MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 15. Probability

01. Some Definitions

Random or Probability Experiments *If an experiment, when repeated under identical conditions, do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes then such an experiment is known as a probabilistic experiment or a random experiment. In other words, an experiment whose outcomes cannot be predicted or determined in advance is called a random experiment. For example, in tossing of a coin one is not sure if a head or a tail will be obtained os it is random experiment.*

Elementary Event *If a random experiment is performed, then each of its outcomes is known as an elementary event or outcomes of a random experiment are known as elementary events associated to it. Elementary events are also known as simple events.*

Sample Space *The set of all possible outcomes of a random experiment is called the sample space associated with it and it is generally denoted by S.*

In other words, the set of all elementary events associated to a random experiment is called its sample space. If $E_1, E_2, E_3, \ldots E_n$ are the possible outcomes (or elementary events) of a random experiment, then $S = \{E_1, E_2, \ldots E_n\}$ is the sample space associated to it.

Example Consider the experiment of throwing a die. Let the six faces of a die be marked as 1, 2, 3, 4, 5 and 6. If the die is thrown, then any one of the six faces may come upward. So there are six possible outcomes of this experiment, namely, 1, 2, 3, 4, 5, 6. Thus, if we define E_i = Getting a face marked with number i, where $i = 1, 2, 3, 4, 5, 6$

Then, $E_1, E_2, \ldots, E_6$ are six elementary events associated to this experiment. The sample space associated to this experiment is

$$S = \{E_1, E_2, \ldots, E_6\}$$

In this experiment, elementary even E_i is denoted by i, where $i = 1, 2, \ldots, 6$. Thus, we have

$$S = \{1, 2, 3, 4, 5, 6\}.$$

02. Event

Event *A subset of the sample space associated with a random experiment is called an event.*

Example Consider the experiment of tossing there coins at a time. The sample space S associated with this experiment is

$$S = \{HHH, HHT, THH, HTH, TTH, THT, THT, TTT\}$$

Let

$A = \{HHT, HTH, THH\}$ $B = \{HHH, HHT, HTH, THH\}$

$C = \{HHH, HHT, HTH, THH, TTH, HTT, THT\}$

and,

$D = \{HHH, TTT, HTH\}$

Clearly, A, B, C and D, being subsets of S, are events associated with the random experiment of tossing three coins (or tossing a coin three times). These events can also be described in words as follows:

A = Getting two heads, B = Number of heads exceeds the number of tails,

C = Getting at least on head.

But, event D cannot be described in words.

Occurrence Of An Event *An event A associated to a random experiment is said to occur if any one of the elementary events associated to it is an outcome.*

Example Suppose a die is thrown and the outcome of the trial is 4. Then, we can say that each of the following events have occurred.

(i) Getting a number greater than or equal to 2, represented by the set $\{2, 3, 4, 5, 6\}$

(ii) Getting a number less than or equal to 5, represented by the set $\{1, 2, 3, 4, 5\}$.

On the basis of the same outcome, we can also say that the following events have not occurred:

(i) Getting an odd number represented by the set $\{1, 3, 5\}$

(ii) Getting a multiple of 3, represented by the set $\{3, 6\}$.

03. Algebra of Events

(i) **A ∪ B**

Let A and B be two events associated to a random experiment with sample space S we define the event "A or B" which is said to occur if an elementary event favourable to either A or B or both is an outcome. In other words, the event "A or B" occurs if either A or B or both occur i.e. at least one of A and B occurs. Thus, "A or B" is represented by the subset $A \cup B$ of the sample space S.

Example In a single throw of a die, if we define

A = Getting an even number, B = Getting a multiple of 3.

These two events are described by the sets $\{2, 4, 6\}$ and $\{3, 6\}$ respectively. In this case, we have

$A \cup B$ = Getting a number which is either even or a multiple of 3 or both

$= \{2, 3, 4, 6\}$

(ii) **A ∩ B**

The event "*A* and *B*" is said to occur if an elementary event favourable to both *A* and *B* is an outcome. In other words, the event "*A* or *B*" occurs if *A* and *B* both occur. The event *A* and *B* is denoted by $A \cap B$.

Example In a single throw of a pair of dice if we define

A = Getting an even number on first die

and,

B = Getting 8 as the sum of the numbers on two dice,

Then,

$A \cap B$ = Getting an even number on first die such that the sum of the numbers is 8

= {(2, 6), (6, 2), (4, 4)}.

(iii) **Negation of An Event**

Corresponding to every event A associated to a random experiment, we define an event "not A" which is said to occur when and only when A does not occur.

Example In a single throw of a die if A denotes the event that the outcome is an odd number. Then *A* = {1, 3, 5} and *A* does not occur if the outcome is any one of the outcomes 2, 4, 6. Thus, the event "not *A*" is represented by the set $\overline{A}$ and is called the *complementary event of A or negation of A.*

04. Types of Events

a. **Certain (Or Sure) Event** *An event associated with a random experiment is called a certain event if it always occurs whenever the experiment is performed.*

Example Associated with the random experiment of rolling a die, the event *A* "Getting an even number or an odd number" is a certain event. Clearly, this event is represented by the set {1, 2, 3, 4, 5, 6} which is the sample space of the experiment.

The sample space associated with a random experiment defines a certain event.

b. **Impossible Event** *An event associated with a random experiment is called an impossible event if it never occur whenever the experiment is performed.*

Example Consider the experiment of rolling a die. Let *A* be the event "the number truns up is divisible by 7". Clearly, none of the possible outcomes 1, 2, 3, 4, 5, 6 is divisible by 7. So, the event *A* cannot occur at all. In other words, there is no outcome belonging to set *A*. So the set *A* is the null set.

The event represented by φ is an impossible event.