PHYSICS

CLASS NOTES FOR CBSE

Chapter 08. System of Particles and Rotational Motion

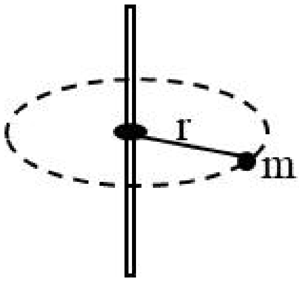
01. Rotational Kinematics

Rotational kinematics is the study of relations and analysis of angular displacement, angular velocity and angular acceleration in different situations. We can list up once again the rotational properties along with comparison with translational motion.

Linear	Rotational
$v=\dfrac{dx}{dt}$	$\omega=\dfrac{d\theta}{dt}$
$a=\dfrac{dv}{dt}=v\dfrac{dv}{dx}$	$\alpha=\dfrac{d\omega}{dt}=\omega\dfrac{d\omega}{d\theta}$
$v=u+at$	$\omega=\omega_0+\alpha t$
$s=ut+\dfrac{1}{2}at^2$	$\theta=\omega_0 t+\dfrac{1}{2}dt^2$
$v^2=u^2+2as$	$\omega^2=\omega_0^2+2\alpha\theta$
$s_n=u+\dfrac{1}{2}a(2n-1)$	$\theta_n=\omega_0+\dfrac{1}{2}\alpha(2n-1)$

02. Moment of Inertia

As we have studied Translational kinematics and dynamics in previous chapters, in rotational motion, we will also study the concepts of rotational dynamics. Before that one important thing required to be discussed in detail is the Moment of Inertia. As “Inertia” plays an important role in definition of Newton’s first law which is also called as inertia law, moment of inertia is the key concept in defining the state of rotation. It is the property of a body rotating or which can rotate about an axis and which resists the change in state of body’s rotational motion. If body is rotating with a constant angular velocity, it continues with the same angular velocity unless some external torque will act on it. Similarly if a body is at rest about an axis of rotation it is impossible to rotate it in an inertial frame without application of an external torque. Moment of inertia gives a measurement of the resistance of the body to a change in its rotational motion. Higher the moment of inertia of a body, it requires a high torque to produce a required change in its motion. If body is at rest, the larger the moment of inertia of a body, the more difficult it is to put that body into rotational motion. Similarly, the larger the moment of inertia of a body, the more difficult it is to stop its rotational motion.



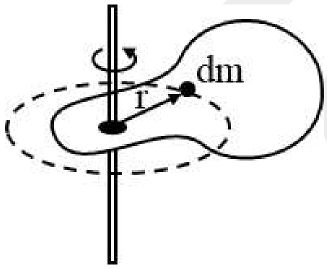
In translation motion, the mass of a body *m* given measure of the inertia of a body. But in rotational motion moment of inertia depends on mass of body as well as on its distribution about the axis of rotation.

For a very simple case of circular motion of a point mass, shown above in figure, the moment of inertia is given as

$$I = mr^2$$

03. Moment of Inertia of a Rigid Body in Rotational Motion

Have a look at figure below. A body of mass M is free to rotate about in axis of rotation passing through the body. We have already discussed that when a body is in rotational motion, its different particles are in circular motion of different radii. Consider an elemental mass dm in the body at a distance x from the axis of rotation. During rotation of body this dm will revolve about the same axis in a circle of radius x. The moment of inertia of the elemental mass dm is dI, it is given as

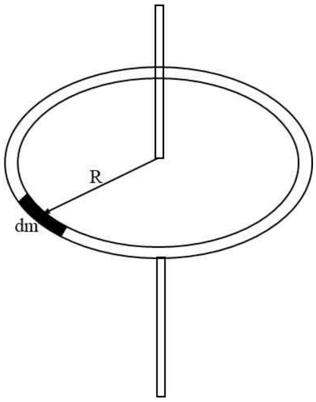


$$dI = dmx^2$$

Now the moment of inertia of the whole body can be evaluated by integrating the above expression for the whole body. Thus moment of inertia of the body is given by

$$I = \int dI = \int dmx^2 \qquad \text{(i)}$$

04. Moment of Inertia of a Ring



Above figure shows a ring of mass M and radius R. To find its moment of inertia, we consider an elemental mass dm on it (see figure). When the ring rotates, the element dm will revolve in a circle of radius R, infect here radius of all the elements taken on ring will be same R.

The moment of inertia of this elemental mass dm is given as

$$dI = dmR^2$$

Moment of inertia of the complete ring is

$$I = \int dI = \int dmR^2$$

$$= R^2 \int dm$$

$$I = MR^2$$

05. Moment of Inertia About a General Axis of Rotation

In previous section, the moment of inertia of different objects, we have evaluated are about the axis of symmetry of the objects, generally passing through the centre of mass. For evaluation of moment of inertia about any randomly selected axis of rotation of body we use axes theorems. There are two axes theorems which are used frequently in problems.

(i) Perpendicular Axes Theorem

This theorem is only valid for laminar objects that is only for two dimensional objects which are rotating about an axis passing through their centre of mass. Consider such a plate like object shown in figure (a). We rotate this body about an axis along x-axis, lying in the plane on body and passing through its centre of mass. Let the moment of inertia about this axis be I_1 and now if it is rotated about y-axis, which is also in its plane and passing through its centre of mass. Let moment of inertia about this axis be I_2. If the body is rotated about a third axis, which is perpendicular to both of the previous axes and also perpendicular to the plane of the body, its moment of inertia I_3 is given by the sum of I_1 and I_2.

$$I_3 = I_1 + I_2 \qquad \text{...(i)}$$