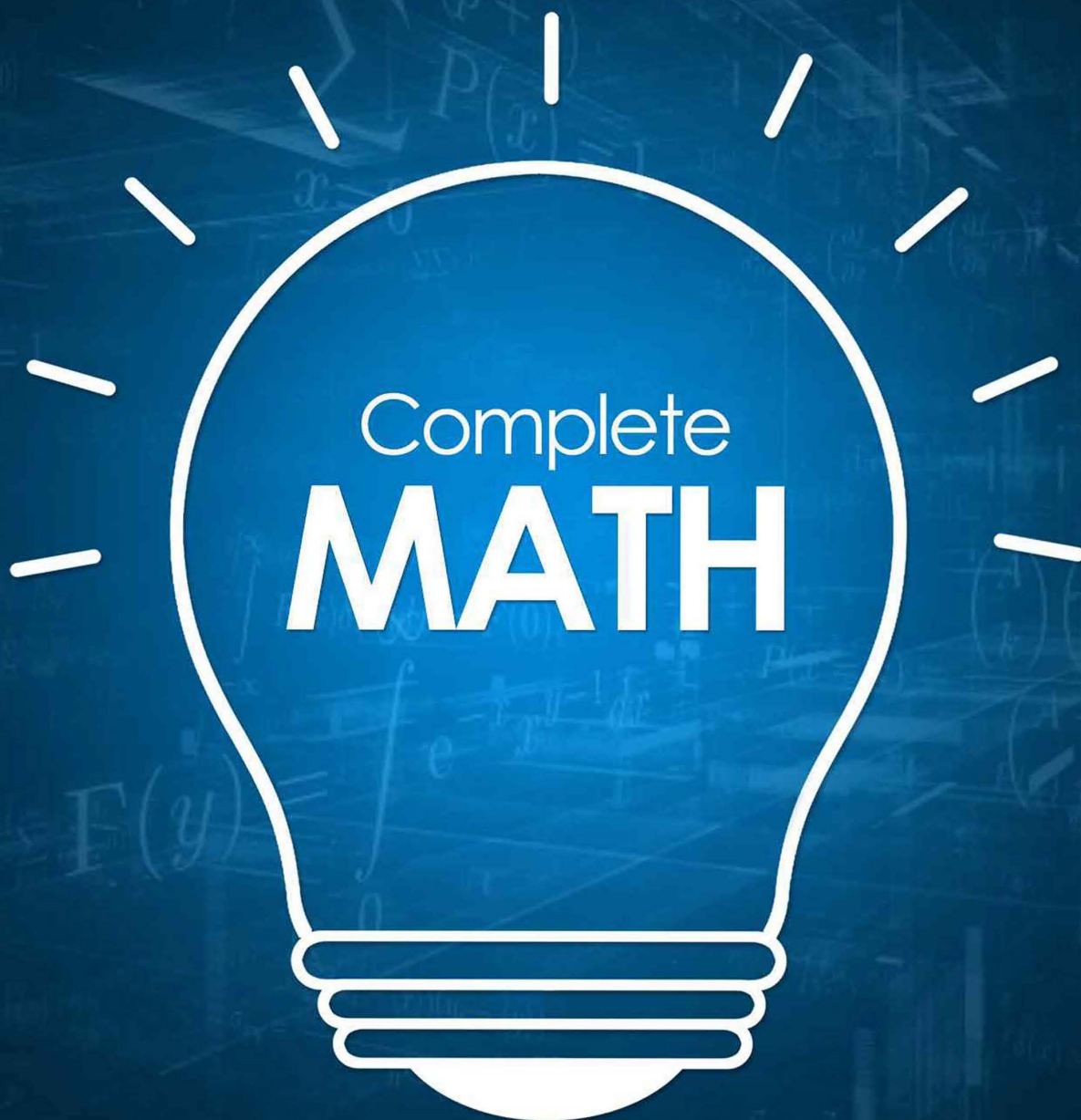




IIT-JEE · CBSE eBOOKS

CLASS 11 & 12th



Learning Inquiry
8929 803 804

CLASS 12th

Integrals

misstudy

$$\frac{dy}{dx} (\phi(x) + C) = f(x) \Leftrightarrow \int f(x) dx = \phi(x) + c$$

where $\phi(x)$ is primitive of $f(x)$ and C is an arbitrary constant known as the constant of integration.

01. Some Standard Results

$$(i) \quad \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$$

$$(ii) \quad \int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b) + c$$

$$(iii) \quad \int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$$(iv) \quad \int a^{px+q} dx = \frac{1}{p \ln a} a^{px+q} (a > 0) + c$$

$$(v) \quad \int \sin(ax+b) dx = -\frac{1}{a} \cos(ax+b) + c$$

$$(vi) \quad \int \cos(ax+b) dx = \frac{1}{a} \sin(ax+b) + c$$

$$(vii) \quad \int \tan(ax+b) dx = \frac{1}{a} \ln \sec(ax+b) + c$$

$$(viii) \quad \int \cot(ax+b) dx = \frac{1}{a} \ln \sin(ax+b) + c$$

$$(ix) \quad \int \sec^2(ax+b) dx = \frac{1}{a} \tan(ax+b) + c$$

$$(x) \quad \int \operatorname{cosec}^2(ax+b) dx = -\frac{1}{a} \cot(ax+b) + c$$

$$(xi) \quad \int \sec(ax+b) \tan(ax+b) dx = \frac{1}{a} \sec(ax+b) + c$$

$$(xii) \quad \int \operatorname{cosec}(ax+b) \cdot \cot(ax+b) dx = -\frac{1}{a} \operatorname{cosec}(ax+b) + c$$

$$(xiii) \quad \int \sec x dx = \ln(\sec x + \tan x) + \cot \ln \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right] + c$$

$$(xiv) \quad \int \operatorname{cosec} x dx = \ln(\operatorname{cosec} x - \cot x) + c \text{ or } \ln \left(\tan \frac{x}{2} \right) + c \text{ or } -\ln(\operatorname{cosec} x + \cot x) + c$$

$$(xv) \quad \int \sinh x dx = \cosh x + c$$

$$(xvi) \quad \int \cosh x dx = \sinh x + c$$

$$(xvii) \quad \int \sec^2 x dx = \tan x + c$$

$$(xviii) \quad \int \operatorname{cosec}^2 x dx = -\cot x + c$$

$$(xix) \int \operatorname{sech} x \cdot \tanh x \, dx = -\operatorname{sech} x + c$$

$$(xx) \int \operatorname{cosech} x \cdot \coth x \, dx = -\operatorname{cosech} x + c$$

02. Techniques of Integration

(i) Substitution or change of independent variable

Integral $I = \int f(x) \, dx$ is changed to $\int f(\phi(x)) \phi'(x) \, dx$, by a suitable substitution $t = \phi(x)$ provided the later integral is easier to integrate.

$$(ii) \int [f(x)]^n f'(x) \, dx \text{ or } \int \frac{f'(x)}{[f(x)]^n} \, dx$$

Put $f(x) = t$ and proceed.

03. Some Special Integrals

$$(i) \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$$

$$(ii) \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$(iii) \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{sech}^{-1} \frac{x}{a} + c$$

$$(iv) \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln [x + \sqrt{x^2 + a^2}] + c$$

$$(v) \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln [x + \sqrt{x^2 - a^2}] + c$$

$$(vi) \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + c$$

$$(vii) \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$$

$$(viii) \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

$$(ix) \int \sqrt{x^2 + a^2} \, dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a} + c$$

$$\text{or } \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln |x + \sqrt{x^2 + a^2}| + c$$