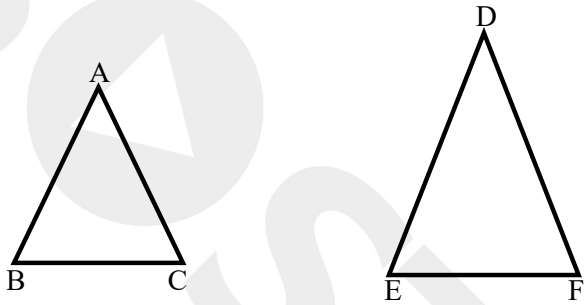
MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 21. Triangles

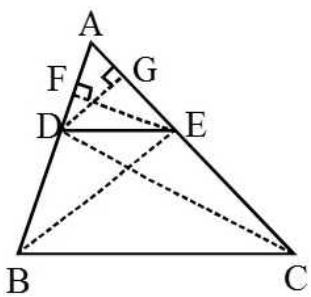
01. Similar Triangles and Their Properties

Definition : Two triangles are said to be similar, if their
(i) corresponding angles are equal and,
(ii) corresponding sides are proportional.
It follows from this definition that two triangles ABC and DEF are similar, if
(i) $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ and (ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



02. Some Basic Results on Proportionality

Result 1 : Basic proportionality Theorem or Thales Theorem. If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.
Given A triangle ABC in which DE || BC, and intersects AB in D and AC in E.
To prove $\frac{AD}{DB} = \frac{AE}{EC}$
Construction Join BE, CD and draw EF $\perp$ BA and DG $\perp$ CA.
Proof : Since EF is perpendicular to AB. Therefore, EF is the height of triangles ADE and DBE.



Now, Area (ΔADE) = $\frac{1}{2}$(base × height) = $\frac{1}{2}$(AD. EF)

and, Area (ΔDBE) = $\frac{1}{2}$(base × height) = $\frac{1}{2}$(DB. EF)

$$\therefore \quad \frac{Area(\triangle ADE)}{Area(\triangle DBE)} = \frac{\frac{1}{2}(AD.EF)}{\frac{1}{2}(DB.EF)} = \frac{AD}{DB} \qquad ...(i)$$

Similarly, we have

$$\frac{Area(\triangle ADE)}{Area(\triangle DEC)} = \frac{\frac{1}{2}(AE.DG)}{\frac{1}{2}(EC.DG)} = \frac{AE}{EC} \qquad ...(ii)$$

But, ΔDBE and ΔDEC are on the same base DE and between the same parallels DE and BC.

∴ Area(ΔDBE) = Area(ΔDEC)

$$\Rightarrow \quad \frac{1}{Area(\triangle DBE)} = \frac{1}{Area(\triangle DEC)}$$

$$\Rightarrow \quad \frac{Area(\triangle ADE)}{Area(\triangle DBE)} = \frac{Area(\triangle ADE)}{Area(\triangle DEC)}$$

$$\Rightarrow \quad \frac{AD}{AB} = \frac{AE}{EC}$$

Corollary If in *a* ΔABC, *a* line DE ∥ BC, intersects AB in D and AC in E, then :

(i) $\frac{AB}{AD} = \frac{AC}{AE}$

(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

Proof (i) From the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{DB}{AD} = \frac{EC}{AE}$$

$$\Rightarrow \quad 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\Rightarrow \quad \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

$$\Rightarrow \quad \frac{AB}{AD} = \frac{AC}{AE}$$

(ii) From the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \quad \frac{AD + DB}{DB} = \frac{AE + EC}{EC}$$

$$\Rightarrow \quad \frac{AB}{DB} = \frac{AC}{EC}$$

The above results can be summarised as follows :

Summary : If in a ΔABC, DE ∥ BC, and intersects AB in D an AC in E, then we have

(i) $\frac{AD}{DB}=\frac{AE}{EC}$ (iii) $\frac{AB}{AD}=\frac{AC}{AE}$ (v) $\frac{AB}{DB}=\frac{AC}{EC}$

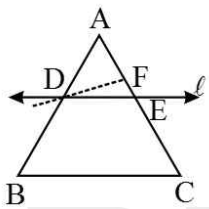
(ii) $\frac{DB}{AD}=\frac{EC}{AE}$ (iv) $\frac{AD}{AB}=\frac{AE}{AC}$ (vi) $\frac{DB}{AB}=\frac{EC}{AC}$

Result 2 : Converse of Basic Proportionality Theorem. If a line divides any two sides of a triangle in the same ratio, then the line must be paralled to the third side.

Given A ΔABC and a line/intersecting AB in D and AC in E, such that $\frac{AD}{DB}=\frac{AE}{EC}$.

To Prove l : BC i.e. DE ∥ BC

Proof : If possible, let DE be not parallel to BC. Then, there must be another line parallel to BC. Let DF ∥ BC.



Since DF ∥ BC. Therefore, from Basic Proportionality Theorem, we get

$$\frac{AD}{DB}=\frac{AF}{FC} \quad ...(i)$$

But, $\frac{AD}{DB}=\frac{AE}{EC}$ (Given) ...(ii)

From (i) and (ii), we get

$$\frac{AF}{FC}=\frac{AE}{EC}$$

$$\Rightarrow \quad \frac{AF}{FC}+1=\frac{AE}{EC}+1 \quad \text{[Adding 1 on both sides]}$$

$$\Rightarrow \quad \frac{AF+FC}{FC}=\frac{AE+EC}{EC}$$

$$\Rightarrow \quad \frac{AC}{FC}=\frac{AC}{EC}$$

$$\Rightarrow \quad FC=EC$$

This is possible only when F and E coincide i.e., DF is the line l itself. But, DF ∥ BC. Hence l ∥ BC.