MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 13. Limits & Derivatives

We can approach to a given number 'a' (say) on the real line either from its left hand side by increasing numbers which are less than 'a' or from right hand side by decreasing numbers which are greater than 'a'. So, there are two types of limits viz. (i) left hand limit and (ii) right hand limit. For some functions at a given point 'a' (say) left and right hand limits are equal whereas for some functions these two limits are not equal and even sometimes either left hand limit or right hand limit or both do not exist.

If $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ i.e., (LHL at $x = a$) = (RHL at $x = a$),

then we say that $\lim_{x\to a} f(x)$ exists. Otherwise, $\lim_{x\to a} f(x)$ does not exist.

01. Evaluation of Left Hand and Right Hand Limits

$x\to a^-$ means that x is tending to a from that left hand side, i.e., x is a number less than a but very close to a. Therefore, $x\to a^-$ is equivalent to $x=a-h$ where $h>0$ such that $h\to 0$.

Similarly, $x\to a^+$ is equivalent to $x=a+h$ where $h\to 0$.

(A) We have the following algorithm for finding left hand limit at $x=a$.

Algorithm

STEP I Write $\lim_{x\to a^-} f(x)$

STEP II Put $x=a-h$ and replace $x\to a^-$ by $h\to 0$ to obtain $\lim_{h\to 0} f(a-h)$.

STEP III Simplify $\lim_{h\to 0} f(a-h)$ by using the formula for the given function.

STEP IV The value obtain in step III is the LHL of $f(x)$ at $x=a$.

(B) To evaluate RHL of $f(x)$ at $x=a$ i.e. $\lim_{x\to a^+} f(x)$ we use the following algorithm.

Algorithm

STEP I Write $\lim_{x\to a^+} f(x)$

STEP II Put $x=a+h$ and replace $x\to a^+$ by $h\to 0$ to obtain $\lim_{h\to 0} f(a+h)$.

STEP III Simplify $\lim_{h\to 0} f(a+h)$ by using the formula for the given function.

STEP IV The value obtain in step III is the RHL of $f(x)$ at $x=a$.

Example *A plane meets the coordinate axes in A, B, C such that the centroid of triangle ABC is the point (p, q, r). Show that the equation of the plane is* $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$.

Solution Let the equation of the required plane be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad \text{...(i)}$$

Then, the coordinates of A, B and C are $A\ (a,\ 0,\ 0)$, $B\ (0,\ b,\ 0)$ and $C(0,\ 0,\ c)$ respectively. So, the centroid of triangle ABC is $(a/3,\ b/3,\ c/3)$. But, the coordinates of the centroid are $(p,\ q,\ r)$.

$$\therefore \quad p=\frac{a}{3},\ q=\frac{b}{3} \text{ and } r=\frac{c}{3} \Rightarrow a=3p,\ b=3q \text{ and } c=3r$$

Substituting the values of a, b and c in (i), we get the required plane as

$$\frac{x}{3p}+\frac{y}{3q}+\frac{z}{3r}=1 \quad \Rightarrow \frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3.$$

02. Difference Between The Value of a Function at a Point and The Limit at a Point

Let $f(x)$ be a function and let a be a point. Then, we have the following possibilities:

(I) $\lim_{x\to a} f(x)$ exists but $f(a)$ (the value of $f(x)$ at $x=a$) does not exist.

(II) The value $f(a)$ exists but $\lim_{x\to a} f(x)$ does not exist.

(III) $\lim_{x\to a} f(x)$ and $f(a)$ both exist but are unequal.

(IV) $\lim_{x\to a} f(x)$ and $f(a)$ both exist and are equal.

03. The Algebra of Limits

Let f and g be two real functions with common domain D.

Following are some results concerning the limits of these functions.
Let $\lim_{x\to a} f(x)=l$ and $\lim_{x\to a} g(x)=m$. If l and m exist, then

(i) $\lim_{x\to a} (f\pm g)(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = l\pm m$

(ii) $\lim_{x\to a} (fg)(x) = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x) = lm$