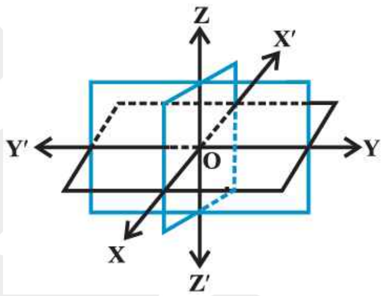
MATHEMATICS

CLASS NOTES FOR CBSE

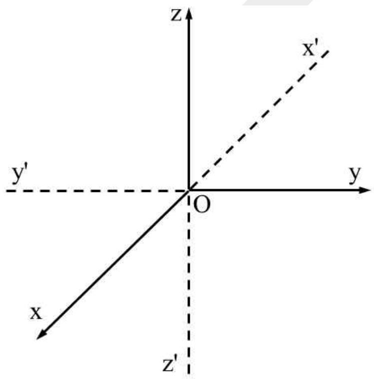
Chapter 12. Introduction to 3-D Geometry

01. Coordinates of A Point In Space

Three mutually perpendicular lines in space define three mutually perpendicular planes which in turn divide the space into eight parts known as *octants* and the lines are known as the coordinate axes.

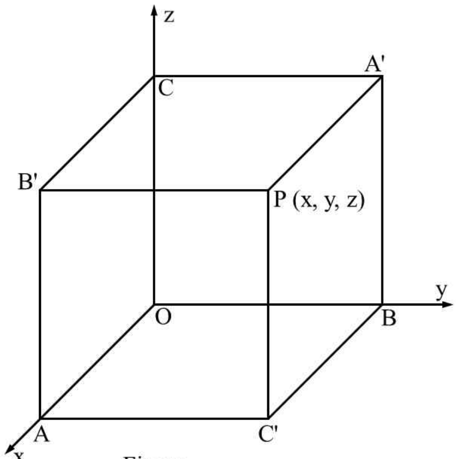


Let *X'OX, Y'OY* and *Z'OZ* be three mutually perpendicular lines intersecting at *O*. Let *O* be the origin and the lines *X'OX, Y'OY* and *Z'OZ* be x-axis, y-axis and z-axis respectively. These three lines are also called the *rectangular axes of coordinates.* The planes containing the lines *X'OX, Y'OY* and *Z'OZ* in pairs determine three mutually perpendicular planes *XOY, YOZ* and *ZOX* or simply *XY, YZ* and *ZX* which are called *rectangular coordinate planes.*



Figure

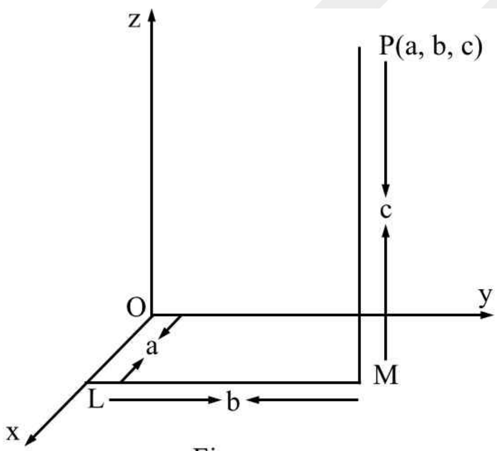
Let *P* be a point in space. Through *P* draw three planes parallel to the coordinate planes to meet the axes in *A, B* and *C* respectively. Let $OA = x$, $OB = y$ and $OC = z$. These three real numbers taken in this order determined by the point *P* are called the coordinates of the point *P*, written as (x, y, z), x, y, z are positive or negative according as they are measured along positive or negative directions of the coordinate axes.



Figure

Also, *the coordinates of the point P are the perpendicular distance from P on the three mutually rectangular coordinate planes YOZ, ZOX and XOY respectively.*
Further, the coordinates of a point are the distances from the origin of the feet of the perpendiculars from the point on the respective coordinate axes.

Alternatively, to find the coordinates of a point P in space, we first draw perpendicular PM on the xy-plane with M as the foot of this perpendicular as shown in Figure. Now, from the point M, we draw perpendicular ML *on* x-axis with L as the foot of this perpendicular. If $OL = a$, $LM = b$ and $PM = c$, then we say that a, b and c are x, y, and z coordinates, respectively, of the point P in space, In such a case, we say that the point P has coordinates (a, b, c).
Thus, there is one-to-one correspondence between the points in space and the ordered triplets (x, y, z) of real numbers.



Figure

Signs of Coordinates of A Point
Distance measured along or parallel to *OX, OY, OZ* will be positive and distances moved along or parallel to *OX', OY', OZ'* will be negative.

As three mutually perpendicular lines *X'OX, Y'OY and Z'OZ* determine three mutually perpendicular coordinate planes which in turn divide the space into eight compartments known as octants. The octant having *OX, OY* and *OZ* as its edges is denoted by *OXYZ.* Similarly, the other octants are denoted by *OX'YZ, OXY'Z, OX'Y'Z, OXYZ', OX'YZ', OXY'Z', OX'Y'Z'*. The signs of the coordinates of a point depend upon the octant in which it lies.

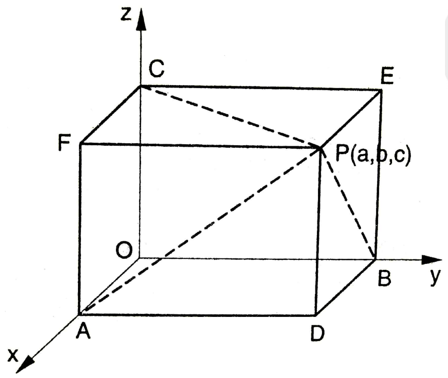
The following table shows the sings of coordinates of points in various octants:

Octant / coordinate	*OXYZ*	*OX'YZ*	*OXY'Z*	*OX'Y'Z*	*OXYZ'*	*OX'YZ'*	*OXY'Z'*	*OX'Y'Z'*
x	+	–	+	–	+	–	+	–
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

Remark 1 *If a point P lies in x y-plane, then by the definition of coordination of a point, z-coordinate of P is zero. Therefore, the coordinates of a point on xy-plane are of the form (x, y, 0) and we may take the equation of xy-plane as z = 0. Similarly, the coordinates of any point in yz and zx-planes are of the forms (0, y, z) and (x, 0, z) respectively and their equations may be taken as x = 0 and y = 0 respectively.*

Remark 2 *If a point lies on the x-axis, then its y and z-coordinates are both zero. Therefore, the coordinates of a point on x-axis are of the form (x, 0, 0) and we may take the equation of x-axis as y = 0, z = 0. Similarly, the coordinates of a point on y and z-axes are of the form (0, y, 0) and (0, 0, z) respectively and their equations may be taken as x = 0, z = 0 and x = 0, y = 0 respectively.*

Example In Figure, if the coordinates of point *P* are (*a, b, c*) then



(i) Write the coordinates of points *A, B, C, D, E* and *F.*
(ii) Write the coordinates of the feet of the perpendiculars from the point *P* to the coordinate axes.
(iii) Write the coordinates of the feet of the perpendicular from the
(iv) Find the perpendicular distances of point *P* from *XY, YZ* and *ZX*-planes.
(v) Find the perpendicular distances of the point *P* from the coordinate axes.
(vi) Find the coordinates of the reflection of *P* in *XY, YZ* and *ZX*-planes.