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Complete MATH

IIT-JEE · CBSE eBOOKS

CLASS 11&12th

CLASS 12th

Continuity & Differentiability

01. Continuity at a Point

DEFINITION:

A function $f(x)$ is said to be continuous at a point $x = a$ of its domain, iff $\lim\limits_{x \to a} f(x) = f(a)$.

Thus, $(f(x)$ is continuous at $x = a)$

$\Leftrightarrow \quad \lim\limits_{x \to a} f(x) = f(a)$

$\Leftrightarrow \quad \lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) = f(a)$

REMARK

(i) If $f(x)$ is not continuous at a point $x = a$, then it is said to be discontinuous at $x = a$.

(ii) If $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x) \neq f(a)$, then the discontinuity is known as the removable discontinuity.

(iii) If $\lim\limits_{x \to a^-} f(x) \neq \lim\limits_{x \to a^+} f(x)$, then $f(x)$ is said to have a discontinuity of first kind.

(iv) A function $f(x)$ is said to have discontinuity of the second kind at $x = a$ iff $\lim\limits_{x \to a^-} f(x)$ or, $\lim\limits_{x \to a^+} f(x)$ or both do not exist.

(v) A function $f(x)$ is said to be left continuous or continuous from the left at $x = a$, iff

(a) $\lim\limits_{x \to a^-} f(x)$ exists and, (b) $\lim\limits_{x \to a^-} f(x) = f(a)$

A function $f(x)$ is said to be right continuous or continuous from the right at $x = a$, iff

(a) $\lim\limits_{x \to a^+} f(x)$ exists and, (b) $\lim\limits_{x \to a^+} f(x) = f(a)$

It follows from the above definitions that
$f(x)$ is continuous at $x = a$ iff it is both left as well as right continuous at $x = a$.

(vi) A function $f(x)$ fails to be continuous at $x = a$ for any of the following reasons.

(a) $\lim\limits_{x \to a} f(x)$ exists but it is not equal to $f(a)$.

(b) $\lim\limits_{x \to a} f(x)$ does not exist.

This happens if either $\lim\limits_{x \to a^-} f(x)$ does not exist or, $\lim\limits_{x \to a^+} f(x)$ does not exist or both $\lim\limits_{x \to a^-} f(x)$ and $\lim\limits_{x \to a^+} f(x)$ exist but are not equal.

(c) f is not defined at $x = a$ i.e. $f(a)$ does not exist.