MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 07. Triangles

01. Congruence of Triangles

Two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.

$\therefore$ *Two triangles are congruent if and only if there axists a correspondence between their vertices such that the corresponding sides and the corresponding angles of the two triangles are equal or congruent.*

If ΔABC is congruent to ΔDEF and the correspondence $ABC \leftrightarrow DEF$ makes the six pairs of corresponding parts of the two triangles congruent, then we write

$$\Delta ABC \cong \Delta DEF$$

Thus, $\Delta ABC \cong \Delta DEF$ if and only if $AB = DE$, $BC = EF$, $CA = FD$, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

NOTE 1 *In the subsequent discussion the order of the letters in the names of two triangles will indicate the correspondence between the vertices of two triangles. For Example,* $\Delta ABC \cong \Delta DEF$ *will indicate the correspondence* $ABC \leftrightarrow DEF$ *and* $\Delta ABC \cong \Delta DFE$ *will indicate the correspondence* $ABC \leftrightarrow DEF$*. Thus, we can easily infer the six equalities between the corresponding parts of two triangles from the notation* $\Delta ABC \cong \Delta DEF$*. We shall use the abbreviation "c.p.c.t" to indicate corresponding parts of congruent triangles.*

NOTE 2 *Note that* $\Delta PQR \cong \Delta VW$ *will mean that*

$$\angle P = \angle U,\ \angle Q = \angle V,\ \angle R = \angle W,\ PQ = UV,\ QR = VW \text{ and } PR = UW.$$

(i) **Congruence Relation :**

From the definition of congruence of two triangles, we obtain the following results :

(a) Every triangle is congruent to itself i.e., $\Delta ABC \cong \Delta ABC$

(b) If $\Delta ABC \cong \Delta DEF$, then $\Delta DEF \cong \Delta ABC$

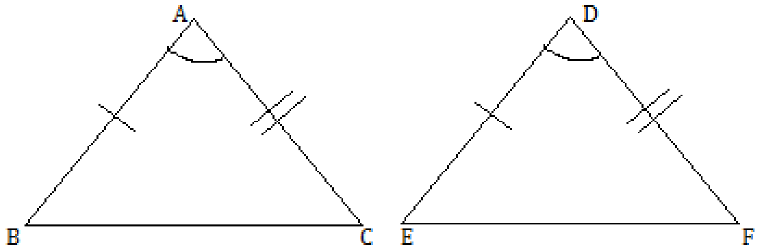
(c) If $\Delta DEF \cong \Delta ABC$, and $\Delta DEF \cong \Delta PQR$, then $\Delta ABC \cong \Delta PQR$

02. Sufficient Conditions (Criteria) for Congruence of Triangles

Side-Angle-Side (SAS) Congruence Criterion :

Result *Two triangles are congruent if two sides and the included angle of one are equal to the corresponding sides and the include angle of the other triangle.*

Given : Two triangles ABC and DEF such that $AB = DE$, $AC = DF$ and $\angle A = \angle D$



(i) (ii)

Figure

To Prove : $\Delta ABC \cong \Delta DEF$

Proof : Place ΔABC over ΔDEF such that the side AB falls on side DE, vertex A falls on vertex D and B on E. Since $\angle A = \angle D$. therefore, AC will fall on DF. But $AC = DF$ and A falls on D. Therefore, C will fall on F. Therefore, BC coincides with DF.

Now, B falls on E and C falls on F. Therefore, BC coincides with EF.

Thus, ΔABC when superposed on ΔDEF, covers in exactly. Hence, by definition of congruence, $\Delta ABC \cong \Delta DEF$.

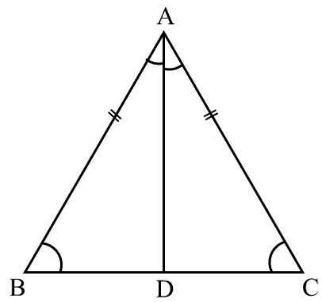
NOTE *It shall be noted that in SAS criterion the equality of included angles is very essential. If two sides and one angle (not included between the two sides) of one triangle are equal to two sides and one angle of the other triangle, then the triangles need not be congruent. So, the equal angle should be the angle included between the sides.*

Result *Angles opposite to two equal sides of a triangle are equal.*

Given : ΔABC in which $AB = AC$

To Prove : $\angle C = \angle B$

Construction : Draw the bisector AD of $\angle A$ which meets BC in D.



Figure

Proof : In Δs ABD and ACD, we have

$AB = AC$ [Given]

$\angle BAD = \angle CAD$ [By construction]

$AD = AD$ [Common side]

Therefore, by SAS criterion of congruence, we have

$\Delta ABD \cong \Delta ACD$

$\Rightarrow$ $\angle B = \angle C$ [Corresponding parts of congruent triangles are equal]