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CLASS 11&12th

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Matrices

01. Meaning

Matrix

A set of mn numbers (real or imaginary) arranged in the form of a rectangular array of m rows and n columns is called an $m \times n$ matrix (to be read as $'m$ by n' matrix). An $m \times n$ matrix is usually written as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}$$

In compact form the above matrix is represented by

$A = [a_{ij}]_{m \times n}$ or, $A = [a_{ij}]$.

The numbers $a_{11}, a_{12}, \dots$ etc. are known as the elements of the matrix A. The element a_{ij} belongs to i^{th} row and j^{th} column and is called the $(i, j)^{\text{th}}$ element of the matrix $A = [a_{ij}]$. Thus, in the element a_{ij} the first subscript i always denotes the number of rows and the second subscript j, number of columns in which the element occurs.

For example, $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 2 \end{bmatrix}$ is a matrix having 2 rows and 3 columns and so it is a matrix of order 2×3 such that $a_{11} = 2, a_{12} = 1, a_{13} = -1, a_{21} = 1, a_{22} = 3, a_{23} = 2$.

02. Types of Matrices

Row Matrix

A matrix having only one row is called a row-matrix or a row-vector.

For example, $A = [1 \; 2 \; -1 \; -2]$ is a row matrix of order 1×4.

Column Matrix

A matrix having only one column is called a column matrix or a column-vector.

For example, $A = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 2 \\ 5 \\ 4 \end{bmatrix}$ are column-matrices or order 3×1 and 4×1 respectively.

Horizontal/Vertical Matrix

A matric is called a horizontal matrix if there are less number of rows then columns and a matrix is called vertical if there are more number of rows then columns.

i.e., $A = [a_{ij}]_{m \times n}$ is a horizontal matrix if $m < n$.

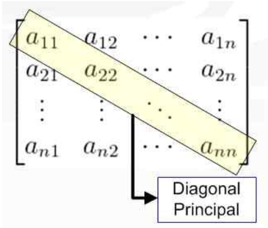
and $A = [a_{ij}]_{m \times n}$ is a vertical matrix if $m > n$.

(where m is number of rows and n is number of columns)

e.g., $A = \begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{bmatrix}, \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$, are respectively horizontal and vertical matrices.

Square Matrix

If in a matrix, number of rows (m) = number of columns (n), then it is said to be a square matrix and the elements $a_{11}, a_{22}, \ldots\ldots\ldots, a_{mn}$ are called diagonal elements and the line passing through then is known as principal or leading diagonal. The other diagonal is known as off diagonal.



For example, $\begin{bmatrix} 2 & 1 & -1 \\ 3 & -2 & 5 \\ 1 & 5 & -3 \end{bmatrix}$ is square matrix of order 3 in which the diagonal elements are $2, -2$ and -3.

Diagonal Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is called a diagonal matrix if all the elements, except those in the leading diagonal, are zero i.e.

$$a_{ij} = 0 \text{ for all } i \neq j$$

A diagonal matrix of order $n \times n$ having $d_1, d_2, \ldots, d_n$ as diagonal elements is denoted by diag $[d_1, d_2, \ldots, d_n]$.

For example, the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is a diagonal matrix, to be denoted by $A = \text{diag}\,[1, 2, 3]$.

Scalar Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is called a scalar matrix if

(i) $a_{ij} = 0$ for all $i \neq j$, and

(ii) $a_{ii} = C$ for all i, where $C \neq 0$.

In order words, a diagonal matrix in which all the diagonal elements are equal is called the scalar matrix.

For example, the matrices $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1-2i & 0 & 0 \\ 0 & 1-2i & 0 \\ 0 & 0 & 1-2i \end{bmatrix}$ are scalar matrices of order 2 and 3 respectively.

Identity Or Unit Matrix

A square matrix $A = [a_{ij}]_{n \times n}$ is called an identity or unit matrix if

(i) $a_{ij} = 0$ for all $i \neq j$ and

(ii) $a_{ii} = 1$ for all i

In order words, a square matrix each of whose diagonal element is unity and each of whose non-diagonal elements is equal to zero is called an identity or unit matrix.
The identity matrix of order n is denoted by I_n.

For example, the matrices $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are identity matrices of order 2 and 3 respectively.

Null Matrix

A matrix whose all elements are zero is called a null matrix or a zero matrix.

For example, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are null matrices of order 2×2 and 2×3 respectively.

Upper Triangular Matrix

A square matrix $A = [a_{ij}]$ is called an upper triangular matrix if $a_{ij} = 0$ for all $i > j$.
Thus, in an upper triangular matrix, all elements below the main diagonal are zero.

For example, $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 0 & 5 & 1 & 3 \\ 0 & 0 & 2 & 9 \\ 0 & 0 & 0 & 5 \end{bmatrix}$ is an upper triangular matrix.

Lower Triangular Matrix

A square matrix $A = [a_{ij}]$ is called a lower triangular matrix if $a_{ij} = 0$ for all $i < j$.
Thus, in a lower triangular matrix, all elements above the main diagonal are zero.

For example, $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$ is a lower triangular matrix of order 3. A triangular matrix $A = [a_{ij}]\, n\times n$ is called a strictly triangular iff.

$$a_{ii} = 0 \text{ for all } i = 1, 2, \ldots, \text{n}.$$

03. Algebra of Matrices

Equality Of Matrices

The matrices $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{r\times s}$ are equal if

(i) $m = r$, i.e., the number of rows in A equals the number of rows in B
(ii) $n = s$, i.e., the number of columns in A equals the number of columns in B
(iii) $a_{ij} = b_{ij}$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$

Addition Of Matrices

Let A, B be two matrices, each of order $m\times n$. Then their sum $A + B$ is a matrix of order $m\times n$ and is obtained by adding the corresponding elements of A and B.
Thus, if $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are two matrices of the same order, their sum $A + B$ is defined to be the matrix of order $m\times n$ such that

$$(A+B)_{ij} = a_{ij} + b_{ij} \text{ for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$