MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 22. Coordinate Geometry

01. Distance Formulae

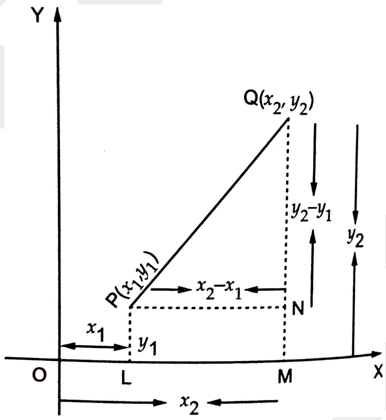
The distance between any two points in the plane is the length of the line segment joining them.

Result The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e. $$PQ = \sqrt{(\text{Difference of abscissae})^2 + (\text{Difference of ordinates})^2}$$

Solution Let $X'OX$ and $Y'OY$ be the coordinate axes. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two given points in the plane. Draw PL and QM perpendicular from P and Q on x-axis. From P draw PN perpendicular to QM. Then,



$OL = x_1$, $OM = x_2$, $PL = y_1$ and $QM = y_2$

$\therefore \quad PN = LM = OM - OL = x_2 - x_1$

and, $QN = QM - NM = QM - PL = y_2 - y_1$

Clearly, ΔPNQ is a right triangle right angled at N. Therefore, by Pythogoras theorem, we have

$$PQ^2 = PN^2 + QN^2$$

$$\Rightarrow \quad PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Hence, distance between any two points is given by

$$\sqrt{(\text{Difference of abscissae})^2 + (\text{Difference of ordinates})^2}$$

Example Show that the points (1, 7), (4, 2), C(–1, –1) and (–4, 4) are the vertices of a square.

Solution Let $A(1, 7)$, $B(4, 2)$, $C(-1, -1)$ and $D(-4, 4)$ be the given points. One way of showing that $ABCD$ is a square is to use the property that all its sides should be equal and both its digonals should also be equal. Now,

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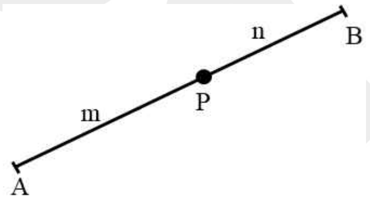
$$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$$
$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$
$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$
$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$
$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$
$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

Since, $AB = BC = CD = DA$ and $AC = BD$, all the four sides of the quadrilateral $ABCD$ are equal and its diagonals AC and BD are also equal. Therefore, $ABCD$ is a square.

NOTE ☞ If O is the origin and $P(x, y)$ is any point, then form the above formula, we have $OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

02. Section Formulae

Let A and B be two point in the plane of the paper as shown in Figure-I and P be a point on the segment joining A and B such that $AP : BP = m : n$. Then, we say that the point P divides segment AB internally in the ratio $m : n$.



Figure

Example In what ratio does the point (– 4, 6) divide the line segment joining the points $A(-6, 10)$ and $B(3, -8)$?

Solution Let (– 4, 6) divide AB internally in the ratio $m_1 : m_2$. Using the section formula, we get

$$(-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right)$$

Recall that if $(x, y) = (a, b)$ then $x = a$ and $y = b$.

So, $-4 = \dfrac{3m_1 - 6m_2}{m_1 + m_2}$ and $6 = \dfrac{-8m_1 + 10m_2}{m_1 + m_2}$

Now, $-4 = \dfrac{3m_1 - 6m_2}{m_1 + m_2}$ gives us

$-4m_1 - 4m_2 = 3m_1 - 6m_2$

i.e., $7m_1 = 2m_2$

i.e., $m_1 : m_2 = 2 : 7$

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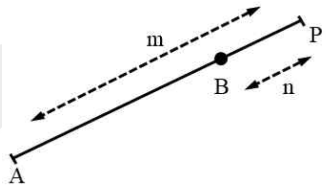
You should verify that the ratio satisfies the y-coordinate also.

Now, $$\frac{-8m_1 + 10m_2}{m_1 + m_2} = \frac{-8\dfrac{m_1}{m_2} + 10}{\dfrac{m_1}{m_2} + 1}$$ (Dividing throughout by m_2)

$$= \frac{-8 \times \dfrac{2}{7} + 10}{\dfrac{2}{7} + 1} = 6$$

Therefore, the point (–4, 6) divides the line segment joining the points A(–6, 10) and B(3, –8) in the ratio 2 : 7.

If P is a point on AB produced such that $AP : BP = m : n$, then point P is said to divide AB externally in the ratio $m : n$.



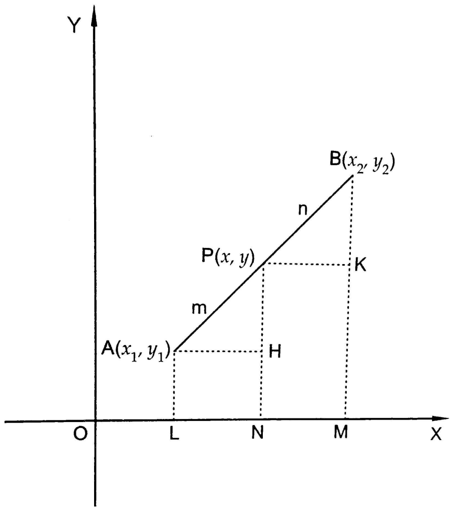
Figure

In this section, we shall develop a formula, generally known as section formula, for finding the coordinates of P when we are given the coordinates of A and B and the ratio in which P divides AB internally.

Result I Prove that the coordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio $m : n$ are given by

$$\left(x = \frac{mx_2 + nx_1}{m+n}, y\frac{my_2 + ny_1}{m+n}\right)$$

Proof Let O be the origin and let OX and OY be the x-axis and y-axis respectively. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the given points. Let (x, y) be the coordinates of the point P which divides AB internally in the ratio $m : n$. Draw $AL \perp OX$, $BM \perp OX$, $PN \perp OX$. Also, draw AH and PK perpendiculars from A and P on PN and BM respectively. Then,



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