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CLASS 11&12th

CLASS 11th

Quadratic Equations

01. Quadratic Equation

In general form of a quadratic equation is $ax^2+bx+c=0$, $a\neq 0$
where a, b, c are numbers (real or complex) and x is a variable.

RESULT
A quadratic equation cannot have more than two roots.

REMARK It follows from the above theorem that if a quadratic equation is satisfied by more than two values of x, then it is satisfied by every value of x and so it is an identity.

(A) QUADRATIC EQUATIONS WITH REAL COEFFICIENTS
Consider the quadratic equation

$$ax^2+bx+c=0 \qquad \text{...(i)}$$

where $a, b, c \in R$ and $a \neq 0$.
Then, the quadratic equation $ax^2+bx+c=0$, where $a, b, c \in R$ and $a \neq 0$ has two roots, say α and β, given by

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

The roots depend upon the value of the quantity b^2-4ac. This quantity is generally denoted by D and is known as the discriminant of the quadratic equation (i).

RESULT 1
If $b^2-4ac=0$ i.e. $D=0$, then

$$\alpha=\beta=-\frac{b}{2a}$$

Thus, if $b^2-4ac=0$, then the quadratic equation has real and equal roots each equal to $-b/2a$.

RESULT 2
If a, b, c are rational numbers and b^2-4ac is positive and a perfect square, then $\sqrt{b^2-4ac}$ is a rational number and hence α and β are rational and unequal.
Thus, if $a, b, c \in Q$ and b^2-4ac is positive and a perfect square, then roots are rational and unequal. If $a, b, c \in R$ and b^2-4ac is positive and a perfect square, then roots are real and distinct.

RESULT 3
If $b^2-4ac>0$ i.e. $D>0$ but it is not a perfect square, then roots are irrational and unequal.

REMARK If $a, b, c \in Q$ and $b^2 - 4ac$ is positive but not a perfect square, then roots are irrational and they always occur in conjugate pair like $2+\sqrt{3}$ and $2-\sqrt{3}$. However, if a, b, c are irrational numbers and $b^2 - 4ac$ is positive but not a perfect square, then the roots may not occur in conjugate pairs. For example, the roots of the equation $x^2 - (5+\sqrt{2})x + 5\sqrt{2} = 0$ are 5 and $\sqrt{2}$ which do not form a conjugate pair.

RESULT 4

If $b^2 - 4ac < 0$ i.e. $D < 0$, then $4ac - b^2 > 0$ and so the roots are imaginary and are given by

$$\alpha = \frac{-b + i\sqrt{4ac - b^2}}{2a} \text{ and } \beta = \frac{-b - i\sqrt{4ac - b^2}}{2a}$$

Clearly, α and β are complex conjugate of each other i.e. $\alpha = \overline{\beta}$ and $\overline{\alpha} = \beta$.

REMARK If $b^2 - 4ac < 0$, then the roots are complex conjugate of each other. In fact, complex roots of an equation with real coefficients always occur in conjugate pairs like $2+3i$ and $2-3i$. However, this may not be true in case of equations with complex coefficients. For example, $x^2 - 2ix - 1 = 0$ has both roots equal to i.

(B) QUADRATIC EQUATIONS WITH COMPLEX COEFFICIENTS

Consider the quadratic equation

$$ax^2 + bx + c = 0$$

where a, b, c are complex numbers and $a \neq 0$.

The roots of equation (i) are given by

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and, } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

These roots are complex as a, b, c are complex numbers.

Since the order relation is not defined in case of complex numbers, therefore we cannot assign positive or negative sign to the discriminant $D = b^2 - 4ac$. However, equation (i) has complex roots which are equal, if $D = b^2 - 4ac = 0$ and unequal roots if $D = b^2 - 4ac \neq 0$.

REMARK In case of quadratic equations with real coefficients imaginary (complex) roots always occur in conjugate paris. However, it is not true for quadratic equations with complex coefficients.

For example, the equation $4x^2 - 4ix - 1 = 0$ has both roots equal to $\frac{1}{2}i$.