MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 17. Polynomials

01. Polynomial

Let x be a variable (literal), n be a positive integer and a_0, a_1, a_2, ..., a_n be constants (real numbers). Then, $a_n\, x_n + a_{n-1}\, x^{n-1} + a_{n-2}\, x^{n-2} + a_1\, x + a_0$ is known as a polynomial in variable x.
$f(x)$, $g(x)$, $h(x)$ etc. is used to denote a polynomial in variable x.

Example $f(x) = 2x^3 + 7x^2 - 4x + 15$ and $g(x) = 3x^4 + 7x^2 - 5$ are polynomials in variable x. However, $7x^3 - 2x^2 + 3\sqrt{x} - 4$ is not a polynomial as the exponent of x in $3\sqrt{x}$ is not positive integer. Also, $x^2 - x + \dfrac{2}{x}$ is not a polynomial in x.

Terms and their Coefficients
If $f(x) = a_n\, x^n + a_{n-1}\, x^{n-1} + a_{n-2}\, x^{n-2} + ... + a_1\, x + a_0$ is a polynomials in variable x, then $a_n\, x^n$, $a_{n-1}\, x^{n-1} + a_{n-2}\, x^{n-2}$, ... , $a_1\, x + a_0$ are known as their coefficients.
The coefficient a_n of the highest degree term is caleld the leading coefficient and a_0 is called the constant term.
In the polynomial $f(x) = 2x^2 - 7x + 8$, $2x^2$, $-7x$ and 8 are its terms and 2, –7 and 8 are coefficients of x^2, x and constant term respectively.

Example In the polynomial $g(x) = 3x^4 - 7x^2 + 2x - 3$, the coefficient of x^3 is 0 whereas the constant term is –3.

Degree of A Polynomial
The exponent of the highest degree term in a polynomial is known as its degree.
$f(x) = 4x^3 - 2x^2 + 8x - 21$ and $g(x) = 7x^2 - 3x + 12$ are polynomials of degree 3 and 2 respectively.

Remark $f(x) = a_n\, x_n + a_{n-1}\, x^{n-1} + a_{n-2}\, x^{n-2} + ... + a_{n-1}\, x + a_n$ is a polynomial of degree n, if $a_n \neq 0$.

Constant Polynomial
A polynomial of degree zero is called a constant polynomial.
For example, $f(x) = 2$, $g(x) = -12$, $h(y) = \dfrac{3}{2}$ etc. are constant polynomials.

The constant polynomial 0 or $f(x) = 0$ is called the zero polynomial. The degree of zero polynomial is not defined, because $f(x) = 0$, $g(x) = 0x$, $h(x) = 0x^2$, $p(x) = 0x^3$ $q(x) = 0x^{10}$ etc. are equal to zero polynomial.

Linear Polynomials A polynomial of degree one is called a linear polynomial.
For example, $f(x) = x - 12$, $g(x) = 12x$, $h(x) = -7x + 8$ are linear polynomials. In general, $f(x) = ax + b$, $a \neq 0$ is a linear polynomial.

Remark A linear polynomial may be a monomial or a binomial. Because, linear polynomial $f(x) = 7x - 15$ is a binomial whereas $g(x) = 3x$ is a monomial.

Quadratic Polynomials A polynomial of degree 2 is known as a quadratic polynomial.
For example, $f(x) = 2x^2 - 3x + 15$, $g(x) = \frac{3}{2}y^2 - 4y + \frac{11}{3}$ etc are quadratic polynomials.
In general, $f(x) = ax^2 + bx + c$, $a \neq 0$ is a quadratic polynomial.

Remark A quadratic polynomial may be a monomial or a binomial or a trinomial. Because, $f(x) = 7x^2$ is a monomial, $g(x) = 2x^2 + 3$ is a binomial and $h(x) = 3x^2 - 2x + 4$ is a trinomial.

Cubic Polynomials A polynomial of degree 3 is called a cubic polynomial.
For example, $f(x) = 12x^3 - 4x^2 + 7x - 6$, $g(x) = 7x^3 + 4x - 12$ etc are cubic polynomials.
Thus, $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$ is a cubic polynomial.

02. Zeros (Roots) of A Polynomial

Value of A Polynomial The value of a polynomial $f(x)$ at $x = \alpha$ is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.
The value of the quadratic polynomial $f(x) = 8x^2 - 3x + 7$ at $x = -1$ and $x = 2$ are given by

$$f(-1) = 8 \times (-1)^2 - 3 \times (-1) + 7 = 8 + 3 + 7 = 18$$

and, $$f(2) = 8 \times (2)^2 - 3 \times 2 + 7 = 32 - 6 + 7 = 33$$

Zero or Root A real number α is a root or zero of a polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0, \text{ if } f(\alpha) = 0$$

i.e., $$a_n \alpha^n + a_{n-1} \alpha^{n-1} + a_{n-2} \alpha^{n-2} + \ldots + a_1 \alpha + a_0 = 0$$

We observe that
$x = 2$ is a root of the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$, because

$$f(2) = 2^3 - 6 \times 2^2 + 11 \times 2 - 6 = 8 - 24 + 22 - 6 = 0$$

$x = -1$ is not a root of this polynomial, because

$$f(-1) = (-1)^3 - 6 \times (-1)^2 + 11 \times -1 - 6 = -1 - 6 - 11 - 6 = -24 \neq 0$$