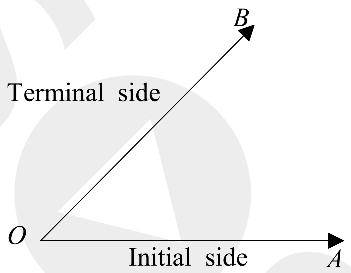
MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 03. Trigonometric Functions

01. Angle

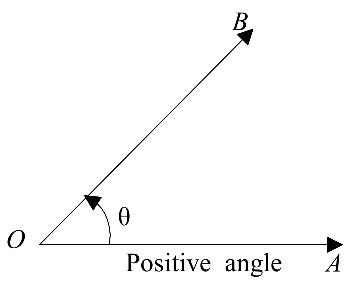
Consider a ray $\overrightarrow{OA}$. If this ray rotates about its end point O and takes the position OB, then we say that the angle $\angle AOB$ has been generated.
An angle is considered as the figure obtained by rotating a given ray about its end-point.

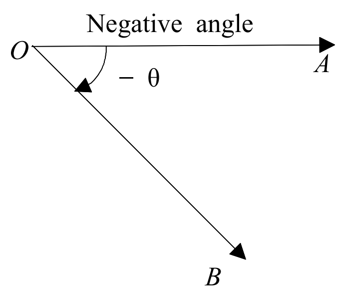


The revolving ray is called the generating line of the angle. The initial position OA is called the initial side and the final position OB is called terminal side of the angle. The end point O about which the ray rotates is called the vertex of the angle.

MEASURE OF AN ANGLE
The measure of an angle is the amount of rotation from the initial side to the terminal side.

SENSE OF AN ANGLE
The sense of an angle is said to be positive or negative according as the initial side rotates in anticlockwise or clockwise direction to get to the terminal side.





RIGHT ANGLE

If the revolving ray starting from its initial position to final position describes one quarter of a circle, then we say that the measure of the angle formed is a right angle.

02. Systems of Measurement of Angles

SEXAGESIMAL SYSTEM

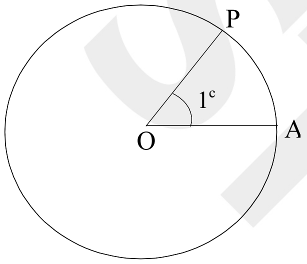
There, 1 right angle = 90 degrees $(= 90°)$
$1° = 60$ minutes $(= 60')$
$1' = 60$ seconds $(= 60'')$

CENTESIMAL SYSTEM

There, 1 right angle = 100 grades $(= 100^g)$
1 grade = 100 minutes $(= 100')$
1 minute = 100 seconds $(= 100'')$

CIRCULAR SYSTEM

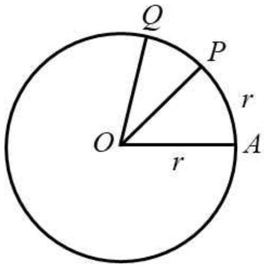
In this system the unit of measurement of radian where one radian, written as 1^c, is the measure of an angle subtended at the centre of a circle by an arc of length equal to the radius of the circle.



RESULT

The number of radians in an angle subtended by an arc of a circle at the center is equal to $\frac{\text{arc}}{\text{radius}}$.

Proof Consider a circle with centre O and radius r. Let $\angle AOQ = \theta^c$ and let arc $AQ = s$.
Let P be a point on the arc AQ such that arc $AP = r$.
Then, $\angle AOP = 1^c$.



Since angles at the centre of a circle are proportional to the arcs subtending them. Therefore,

$$\frac{\angle AOQ}{\angle AOP} = \frac{\text{arc}\, AQ}{\text{arc}\, AP}$$

$$\Rightarrow \qquad \angle AOQ = \left(\frac{\text{arc}\, AQ}{\text{arc}\, AP} \times 1\right)^c \qquad [\because \ \angle AOP = 1^c]$$

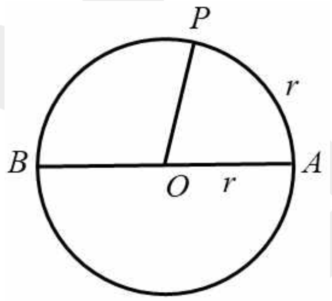
$$\Rightarrow \qquad \theta = \frac{s}{r} \text{ radians.}$$

03. Relation Between Degrees and Radians

Consider a circle with centre O and radius r. Let A be a point on the circle. Join OA and cut off an arc OP of length equal to the radius of the circle. Then, $\angle AOP = 1$ radian. Produce AO to meet the circle at B.

$\therefore \qquad \angle AOB$ = a straight angle = 2 right angles

We know that the angles at the centre of a circle are proportional to the arcs subtending them.



$$\therefore \qquad \frac{\angle AOP}{\angle AOC} = \frac{\text{arc}\, AP}{\text{arc}\, APB}$$

$$\Rightarrow \qquad \frac{\angle AOP}{2 \ right \ angles} = \frac{r}{\pi r} \qquad \left[\because \ \text{arc } APB = \frac{1}{2} \text{ (circumference)}\right]$$

$$\Rightarrow \qquad \angle AOP = \frac{2 \, right \ angles}{\pi}$$

$$\Rightarrow \qquad 1^c \, \frac{180^\circ}{\pi}$$

Hence, Once radian $= \dfrac{180^\circ}{\pi} \Rightarrow \pi$ radians $= 180^\circ$.

REMARK (1) Since $180^\circ = \pi$ radians. Therefore, $1^\circ = \pi/180$ radian.

Hence, $30^\circ = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$ radians, $45^\circ = \frac{\pi}{180} \times 45 = \frac{\pi}{4}$ radians,

$60^\circ = \frac{\pi}{180} \times 60 = \frac{\pi}{3}$ radians, $90^\circ = \frac{\pi}{180} \times 90 = \frac{\pi}{2}$ radians etc,