## MATHEMATICS

## CLASS NOTES FOR CBSE

## Chapter 23. Introduction to Trigonometry

## 01. Trigonometric Ratios



Consider an acute angle $\angle \mathrm{YAX}=\theta$ with initial side AX and terminal side AY . Let P be any point on the terminal side AY. Draw PM perpendicular from P on AX to get the right angled triangle AMP in which $\angle \mathrm{PAM}=\theta$.
In right angled triangle AMP , Base $=\mathrm{AM}=x$, Perpendicular $=\mathrm{PM}=y$, and Hypotenuse $=$ $\mathrm{AP}=r$.
We define the following six trigonometric ratios
(i) $\sin \theta=\frac{\text { Perpendicular }}{\text { Hypotenuse }}=\frac{\mathrm{y}}{\mathrm{r}}$, and is written as $\sin \theta$
(ii) $\cos \sin \theta=\frac{\text { Base }}{\text { Hypotenuse }}=\frac{\mathrm{x}}{\mathrm{r}}$, and is written as $\cos \theta$
(iii) $\operatorname{Tangent} \theta=\frac{\text { Perpendicular }}{\text { Base }}=\frac{\mathrm{y}}{\mathrm{x}}$, and is written as $\tan \theta$
(iv) $\operatorname{Cosecant} \theta=\frac{\text { hypotenuse }}{\text { Perpendicular }}=\frac{\mathrm{r}}{\mathrm{y}}$, and is written as $\operatorname{cossec} \theta$
(v) $\operatorname{Secant} \theta=\frac{\text { Hypotenuse }}{\text { Base }}=\frac{\mathrm{r}}{\mathrm{x}}$, and is written as $\sec \theta$
(vi) $\operatorname{Cotangent} \theta=\frac{\text { Base }}{\text { Perpendicular }}=\frac{x}{y}$, and is written as $\cot \theta$

Remark : It is clear from the definitions of the trigonometric ratios that for any acute angle $\theta$. we have
(i) $\operatorname{cosec} \theta=\frac{1}{\sin \theta}$ or, $\sin \theta=\frac{1}{\operatorname{cosec} \theta}$
(ii) $\sec \theta=\frac{1}{\cos \theta}$ or, $\cos \theta=\frac{1}{\sec \theta}$
(iii) $\cot \theta=\frac{1}{\tan \theta}$ or, $\tan \theta=\frac{1}{\cot \theta}$
(iv) $\cot \theta=\frac{\cos \theta}{\sin \theta}$
(v) $\tan \theta \cdot \cot \theta=1$

Example : In a $\triangle \mathrm{ABC}$, right angled at B , if $\mathrm{AB}=12$ and $\mathrm{BC}=5$. find:
(i) $\sin \mathrm{A}$ and $\tan \mathrm{A}$
(ii) $\quad \cos C$ and $\cot C$

Solution : By pythagoras theorem, we have

(i) When we consider $t$-ratios of $\angle \mathrm{A}$, we have

$$
\text { Base }=\mathrm{AB}=12, \text { Perpendicular }=\mathrm{AB}=12 \text { and Hypotenuse }=\mathrm{AC}=13
$$

$\therefore \quad \sin A=\frac{\text { Perpendicular }}{\text { Hypotenuse }}=\frac{5}{13}$ and, $\tan A=\frac{\text { Perpendicular }}{\text { Base }}=\frac{5}{12}$
(ii) When we consider $t$-ratios of $\angle \mathrm{C}$, we have

Base $=\mathrm{BC}=5$, Perpendicular $=\mathrm{AB}=12$ and Hypotenuse $=\mathrm{AC}=13$
$\therefore \quad \cos C=\frac{\text { Base }}{\text { Hypotenuse }}=\frac{5}{13}$ and, $\cot C=\frac{\text { Base }}{\text { Perpendicular }}=\frac{5}{12}$

## 02. Trigonometric Ratios of Some Specific Angles

## Trigonometric Ratios of $0^{\circ}$ and $90^{\circ}$ :

Let $\angle \mathrm{XAY}=\theta$ be an acute angle and let P be a point on its terminal side AY. Draw perpendicular PM from P on AX .


In $\triangle \mathrm{AMP}$, we have

$$
\sin \theta=\frac{P M}{A P}, \cos \theta=\frac{A M}{A P} \text { and } \tan \theta=\frac{P M}{A M}
$$

It is evident from $\triangle \mathrm{AMP}$ that as $\theta$ becomes smaller and smaller, line segment PM also becomes smaller and smaller; and finally when $\theta$ becomes $0^{\circ}$; the point P will coincide with M. Consequently, we have
$\mathrm{PM}=0$ and $\mathrm{AP}=\mathrm{AM}$
$\therefore \quad \sin 0^{\circ}=\frac{P M}{A P}=\frac{0}{A P}=0, \quad \cos 0^{\circ}=\frac{A M}{A P}=\frac{A P}{A P}=1$

