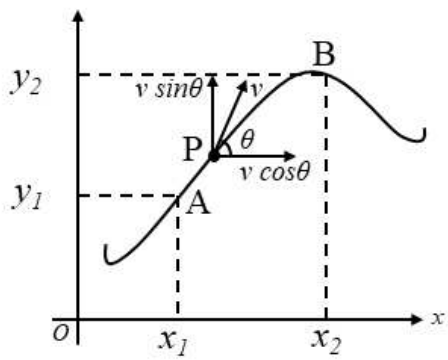
PHYSICS

CLASS NOTES FOR CBSE

Chapter 05. Motion in a Plane

01. Motion in Two Dimension

Now we change our kinematics analysis from one dimension to two dimensions. In previous sections, we've discussed about the motion of an object along a straight line. Now we discuss, what happens when a particle moves in a plane. Have a look at figure, which shows a particle moving in X-Y plane, along a two dimensional path, known as trajectory of the particle. We discuss the motion of the particle between two points of the curve *A* and *B*. If the particle is moving along the curve and its velocity at an instant is *v* at an intermediate position of particle at point *P*. In two dimensional motion, direction of velocity of a particle is always tangential to its trajectory curve. As the particle moves from point $A(x_1,y_1)$ to point $B(x_2, y_2)$. Its projection on *x*-axis changes from x_1 to x_2, and its projection of *y*-axis changes from y_1 to y_2. The velocities of the projections of the particle along *x* and *y* direction can be found by resolving the velocity of the particle in *x* and *y* direction.



If along the curve particle moves a distance *dr* in time *dt*, we define *v=dr/dt*. Similarly, when particle moves dr along the curve, its *x*-coordinate changes by *dx* and *y*-coordinate changes by *dy*. Thus the velocity projections can be written as

$$v_x = \frac{dx}{dt} = v cos\theta \qquad ...(i)$$

and

$$v_y = \frac{dy}{dt} = v sin\theta \qquad ...(ii)$$

In standard unit vector notification we can write the instantaneous velocity of particle as

$$v = v_x\,\hat{i} + v_y\,\hat{j}$$

Squaring and adding equations (i) and (ii), gives net velocity of the particle as

$$v = \sqrt{v_x^2 + v_y^2} \qquad ...(iii)$$

Dividing above equations will give the angle formed by the trajectory with the positive *x*-direction or the slope angle of the trajectory as

$$\tan\theta = \frac{v_y}{v_x}$$

or $$\theta = \tan^{-1}\frac{v_y}{v_x} \quad ...(iv)$$

02. Acceleration in Two Dimensional Motion

The total or net acceleration of a particle moving in two dimensions, can be resolved in two mutually perpendicular directions x and y and these two projections termed as a_x and a_y, and mathematically these can be given as

$$a_x = \frac{dv_x}{dt} = v_x\frac{dv_x}{dx} = \frac{d^2x}{dt^2} \quad ...(i)$$

and $$a_y = \frac{dv_y}{dt} = v_y\frac{dv_y}{dy} = \frac{d^2y}{dt^2} \quad ...(ii)$$

From equations (i) and (ii), total acceleration is given as

$$a_{net} = a_x\hat{i} + a_y\hat{j}$$

The magnitude of which is given as $a_{net} = \sqrt{a_x^2 + a_y^2}$ ...(iii)

If we find the direction of net acceleration vector is given as $\phi = \tan^{-1}\frac{a_y}{a_x}$

03. Trajectory of a Particle in Two dimension

Path traced by a moving particle in space is called trajectory of the particle, as in one dimensional motion the trajectory of a particle is straight line. In two dimensional motion the trajectory of a moving particle will be a two dimensional curve e.g. circle, ellipse, parabola, hyperbola, spiral, cycloid and so many more paths, including random paths. Shape of trajectory is decided by the forces acting on the particle, for a specific shape a particular type of force or a group of forces are required. This we'll discuss after the chapter of forces.
When a coordinate system is associated with a particle's motion, the curve equation in which the particle moves $[y = f(x)]$ is called equation of trajectory. It is just giving us the relation among x and y coordinates of the particle (locus of particle).
To find equation of trajectory of a particle there are several methods but the simple way is to find first x and y coordinates of the particle as a function of time and eliminate the time factor.

Example A particle is moving in XY plane such that its velocity in x-direction remains constant at 5 m/s and its velocity in y-direction varies with time as v=3 t m/s, where t is time in seconds. Find.
(a) Speed of particle after time t = 10 s.
(b) Direction of motion of particle at that time.
(c) Acceleration of particle at t = 5 s and its direction.

(d) Displacement of particle at this instant.
(e) Equation of trajectory of particle if it starts at time t = 0 from rest at origin.

Solution

(a) At time t = 10 s particle velocity in x and y directions are v_x = 5 m/s and v_y = 3(10) = 30 m/s

Magnitude of instantaneous velocity is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(5)^2 + (30)^2} = 30.41\ m/s$$

(b) As we know that the direction of motion of particle is along the instantaneous velocity of it thus we have the inclination of particle's velocity from horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{30}{5}\right) = 80.53^o$$

(c) Acceleration of particle in x and y directions are

$$a_x = \frac{d}{dt}(v_x) = 0 \qquad [\text{As } v_x = 3\ t\ m/s\ constant]$$

and

$$a_y = \frac{d}{dt}(v_{y)} = 3 m/s^2 \quad [\text{As } v_y = 3\ m/s]$$

Thus net acceleration of particle is constant and is given as $a = \sqrt{a_x^2 + a_y^2} = 3\ m/s^2$

(d) To find displacement of particle in x and y direction, we used respective velocities as, we have. In x-direction particle velocity is constant $v_x = 5\ m/s$, thus

Displacement of particle along x-direction is

$$x = v_x t = 5\ t = 5(5) = 25\ m$$

In y-direction particle velocity is given as

$$v_y = 3t\ m/s \text{ thus we have}$$

$$\frac{dy}{dt} = 3\ t$$

or

$$dy = 3\ t\ dt$$

Integrating within proper limits, we have

$$\int_0^y dy = \int_0^t 3t\,dt$$

or

$$y = \left[\frac{3}{2}t^2\right]_0^t = \frac{3}{2}t^2 = \frac{3}{2}(5)^2 = 37.5\ m$$

Note that here acceleration in y-direction is constant and is $a_y = 3\ m/s^2$, so this can be obtained directly using speed equations as

$$y = \frac{1}{2}a_y t^2 = \frac{1}{2}(3)(5)^2 = 37.5\ m$$

$$r = 25\ \hat{i} + 37.5\ \hat{j} = 45.07/\underline{56.31^o} m$$

(e) For finding equation of trajectory we should require x and y coordinates of particle as a function of time, and here we have

$$x = 5t \qquad ...(i)$$

$$y = \frac{3}{2}t^2 \qquad ...(ii)$$

Eliminating t from equations (i) and (ii), we have

$$y = \frac{3}{50}x^2 \qquad ...(iii)$$

Equation (iii) gives the equation of trajectory of the particle.