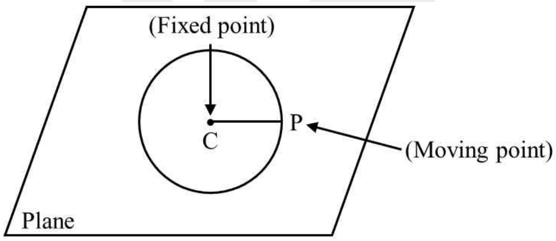
MATHEMATICS

CLASS NOTES FOR CBSE

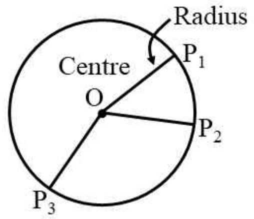
Chapter 11. Conic Sections

01. The Circle

A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.
The fixed point is called the centre of the *circle* and the constant distance is called the *radius* of the circle.
In Figure, P is the moving point, C is the fixed point and CP is equal to the radius.



Figure



$OP_1 = OP_2 = OP_3$

Figure (i)

Standard Equation of A Circle

Result *The equation of a circle whose centre is at (h, k) and radius a is given.*

$$(x-h)^2 + (y-k)^2 = a^2$$

Proof Proof Let C (h, k) be the centre and r the radius of circle. Let P(x, y) be any point on the circle (Figure ii).

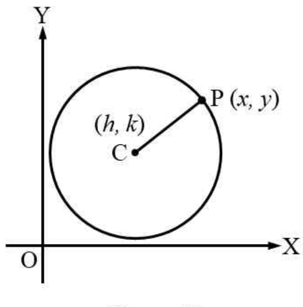


Figure (ii)

Then, by the definition, $|CP| = r$. By the distance formula, we have

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

i.e. $$(x-h)^2 + (y-k)^2 = r^2$$

NOTE ☞ *If the centre of the circle is at the origin and radius is a, then from the above form the equation of the circle is* $x^2 + y^2 = a^2$

Example I Find the equation of the circle with centre (–3, 2) and radius 4.

Solution Here $h = -3$, $k = 2$ and $r = 4$. therefore, the equation of the required of the required circle is $(x + 3)^2 + (y - 2)^2 = 16$

Example II Find the centre and the radius of the circle $x^2 + y + 8x + 10y - 8 = 10$

Solution The given equation is

$$(x^2 + 8x) + (y^2 + 10y) = 8$$

Now, completing the squares within the parenthesis, we get

$$(x^2 + 8x + 16)\ (y^2 + 10y + 25) = 8 + 16 + 25$$

i.e. $$(x + 4)^2 + (y + 5)^2 = 49$$

i.e. $$\{x - (-4)^2 + \{y - (-5)\}^2 = 7^2$$

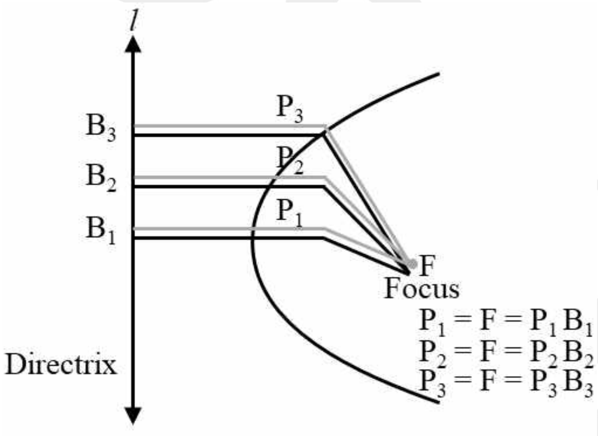
Therefore, the given circle has centre at (–4, –5) and radius 7.

02. The Parabola

Definition

A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in the plane.

The fixed point F is called the focus of the parabola and the fixed line is known as directrix of the parabola.



Figure

Some Useful Terms

Axis *The straight line passing through line focus and perpendicular to the directrix is called the axis of the conic section.*

Vertex *The point(s) of intersection of the conic section and the axis is (are) called the vertex (vertices) of the conic section.*

Latus-Rectum *The latus-rectum of a conic is the chord passing through the focus and perpendicular to the axis of the parabola.*

Standard Equations of Parabola

The equation of a *parabola* is simplest if the vertex is at the origin and the axis of symmetry is along the *x*-axis or *y*-axis. The four possible such orientations of parabola are shown below in Figure (a) to (d).

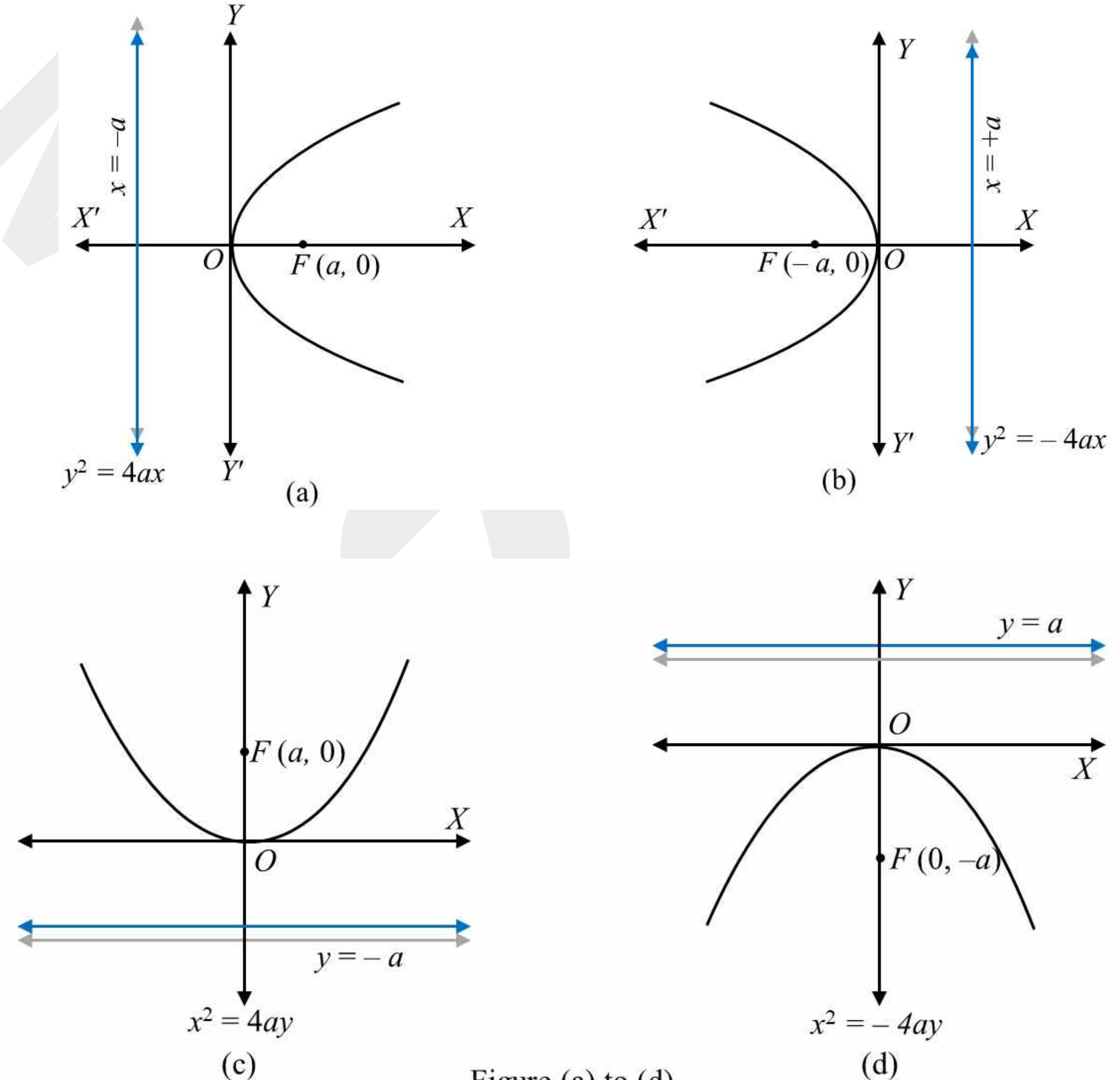


Figure (a) to (d)

Derivation of the equation of the parabola with focus at (a, 0) $a > 0$ and directrix $x = -a$ (Figure *a*)

Let F be the *focus* and *l* the *directrix*. Let FM be perpendicular to the *directrix* and bisect FM at the point O. Produce MO to X. By the definition of parabola, the mid-point O is on the parabola and is called the *vertex* of the parabola. Take O as origin, OX the *x*-axis and OY perpendicular to it as the *y*-axis and OY perpendicular to it as the *y*-axis. Let the distance from the directrix to the focus be $2a$. Then, the coordinates of the *focus* be $2a$. Then, the coordinates of the focus are (a, 0), and the equation of the focus are (a, 0), and the equation of the directrix is $x + a = 0$, and the equation of the directrix $x + a = 0$ as in Figure.