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# Complete MATH

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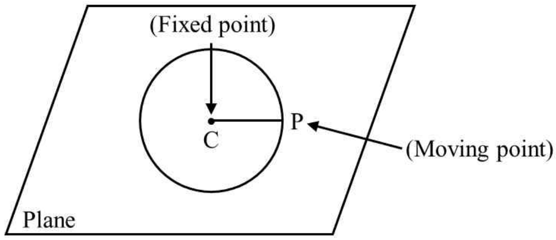
# Conic Sections

## 01. The Circle

*A circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane is always constant..*

The fixed point is called the centre of the *circle* and the constant distance is called the *radius* of the circle.
In Figure, $P$ is the moving point, $C$ is the fixed point and $CP$ is equal to the radius.




Figure

**Standard Equation of A Circle**
**Result** *The equation of a circle whose centre is at (h, k) and radius a is given.*

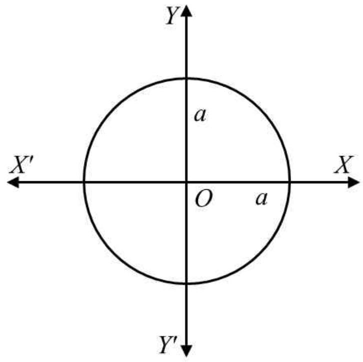
$$(x - h)^2 + (y - k)^2 = a^2$$

***NOTE*** ☞ *If the centre of the circle is at the origin and radius is a, then from the above form the equation of the circle is* $x^2 + y^2 = a^2$

**Some Particular Cases of the Central Form**
The equation of a circle with centre at $(h, k)$ and radius equal to $a$, is

$$(x - h)^2 + (y - k)^2 = a^2 \qquad ...(i)$$

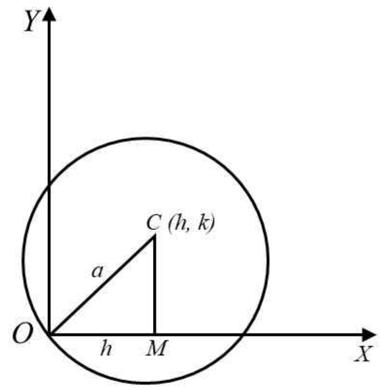
(I) *When the centre of the circle coincides with the origin* i.e., $h = k = 0$




Figure

Then the equation (i) reduces to $x^2 + y^2 = a^2$

(II) *When the circle passes through the origin*

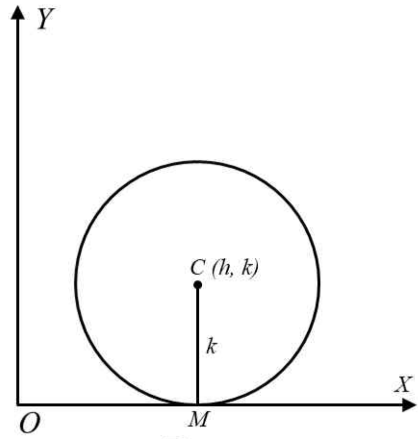



Figure

The equation of the circle (i) then becomes

$(x - h)^2 + (y - k)^2 = h^2 + k^2$ or, $x^2 + y^2 - 2hx - 2ky = 0.$

(III) *When the circle touches x-axis*



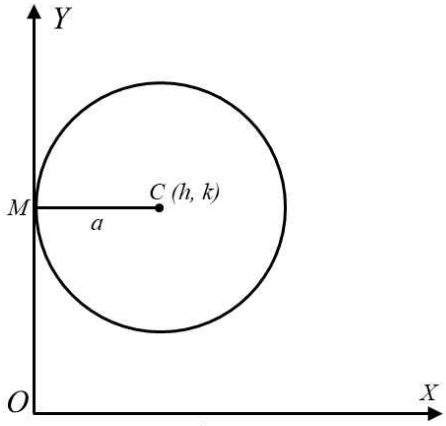

Figure

The equation of the circles is $(x - h)^2 + (y - a)^2 = a^2$ or, $x^2 + y^2 - 2hx - 2ay + h^2 = 0$

(IV) *When the circle touches y-axis*

The equation of the circle is

$(x - a)^2 + (y - k)^2 = a^2$ or, $x^2 + y^2 - 2ax - 2ky + k^2 = 0$




Figure

## General Equation of A Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ always represents a circle whose centre is at $(-g, -f)$ and, Radius $= \sqrt{g^2 + f^2 - c}$

**NOTE 1☞** *The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle of radius $\sqrt{g^2 + f^2 - c}$.*

*If $g^2 + f^2 - c > 0$ then the radius of the circle is real and hence the circle is also real.*

*If $g^2 + f^2 - c = 0$ then the radius of the circle is zero. Such a circle is known as a point circle.*

*If $g^2 + f^2 - c < 0$, then the radius $\sqrt{g^2 + f^2 - c}$ of the circle is imaginary but the centre is real. Such a circle is called an imaginary circle as it is not possible to draw such a circle.*

**NOTE 2☞** *Special features of the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$ of the circle are:*

(i) *It is quadratic in both x and y.*

(ii) *Coefficient of $x^2$ = Coefficient of $y^2$.*

*In solving problems it is advisable to keep the coefficient of $x^2$ and $y^2$ unity.*

(iii) *There is no term containing xy i.e., the coefficient of xy is zero.*

(iv) *It contains three arbitrary constants viz. g, f and c.*

**NOTE 3☞** *The equation $ax^2 + ay^2 + 2gx + 2fy + c = 0$, $a \neq 0$ also represents a circle. This equation can also be written as*

$$x^2 + y^2 + \frac{2g}{a}x + \frac{2f}{a}y + \frac{c}{a} = 0.$$

*The coordinates of the centre are* $\left(-\frac{g}{a}, -\frac{f}{a}\right)$

*and, Radius* $= \sqrt{\frac{g^2}{a^2} + \frac{f^2}{a^2} - \frac{c}{a}}$

**NOTE 4☞** *On comparing the general equation*

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

*of a circle with the general equation of second degree*

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

*we find that it represents a circle if $a = b$*

*i.e., Coefficient of $x^2$ = Coefficient of $y^2$*

*and, $h = 0$ i.e., Coefficient of xy = 0.*

## Circle Passing Through Three Points

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

be the circle passing through three non-collinear points $P(x_1, y_1)$, $Q(x_2, y_2)$, and $R(x_3, y_3)$. Then,

$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0 \qquad \text{...(ii)}$$