and sign of trigonometric function will be according to value of that function in quadrant.

$$\sin\left(\frac{\pi}{2}+\theta\right) = +\cos\theta \qquad \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta \qquad \tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$$

$$\sin\left(\frac{\pi}{2}-\theta\right) = +\cos\theta \qquad \cos\left(\frac{\pi}{2}-\theta\right) = +\sin\theta \qquad \tan\left(\frac{\pi}{2}-\theta\right) = +\cot\theta$$

(iv) Trigonometric function of an angle $-\theta$ (negative angles)

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = +\cos\theta \qquad \tan(-\theta) = -\tan\theta$$