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Complete MATH

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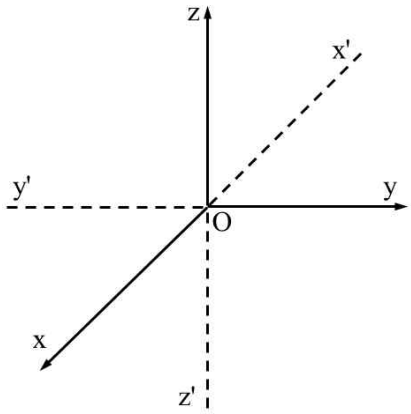
CLASS 11&12th

CLASS 12th

3-D Geometry

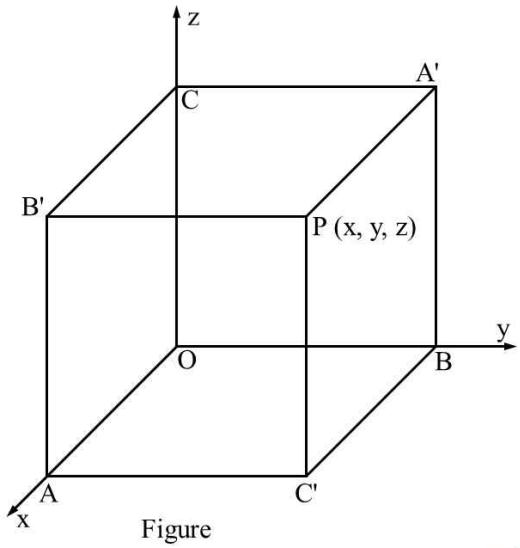
01. Coordinates of A Point In Space

Three mutually perpendicular lines in space define three mutually perpendicular planes which in turn divide the space into eight parts known as *octants* and the lines are known as the coordinate axes.



Figure

Let *X'OX, Y'OY* and *Z'OZ* be three mutually perpendicular lines intersecting at *O*. Let *O* be the origin and the lines *X'OX, Y'OY* and *Z'OZ* be *x*-axis, *y*-axis and *z*-axis respectively. These three lines are also called the *rectangular axes of coordinates.* The planes containing the lines *X'OX, Y'OY* and *Z'OZ* in pairs determine three mutually perpendicular planes *XOY, YOZ* and *ZOX* or simply *XY, YZ* and *ZX* which are called *rectangular coordinate planes.*



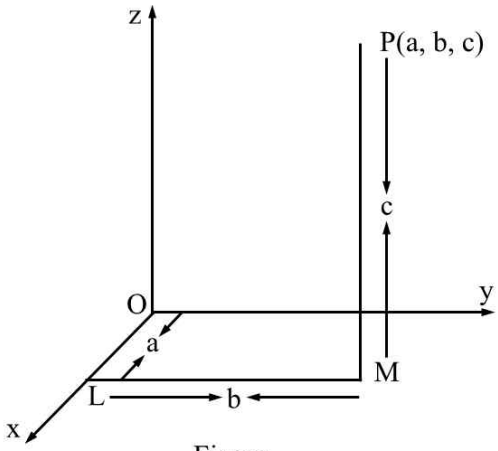
Figure

Let *P* be a point in space. Through *P* draw three planes parallel to the coordinate planes to meet the axes in *A, B* and *C* respectively. Let $OA = x$, $OB = y$ and $OC = z$. These three real numbers taken in this order determined by the point *P* are called the coordinates of the point *P*, written as (x, y, z), x, y, z are positive or negative according as they are measured along positive or negative directions of the coordinate axes.

Also, *the coordinates of the point P are the perpendicular distance from P on the three mutually rectangular coordinate planes YOZ, ZOX and XOY respectively.*
Further, the coordinates of a point are the distances from the origin of the feet of the perpendiculars from the point on the respective coordinate axes.

Alternatively, to find the coordinates of a point P in space, we first draw perpendicular PM on the xy-plane with M as the foot of this perpendicular as shown in Figure. Now, from the point M, we draw perpendicular ML *on* x-axis with L as the foot of this perpendicular. If $OL = a$, $LM = b$ and $PM = c$, then we say that a, b and c are x, y, and z coordinates, respectively, of the point P in space, In such a case, we say that the point P has coordinates (a, b, c).

Thus, there is one-to-one correspondence between the points in space and the ordered triplets (x, y, z) of real numbers.



Figure

Signs of Coordinates of A Point

Distance measured along or parallel to OX, OY, OZ will be positive and distances moved along or parallel to OX', OY', OZ' will be negative.

As three mutually perpendicular lines $X'OX$, $Y'OY$ *and* $Z'OZ$ determine three mutually perpendicular coordinate planes which in turn divide the space into eight compartments known as octants. The octant having OX, OY and OZ as its edges is denoted by $OXYZ$. Similarly, the other octants are denoted by $OX'YZ$, $OXY'Z$, $OX'Y'Z$, $OXYZ'$, $OX'YZ'$, $OXY'Z'$, $OX'Y'Z'$. The signs of the coordinates of a point depend upon the octant in which it lies.

The following table shows the sings of coordinates of points in various octants:

Octant / coordinate	$OXYZ$	$OX'YZ$	$OXY'Z$	$OX'Y'Z$	$OXYZ'$	$OX'YZ'$	$OXY'Z'$	$OX'Y'Z'$
x	+	–	+	–	+	–	+	–
y	+	+	–	–	+	+	–	–
z	+	+	+	+	–	–	–	–

Remark 1 *If a point P lies in x y-plane, then by the definition of coordination of a point, z-coordinate of P is zero. Therefore, the coordinates of a point on xy-plane are of the form (x, y, 0) and we may take the equation of xy-plane as z = 0. Similarly, the coordinates of any point in yz and zx-planes are of the forms (0, y, z) and (x, 0, z) respectively and their equations may be taken as x = 0 and y = 0 respectively.*