MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 06. Linear Inequations

01. Inequations

An equation is defined as a statement involving variable (s) and the sign of equality (=). Similarly, we define the term inequation as follows:

Inequation *A statement involving variable (s) and the sign of inequality viz, >, <, ≥ or ≤ is called an inequation or an inequality.*

An inequation may contain one or more variables. Also, it may be linear or quadratic or cubic etc.

Following are some examples of inequations:

(i) $3x - 2 < 0$
(ii) $2x^2 + 3x + 4 > 0$
(iii) $2x + 5y \geq 4$
(iv) $x^2 - 5x + 4 \leq 0$
(v) $x^3 + 6x^2 + 11x + 6 \leq 0$

Linear Inequation in One Variable *Let a be non-zero real number and x be a variable. Then inequations of the form $ax + b < 0$, $ax + b \leq 0$, $ax + b > 0$ and $ax + b \geq 0$ are known as linear inequations in one variable x.*

Linear Inequation in Two Variable *Let a and b be non-zero real numbers and x, y be variable. Then inequations of the form $ax + by < c$, $ax + by \leq c$, $ax + by > c$ and $ax + by \geq c$ are known as linear inequations in two variables x and y.*

(i) $3x - 2 < 0$
(ii) $5x - 3 > 0$
(iii) $2x + 3y < 1$
(iv) $4x - 6y > 5$
(v) $2x^2 + 3x + 4 > 0$
(vi) $x^2 + 3x + 2 < 0$
(vii) $5x + 4y \leq 3$
(viii) $x^2 - 5x + 4 \leq 0$
(ix) $x^3 + 6x^2 + 11x + 6 \leq 0$.

Solutions of An Inequation

Definition *A solution of an inequation is the value (s) of the variable (s) that makes it a true statement.*

Example Consider the inequation

$$\frac{3-2x}{5} < \frac{x}{3} - 4$$

Left hand side (LHS) of this inequation is $\frac{3-2x}{5}$ and right hand side (RHS) is $\frac{x}{3} - 4$.

We observe that :

For $x = 9$, we have, LHS $= \frac{3-2\times 9}{5} = -3$ and, RHS $= \frac{9}{3} - 4 = -1$

Clearly, $-3 < -1$

$\Rightarrow$ LHS < RHS, which is true.

So, $x = 9$ is a solution of the given inequation

For $x = 6$, we have

$$\text{LHS} = \frac{3-2\times 6}{5} = -\frac{9}{5} \text{ and RHS} = \frac{6}{3} - 4 = -2$$

Because, $-\frac{9}{5} < -2$ is not true.

So, $x = 6$ is not a solution of the given inequatins.

We can verify that any real number greater than 7 is a solution of the given inequation.

Example Consider the inequation

$$x^2 + 1 < 0$$

We know that

$$x^2 \geq 0 \text{ for all } x \in R$$

$$\therefore \quad x^2 + 1 \geq 1 \text{ for all } x \in R$$

So, there is no real value of x which makes the given inequation a true statement. Hence, it has no solution.

Solving An Inequation *It is the process of obtaining all possible solutions of an inequation.*

Solution Set *The set of all possible solutions of an inequation is known as its solution set.*

The solution set of the inequation $x^2 + 1 \geq 0$ is the set R of all real numbers whereas the solution set of the inequation $x^2 + 1 < 0$ is the null set ϕ.

Solution Linear Inequations in One Variable

Solving an inequation is the process of obtaining its all possible solutions.

Rule I *Same number may be added to (or subtracted from) both sides of an inequation without change in the sign of inequality.*

Rule II *Both sides of an inequation can be multiplied (or divided) by the same positive real number without changing the sign of inequality. However, the sign of inequality is reversed when both sides of an inequation are multiplied or divided by a negative number.*

Rule III *Any term of an inequation may be taken to the other side with its sign changed without affecting the sign of inequality.*

A linear inequation in one variable is of the form

$$ax + b < \text{ or, } ax + b \leq 0 \text{ or, } ax + b > 0 \text{ or, } ax + b \geq 0.$$

We follow the following algorithm to solve a linear inequation in one variable.

Algorithm

Step I *Obtain the linear inequation.*

Step II *Collect all terms involving the variable on one side of the inequation and the constant terms on the other side.*

Step III *Simplify both sides of inequality in their simplest forms to reduce the inequation in the form*

$$ax < b, \text{ or } ax \leq b, \text{ or } ax > b, \text{ or } ax \geq b$$

Step IV *Solve the inequation obtained in step III by dividing both sides of the inequation by the coefficient of the variable.*

Step V *Write the solution set obtained in step IV in the form of an interval on the real line.*

Example Solve $5x - 3 < 3x + 1$

$\Rightarrow \quad 5x - 3x < 3 + 1$ [Transposing $3x$ on LHS and -3 on RHS]

$\Rightarrow \quad 2x < 4$

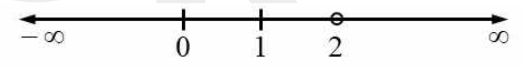
$\Rightarrow \quad \dfrac{2x}{2} < \dfrac{4}{2}$ [Dividing both sides by 2]

$\Rightarrow \quad x < 2$

(i) If $x \in R$, then

$x < 2 \Rightarrow x \in (-\infty, 2)$

Hence, the solution set is $(-\infty, 2)$ as shown in Figure.



Figure

(ii) If $x \in Z$, then

$x < 2 \Rightarrow x = 1, 0, -1, -2, -3, -4, \ldots..$

So, the solution set is $\{\ldots.. -4, -3, -2, -1, 0, 1\}$

(iii) If $x \in N$, then

$x < 2 \Rightarrow x = 1$

So, the solution set is $\{1\}$.

Solution of System of Linear Inequations in One Variable

The solution set of a system of linear inequations in one variable is the intersection of the solution sets of the linear inequations in the given system.

We use the following algorithm to solve a system of linear inequations in one variable.

Algorithm

Step I *Obtain the system of linear inequations.*

Step II *Solve each inequation and obtain their solution sets. Also, represent them on real time.*

Step III *Find the intersection of the solution sets obtained in step II by taking the help of the graphical representation of the solution sets in step II.*

Step IV *The set obtained in step III is the required solution set of the given system of inequations.*