MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 19. Quadratic Equations

01. Quadratic Equations

Quadratic Equation If $p(x)$ is a quadratic polynomial, then $p(x) = 0$ is called a quadratic equation.

Roots of A Quadratic Equation Let $p(x) = 0$ be a quadratic equation, then the zeros of the polynomial $p(x)$ are called the roots of the equation $p(x) = 0$ be quadratic equation, then the zeros of Thus, $x = \alpha$ is a roots of $p(x) = 0$ if and only if $p(\alpha) = 0$.

Finding the roots of a quadratic equation is known as solving the quadratic equation.

Example I Which of the following are quadratic equations?

(i) $x2 - 6x + 4 = 0$

Solution Let $p(x) = x^2 - 6x + 4$

Clearly, $p(x) = x^2 - 6x + 4$ is a quadratic polynomial. Therefore, $x^2 - 6x + 4 = 0$ is a quadratic equation.

(ii) $x + \dfrac{3}{x} = x^2$

Solution We have,

$$x + \frac{3}{x} = x^2 \Rightarrow \frac{x^2 + 3}{x} = x^2 \Rightarrow x^2 + 3 = x^3 \Rightarrow x^3 - x^2 - 3 = 0$$

Clearly, $x^3 - x^2 - 3$, being a polynomial of degree 3, is not a quadratic polynomial. So, the given equation is not a quadratic equation.

02. Formulation of Quadratic Equations

Following examples will illustrate the formulation of quadratic equations.

Example I The product of two consecutive positive integers is 240. Formulate the quadratic equation whose roots are these integers.

Solution Let two consecutive positive integers be x and $x + 1$. Then, their product is $x(x + 1)$.

It is given that the product is 240.

$\therefore \quad x(x + 1) = 240 \Rightarrow x^2 + x - 240 = 0$

This is the required quadratic equation.

Example II The are of a rectangular plot is 528 m^2. The length of the plot (in meters) is one more than twice its breadth. Formulate the quadratic equation to determine the length and breadth of the plot.

Solution Let the breadth of the plot be x meters.

It is given that the length of the plot is one more than twice its breadth.

$\therefore$ Length = $(2x + 1)$ meters

Now, Area of the plot = 528 m^2

$\Rightarrow$ Length × Breadth = 528 m^2

$\Rightarrow (2x + 1) \times x = 528 \Rightarrow 2x^2 + x - 528 = 0$

This is the required quadratic equation.

03. Solution of A Quadratic Equation By Factorization Method

Example I Solve the following quadratic equations by factorization:

(i) $x^2 + 6x + 5 = 0$ (ii) $8x^2 - 22x - 21 = 0$ (iii) $9x^2 - 3x - 2 = 0$

Solution (i) We have,

$x^2 + 6x + 5 = 0$

$\Rightarrow x^2 + 5x + x + 5 = 0$

$\Rightarrow x(x + 5) + (x + 5) = 0$

$\Rightarrow (x + 5)(x + 1) = 0 \Rightarrow x + 5 = 0$ or, $x + 1 = 0 \Rightarrow x = -5$ or, $x = -1$

Thus, $x = -5$ and $x = -1$ are two roots of the equation $x^2 + 6x + 5 = 0$

(ii) We have,

$8x^2 - 22x - 21 = 0$

$\Rightarrow 8x^2 - 28x + 6x - 21 = 0$

$\Rightarrow 4x(2x - 7) + 3(2x - 7) = 0$

$\Rightarrow (2x - 7)(4x + 3) = 0 \Rightarrow 2x - 7 = 0$ or, $4x + 3 = 0 \Rightarrow x = \frac{7}{2}$ or,

$x = -\frac{3}{4}$

Thus, $x = \frac{7}{2}$ and $x = -\frac{3}{4}$ are two roots of the equation $8x^2 - 22x - 21 = 0$

(iii) We have,

$9x^2 - 3x - 2 = 0$

$\Rightarrow 9x^2 - 6x + 3x - 2 = 0$

$\Rightarrow 3x(3x - 2) + (3x - 2) = 0$

$\Rightarrow (3x - 2)(3x + 1) = 0$

$\Rightarrow 3x - 2 = 0$ or, $3x + 1 = 0 \Rightarrow x = \frac{2}{3}$ or, $x = -\frac{1}{3}$

Thus, $x = \frac{2}{3}$ and $x = -\frac{1}{3}$ are two roots of the equation $9x^2 - 3x - 2 = 0$

Algorithm

Step I Factorize the constant term of the given quadratic equation.

Step II Express the coefficient of middle term as the sum or difference of the factors obtained in Step I. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step III Split the middle term in two parts obtained in step II.

Step IV Factorize the quadratic equation obtained in step III by grouping method.

04. Solution of A Quadratic Equation by Completing The Square

Algorithm

Step I Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0$, $a \neq 0$.

Step II Make the coefficient of x^2 unity by diving throughout by it, if it is not unity. i.e., obtain $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Step III Shift the constant term $\frac{c}{a}$ on RHS to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

Step IV Add square of half of the coefficient of x i.e. $\left(\frac{b}{2a}\right)^2$ on both sides to obtain

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Step V Write LHS as the perfect square of a binomial expression and simplify RHS to get $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Step VI Take square root of both sides to get $x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

Step VII Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on RHS.

Example I Solve the quadratic equation $9x^2 - 15x + 6 = 0$ by the method of completing the square.

Solution We have, $9x^2 - 15x + 6 = 0$

$\Rightarrow \quad x^2 - \frac{15}{9}x + \frac{6}{9} = 0$ [Dividing throughout by 9]

$\Rightarrow \quad x^2 - \frac{5}{3}x + \frac{2}{3} = 0$

$\Rightarrow \quad x^2 - \frac{5}{3}x = -\frac{2}{3}$ [Shifting the constant term on RHS]

$\Rightarrow \quad x^2 - 2\left(\frac{5}{6}\right)x + \left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2 - \frac{2}{3}$ [Adding Square of half of coefficient of x on both sides]