# MATHEMATICS

## CLASS NOTES FOR CBSE

## Chapter 16. Real Numbers

### 01. Euclid's Division Lemma

**Euclid's Division Lemma :** Let a and b be any two positive integers. Then, there exist unique integers q and r such that

$$a = bq + r,\ 0 \le r < b$$

*If* $b \mid a$, *then* $r = 0$. *Otherwise,* $r$ *satisfies the stronger inequality* $0 < r < b$.

**Remark I** *The above Lemma is nothing but a restatement of the long division process we have been doing all these years, and that the integers q and r are called the quotient and remainder, respectively.*

**Remark II** *The above Lemma has been stated for positive integers only. But, it can be extended to all integers as stated below :*

*Let a and b any two integers with* $b \neq 0$. *Then, there exist unique integers q and r such that*

$$a = bq + r, \text{ where } 0 \le\ < |b|$$

**Remark III**

(i) *When a positive integer is divided by 2, the remainder is either 0 or 1. So, any positive integer is of the form* $2m$, $2m + 1$ *for some interger m.*

(ii) *When any positive integer is divided by 3, the remainder is 0 or 1 or 2. So, any positive integer can be written in the form* $3m$, $3m + 2$ *for some integer m.*

(iii) *When a positive integer is divided by 4, the remainder can be 0 or 1 or 2 or 3. So, any positive integer is of the form* $4q$ *or,* $4q + 1$ *or,* $4q + 3$.

**Example** *Show that any positive odd integer is of the form* $4q + 1$ or $4q + 3$, *where q is some integer.*

**Solution** Let $a$ be any odd positive integer and $b = 4$. By division Lemma there exists integers $q$ and $r$ such that

$$a = 4q + r, \text{ where } 0 \le r < 4$$

$$\Rightarrow \quad a = 4q \text{ or, } a = 4q + 1 \text{ or, } a = 4q + 2 \text{ or, } a = 4q + 3$$

$$[\because\ 0 \le r < 4 \Rightarrow r = 0,1,2,3]$$

$$\Rightarrow \quad a = 4q + 1 \text{ or, } a = 4q + 3$$

$$[\because\ a \text{ is an odd integer } \therefore\ a \neq 4q,\ a \neq 4q + 2]$$

Hence, any odd integer is of the form $4q + 1$ or, $4q + 3$.

## 02. Euclid's Division Algorithm

In order tlo compute the *HCF* of two positive intgers, say $a$ and $b$, with $a > b$ we may follow the following steps :

**Step I** *Apply Euclid's division lemma to $a$ and $b$ and obtain whole numbers $q_1$ and $r_1$ such that $a = bq_1 + r_1$, $0 \le r_1 < b$.*

**Step II** *If $r_1 = 0$, $b$ is the HCF of $a$ and $b$*

**Step III** *If $r_1 \neq 0$, apply Euclid's division lemma to $b$ and $r_1$ and obtain two whole numbers $q_1$ and $r_2$ such that $b = q_1 r_1 + r_2$.*

**Step IV** *If $r_2 = 0$, then $r_2$ is the HCF of $a$ and $b$.*

**Step V** *If $r_2 \neq 0$, then apply Euclid's division lemma to $r_1$ and $r_2$ and continue the above process till the remainder $r_n$ is zero. The divisor at this stage i.e. $r_{n-1}$, or the non-zero remainder at the previous stage, is the HCF of $a$ and $b$.*

**Example** *Use Euclid's division algorithm to find the HCF of* 4052 *and* 12576.

**Solution** Given integers are 4052 and 12576 such that 12576 > 4052. Applying Euclid's division lemma to 12576 and 4052, we get

$$12576 = 4052 \times 3 + 420 \quad \text{...(i)} \qquad \left[\because\ 4052\overline{)12576}^{\,3},\ \ 12576 - 12156 = 420\right]$$

Since the remainder $420 \neq 0$. So, we apply the division lemma to 4052 and 420, to get

$$4052 = 420 \times 9 + 272 \quad \text{...(ii)} \qquad \left[\because\ 420\overline{)4052}^{\,9},\ \ 4052 - 3780 = 272\right]$$

We consider the new divisor 420 and the new remainder 272 and apply division lemma to get

$$420 = 272 \times 1 + 148 \quad \text{...(iii)} \qquad \left[\because\ 272\overline{)420}^{\,1},\ \ 420 - 272 = 148\right]$$

Let us now consider the new divisor 272 and the new remainder 148 and apply division lemma to get

$$272 = 148 \times 1 + 124 \quad \text{...(iv)} \qquad \left[\because\ 148\overline{)420}^{\,1},\ \ 148,\ \ 124\right]$$