MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 07. Permutations & Combinations

01. Factorial

Factorial *The continued product of first n natural numbers is called the "n factorial" and is denoted by* $\lfloor n$ *or* $n!$ *i.e.*
$n! = 1 \times 2 \times 3 \times 4 \times \ldots \times (n-1) \times n$

$3! = 1 \times 2 \times 3 = 6$; $4! = 1 \times 2 \times 3 \times 4 = 24$, $5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$ etc.
$n!$ is defined for positive integers only.

Deduction We have,

$$n! = 1 \times 2 \times 3 \times 4 \ldots \times (n-1) \times n$$
$$\Rightarrow \quad n! = [1 \times 2 \times 3 \times 4 \ldots \times (n-1)]n$$
$$\Rightarrow \quad n! = [(n-1)!]n = n \times (n-1)!$$

Thus, $n! = n \times (n-1)!$
For example, $8! = 8\,(7!)$, $5! = 5\,(4!)$ and $2! = 2(1!)$

02. Fundamental Principles of Counting

Fundamental Principle of Multiplication *If there are two jobs such that one of them can be completed in m ways, and when it has been completed in any one of these m ways, second job can be completed in n ways; then the jobs in succession can be completed in* $m \times n$ *ways.*

Example I *In a class there are 10 boys and 8 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this selection?*

Solution Here the teacher is to perform two jobs:
(i) Selecting a boy among 10 boys, and
(ii) Selecting a girl among 8 girls.
The first of these can be performed in 10 ways and the second in 8 ways. Therefore by the fundamental principle of multiplication, the required number of ways is $10 \times 8 = 80$.

Remark *The above principle can be extended for any finite number of jobs as stated below: If there are n jobs* $J_1, J_2, \ldots J_n$ *such that job* J_i *can be performed independently in* m_i *ways in which all the jobs can be performed is* $m_1 \times m_2 \times m_3 \times \ldots \times m_n$.

Fundamental Principle of Addition *If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in (m + n) ways.*

Example II *In a class there are 10 boys and 8 girls. The teacher wants to select either a boy or a girl to represent the class in a function. In how many ways the teacher can make this selection?*

Solution Here the teacher is to perform either of the following two jobs:

(i) Selecting a boy among 10 boys. or

(ii) Selecting a girl among 8 girls.

The first of these can be performed in 10 ways and the second in 8 ways. Therefore, by fundamental principle of addition either of the two jobs can be performed in (10 + 8) = 18 ways. Hence, the teacher can make the selection of either a boy or a girl in 18 ways.

03. Some Useful Symbols

If n is a natural number and r is a positive integer satisfying $0 \le r \le n$, then the natural number $\frac{n!}{(n-r)!}$ is denoted by the symbol ${}^nP_r$ or, $P\ (n,\ r)$.

i.e., ${}^nP_r = P(n,r) = \frac{n!}{(n-r)!}$

If n is *a* natural number and r is a positive integer satisfying $0 \le r \le n$, then the natural number $\frac{n!}{(n-r)!\ r!}$ is denoted by the symbol ${}^nC_r$, or, $C(n,\ r)$. Thus,

$${}^nC_r = C\ (n,\ r) = \frac{n!}{(n-r)!\ r!}$$

Property I ${}^nC_r = {}^nC_{n-r}$, *for* $0 \le r \le n$.

Remark The above property can be restated as follow:
If x and y are non-negative integers such that ${}^nC_x = {}^nC_y$, then $x = y$ or, $x + y = n$.

Property II *Let n and r be non-negative integers such that* $1 \le r \le n$.

Then, ${}^nC_r = \frac{n}{r}\,.\,{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\ \ {}^nC_{r-1} = {}^{n+1}C_r$

Property III *Let n and r be non-negative integers such that* $1 \le r \le n$.

Then, ${}^nC_r = \frac{n}{r}\,.\,{}^{n-1}C_{r-1}$

Property IV *If* $1 \le r \le n$, *then* $n.\ {}^{n-1}C_{r-1} = (n - r + 1)\,{}^nC_{r-1}$

Property V *If n is even, then the greatest value of* ${}^nC_r$ $(0 \le r \le n)$ *is* ${}^nC_{n/2}$.

Property VI *If n is odd, then the greatest value of* ${}^nC_r$ $(0 \le r \le n)$ is ${}^nC_{\frac{n+1}{2}}$ or, ${}^nC_{\frac{n-1}{2}}$