MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 20. Arithmetic Progressions

01. Arithmetic Progression

A sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called an arithmetic progression, if there exists a constant number d such that

$$\begin{aligned} a_2 &= a_1 + d \\ a_3 &= a_2 + d \\ a_4 &= a_3 + d \\ &\vdots \\ a_n &= a_{n-1} + d \text{ and so on.} \end{aligned}$$

The constant 'd' is called the common difference of the A.P.

Thus, if the first term is a and the common difference is d, then

$$a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ \ldots\ldots\ldots\ldots$$

is an arithmetic progression.

In other words, a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$ is called an arithmetic progression if the difference of a term and the preceding term is always constant. This constant is called the common difference of the A.P.

Thus, if $a_1, a_2, a_3, \ldots, a_n, \ldots$ is an A.P. with common difference 'd', then,

$$\begin{aligned} a_2 - a_1 &= d \\ a_3 - a_2 &= d \\ a_4 - a_3 &= d \\ &\vdots \\ a_n - a_{n-1} &= d \text{ and so on.} \end{aligned}$$

It follows from the above discussion that the sequence $a_1, a_2, a_3, \ldots, a_n, a_{n+1} \ldots$ is an A.P. with common difference 'd' if and only if

$$a_{n+1} - a_n = d \text{ for } n = 1, 2, 3, 4, \ldots$$

This suggests us the following algorithm to determine whether a sequence is an A.P. or not when we are given an algebraic formula for the general term of the sequence.

Algorithm

Step I. Obtain a_n

Step II. Replace n by $(n+1)$ is a_n to get a_{n+1}

Step III. Calculate $a_{n+1} - a_n$

Step IV. Check the value of $a_{n+1} - a_n$. If $a_{n+1} - a_n$ is independent of n, then the given sequence is an A.P. Otherwise it is not an A.P.

Example : Which of the following list of numbers form an AP? If they form an AP, write the next two terms.
(i) 1, −1, −3, −5, ...
(ii) 1, 1, 1, 2, 2, 2, 3, 3, 3, ...

Solution : (i) $a_2 - a_1 = -1 - 1 = -2$
$a_3 - a_2 = -3 - (-1) = -3 + 1 = -2$
$a_4 - a_3 = -5 - (-3) = -5 + 3 = -2$
i.e., $a_{k+1} - a_k$ is the same every time.
So, the given list of numbers forms an AP with the common difference $d = -2$. The next two terms are :
$-5 + (-2) = -7$ and $-7 + (-2) = -9$
(ii) $a_2 - a_1 = 1 - 1 = 0$
$a_3 - a_2 = 1 - 1 = 0$
$a_4 - a_3 = 2 - 1 = 0$
Here, $a_2 - a_1 = a_3 - a_2 \neq a_4 - a_3$.
So, the given list of numbers does not form an AP.

02. General Term of An A.P.

Result : Let a be the first term and d be the common difference of an A.P. Then, its nth term or general term is given by

$$a_n = a + (n - 1)\, d.$$

Remark : It is evident from the above theorem that
General term of an A.P. = First term + (Term number − 1) × (Common difference)

n^{th} Term of an A.P. from the end

Let there be an A.P. with first term a and common difference d. If there are m terms in the A.P., then

nth term from the end = $(m - n + 1)$th term the beginning
$\Rightarrow$ nth term from the end = a_{m-n+1}
$\Rightarrow$ nth term from the end = $a + (m - n + 1 - 1)\, d$
$\Rightarrow$ nth term from the end = $a + (m - n)\, d$

Also, if l is the last term of the A.P., then nth term from the end is the nth term of an A.P. whose first term is l and common difference is $-d$.
$\therefore$ nth term from the end = Last term + $(n - 1)(-d)$
$\Rightarrow$ nth term from the end = $l - (n - 1)\, d$

Middle Term(s) of a Finite A.P.

Let there be a finite A.P. with first them a, common difference d and number of terms n.

If n is odd, then $\left(\frac{n+1}{2}\right)^{th}$ term is the middle term and is given by $a+\left(\frac{n+1}{2}-1\right)d$.

If n is even, then $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ are middle terms given by $a+\left(\frac{n}{2}-1\right)d$ and $a+\left(\frac{n}{2}+1-1\right)d = a+\frac{n}{2}d$ respectively.