

MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 04. Mathematical Induction

01. Statements

A sentence or description which can be judged to be true or false is called a statement.

Mathematical Statements : Statements involving mathematical relations are known as the mathematical statements.

We shall be using notations $P(n)$ or $P_1(n)$ or $P_2(n)$ etc. to denote such statements.

02. The Principle of Mathematical Induction

Let $P(n)$ be a statement involving the natural number n such that

(I) $P(1)$ is true i.e. $P(n)$ is true for $n = 1$

and (II) $P(m + 1)$ is true, whenever $P(m)$ is true i.e. $P(m)$ is true $\Rightarrow P(m + 1)$ is true.

Then, $P(n)$ is true for all natural numbers n .

In order to prove that a statement is true for all natural numbers using principle of mathematical induction, we may use the following algorithm:

Algorithm

Step I Obtain $P(n)$ and understand its meaning.

Step II Prove that the statement $P(1)$ is true i.e. $P(n)$ is true for $n = 1$.

Step III Assume that the statement $P(n)$ is true for $n = m$ (say) i.e. $P(m)$ is true.

Step IV Using assumption in step III prove that $P(m + 1)$ is true.

Step V Combining the results of step II and step IV, conclude by the first principle of mathematical induction that $P(n)$ is true for all $n \in N$.

Application of The Principle of Mathematical Induction

Example - Prove by the principle of mathematical induction that for all $n \in N$:

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n (n + 1) (2n + 1)$$

Solution Let $P(n)$ be the statement given by

$$P(n) : 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n (n + 1) (2n + 1)$$

Step I We have,

$$P(1) : 1^2 = \frac{1}{6} (1) (1 + 1) (2 \times 1 + 1)$$



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