MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 01. Sets

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01. Set

Set as "a well defined collection of objects".

Example The collection of vowels in English alphabets. This set contains five elements, namely, a, e, i, o, u.

NOTE ☞ Collection of good teachers in a school is not a set.

02. Reserved Symbols

We reserve some symbols for these set:

① **N** : for the set of natural numbers.
② **Z** : for the set of integers.
③ $\mathbf{Z}^+$: for the set of all positive integers.
④ **Q** : for the set of all rational numbers.
⑤ $\mathbf{Q}^+$: for the set of all positive rational numbers.
⑥ **R** : for the set of all real numbers.
⑦ $\mathbf{R}^+$: for the set of all positive real numbers.
⑧ **C** : for the set of all complex numbers.

03. Description of a Set

A set is often described in the following two forms. One can make use of any one of these two ways according to his (her) convenience.

(i) Roster form or Tabular form
(ii) Set-builder form

ROSTER FORM

In this form a set is described by listing elements, separated by commas, within braces {}.

Example I *The set of even natural numbers can be described as {2, 4, 6, ...}. Here the dots stands for 'and so an'.*

Example II *If A is the set of all prime numbers less than* 11*, then* $A = \{2, 3, 5, 7\}$.

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NOTE ☞ (1) The order in which the elements are written in a set makes no difference.
(2) Also, the repetition of an element has no effect.

SET-BUILDER FORM

In this form, a set is described by a characterizing property $P(x)$ of its elements x. In such a case the set is described by $\{x : P(x) \text{holds}\}$ or, $\{x | P(x) \text{holds}\}$, which is read as 'the set of all x such that $P(x)$ holds'. The symbol $'|'$ or $':'$ is read as 'such that'.

Example I *The set E of all even natural numbers can be written as*

$E = \{x : x \text{ is a natural number and } x = 2n \text{ for } n \in N\}$

or, $E = \{x : x \in N, x = 2n, n \in N\}$ or, $E = \{x \in N : x = 2n, n \in N\}$

04. Types of Sets

(1) EMPTY SET A set is said to be empty or null or void set if it has no element and it is denoted by ϕ(phi).

In Roster method, ϕ is denoted by $\{\}$.

Example I $\{x \in R : x^2 = -2\} = \phi$.

Example II $\{x \in N : 5 < x < 6\} = \phi$.

Example III *The set A given by $A = \{x : x \text{ is an even number greater than } 2\}$ is an empty set because 2 is the only even prime number.*

A set consisting of at least one element is called a non-empty or non-void set.

(2) SINGLETON SET A set consisting of a single element is called a singleton set.

Example I *The set* $\{5\}$ is a singleton set.

Example II *The set $\{x : x \in N \text{ and } x^2 = 9\}$ is a singleton set equal to* $\{3\}$.

(3) FINITE SET A set is called a finite set if it is either void set or its elements can be listed (counted, labelled) by natural numbers $1, 2, 3, ...$ and the process of listing terminates at a certain natural number n (say).

(4) CARDINAL NUMBER OF A FINITE SET The number n in the above definition is called the cardinal number or order of a finite set A and is denoted by $n(A)$.

(5) INFINITE SET A set whose elements cannot be listed by the natural numbers $1, 2, 3, ..., n,$ for any natural number n is called an infinite set.

Example I *Each one of the following sets is a finite set*:

(i) *Set of even natural numbers less than* 100.
(ii) *Set of soldiers in Indian army.*
(iii) *Set of even prime natural numbers.*
(iv) *Set of all persons on the earth.*

Example II *Each one of the following sets is an infinite set*:

(i) *Set of all points in a plane.*
(ii) *Set of all lines in a plane.*

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(6) EQUIVALENT SETS Two finite sets A and B are equivalent if their cardinal numbers are some i.e. $n(A) = n(B)$.

(7) EQUAL SETS Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

Example I *If* $A = \{1, 2, 5, 6\}$ *and* $B = \{5, 6, 2, 1\}$. *Then* $A = B$, because each element of A is an element of B and vice-versa. Note that the elements of a set may be listed in any order, It follows from the above definition and the definition of equivalent sets that equal sets are equivalent but equivalent sets need not be equal. For example, $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$ are equivalent sets but not equal sets.

05. Subsets

Let A and B be two sets. If every element of A is an element of B, then A is called a subset of B.
If A is a subset of B, we write $A \subseteq B$, which is read as "A is a subset of B" or "A is contained in B".
Thus, $A \subseteq B$ if $a \in A \Rightarrow a \in B$.
If A is not a subset of B, we write $A \not\subseteq B$. every set is a subset of itself and the empty set is subset of every set. These two subsets are called ***improper subsets***. A subset A of B is called ***a proper subset*** of B if $A \neq B$ and we write $A \subset B$. In such a case, we also say that B is a **super set** of A.
Thus, if A is a proper subset of B, then there exists an element $x \in B$ such that $x \notin A$.
It follows immediately from this definition and the definition of equal sets that two sets A and B are equal sets that two sets A and B are equal iff $A \subseteq B$ and $B \subseteq A$.
Example I Clearly $\{1\} \subseteq \{1, 2, 3\}$, but $\{1, 4\} \not\subseteq \{1, 2, 3\}$.

SOME RESULTS ON SUBSETS

RESULT 1 Every set is a subset of itself.
RESULT 2 The empty set is a subset of every set.
RESULT 3 The total number of subsets of a finite set containing n elements is 2^n.

REMARK SUBSETS OF THE SET **R** OF REAL NUMBERS

i) The set of all natural numbers $N = \{1, 2, 3, 4, 5, 6, ...\}$

ii) The set of all integers $Z = \{...-3, -2, -1, 0, 1, 2, 3, ...\}$

iii) The set of all rational numbers $Q = \{x : x = \dfrac{m}{n}, m, n \in Z, n \neq 0\}$.

iv) The set of all irrational numbers. It is denoted by T.

Thus,
$T = \{x : x \in R \text{ and } x \notin Q\}$
Clearly, $N \subset Z \subset Q \subset R$, $T \subset R$ and $N \not\subset T$.

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