MATHEMATICS

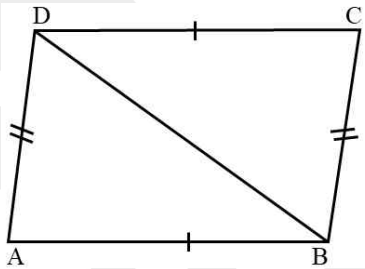
CLASS NOTES FOR CBSE

Chapter 09. Areas of Parallelograms and Triangles

01. Parallelograms on the Same Base and Between The Same

Base : *For base of a parallelogram is any side of it.*
Altitude : *For each base of a parallelogram, the corresponding altitude is the line segment form a point on the base, perpendicular to the line containing the opposite side.*

Result *A diagonal of a parallelogram divides it into two triangles of equal area.*
Given : A parallelogram $ABCD$ in which BD is one of the diagonals.
To Prove : $ar\ (\Delta ABD) = ar\ (\Delta CDB)$



Figure

Proof : Since two congruent geometrical figures have equal area. Therefore, in order to prove that $ar\ (\Delta ABD) = ar\ (\Delta CDB)$ it is sufficient to show that

$$\Delta ABD \cong \Delta CDB$$

In Δs ABD and CDB, we have

$AB = CD$ [$\because$ $ABCD$ is a $\|^{gm}$ $\therefore$ $AD = CD$]

$AD = CB$ [$\because$ $ABCD$ is a $\|^{gm}$ $\therefore$ $AD = CB$]

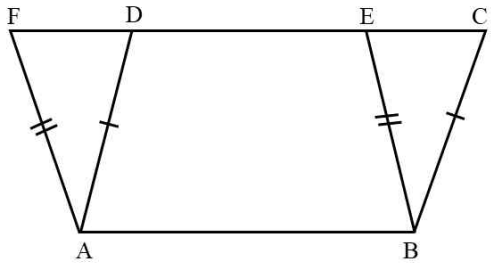
and, $BD = DB$ [Common side]

So, by SSS criterion of congruence, we obtain

$$\Delta ABD \cong \Delta CDB$$

Hence, $ar\ (\Delta ABD) = ar\ (\Delta CDB)$

Result *Parallelograms on the same base and between the same parallels are equal in area.*
Given : Two parallelograms $ABCD$ and $ABEF$, which have the same base AB and which are between the same parallel lines AB and FC.
To Prove : $ar\ (\|^{gm}\ ABCD) = ar\ (\|^{gm}\ ABCD)$



Figure

Proof : In Δs ADF and BCE, we have

$AD = BC$ [$\because$ $ABCD$ is a $\|^{gm}$ $\therefore$ $AD = BC$]

$AF = BE$ [$\because$ $ABEF$ is a $\|^{gm}$ $\therefore$ $AF = BE$]

and, $\angle DAF = \angle CBE$ [$\because$ $AD \parallel BC$ and $AF \parallel BE$ $\therefore$ Angle between AD and AF Angle between BC and $BE \Rightarrow \angle DAF = \angle CBE$]

So, by SAS criterion of congruence, we obtain

$\Delta ADF = \Delta BCE$

$ar\ (\Delta ADF) = ar\ (\Delta BCE)$ [By Congruence area axiom] ...(i)

Now,

$ar\ (\|^{gm}\ ABCD) = ar\ (\square ABED) + ar\ (\Delta BCE)$ [By Area addition axiom]

$\Rightarrow$ $ar\ (\|^{gm}\ ABCD) = ar\ (\square ABED) + ar\ (\Delta ADF)$ [Using (i)]

$\Rightarrow$ $ar\ (\|^{gm}\ ABCD) = ar\ (\|^{gm}\ ABEF)$

Hence, $ar\ (\|^{gm}\ ABCD) = ar\ ((\|^{gm}\ ABEF)$

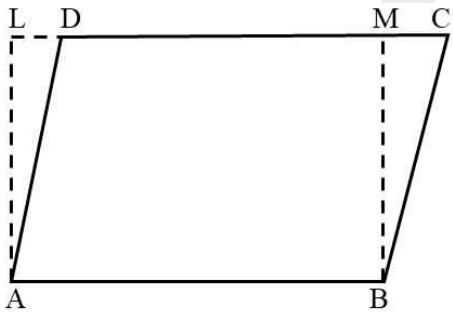
Corollary : *A parallelogram and a rectangle on the same base and between the same parallels are equal in area.*

<u>Result</u> *The area of a parallelogram is the product of its base and the corresponding altitude.*

Given : A parallelogram $ABCD$ in which AB is the base and AL the corresponding altitude.

To Prove : $ar\ (\|^{gm}\ ABCD) = AB \times AL$

Construction : Complete the rectangle $ALMB$ by drawing $BM \perp CD$.



Figure

Poof : Since ($ar\ \|^{gm}\ ABCD$) and rectangle $ALMB$ are on the same base and between the same parallels.

$\therefore$ $ar\ (\|^{gm}\ ABCD)$

$= ar$ (rect. $ALMB$)

$= AB \times AL$ [By rectangle area axiom, area of a rectangle = Base × height]

Hence, $(ar\ \|^{gm}\ ABCD) = AB \times AL$