MATHEMATICS

CLASS NOTES FOR CBSE

Chapter 08. Binomial Expansion

01. Binomial Theorem

Theorem *If x and a are real numbers , then for all $n \in N$,*

$$(x+a)^n = {}^nC_0 x^n a^0 + {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 + \ldots\ldots$$

$$+ \ldots + {}^nC_r x^{n-r} a^r + \ldots + {}^nC_{n-1} x^1 a^{n-1} + {}^nC_n x^0 a^n$$

$$i.e. \quad (x+a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r$$

Remark *The above expansion is also valid when x and a are complex numbers.*

Properties of Binomial Expansion

Property I *We have,*

$$(x+a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r} a^r$$

Since r can have values from 0 to n, the total number of terms in the expansion is (n + 1).

Property II *The sum of indices of x and a in each term is n.*

Property III *We have,*

$${}^nC_r = {}^nC_{n-r'} \qquad r = 0,1,2,\ldots,n$$

$$\Rightarrow {}^nC_0 = {}^nC_n, \; {}^nC_1 = {}^nC_{n-1}, \; {}^nC_2 = {}^nC_{n-2} = \ldots$$

So, the coefficients of terms equidistant from the beginning and the end are equal. These coefficients are known as the binomial coeffi-cients.

Property IV *Replacing a by $-a$, in expansion of $(x+a)^n$, we get*

$$(x-a)^n = {}^nC_0 x^n a^0 - {}^nC_1 x^{n-1} a^1 + {}^nC_2 x^{n-2} a^2 - {}^nC_3 x^{n-3} a^3$$

$$\ldots\ldots + \ldots + (-1)^r {}^nC_r x^{n-r} a^r + \ldots + (-1)^n {}^nC_n x^0 a^n.$$

$$\text{i.e.,} \qquad (x-a)^n = \sum_{r=0}^{n} (-1)^r {}^nC_r x^{n-r} a^r$$

Property V Putting $x = 1$ and $a = x$ in the expansion of $(x+a)^n$, we get

$$(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots + {}^nC_r x^r + \ldots + {}^nC_n x^n$$

$$\text{i.e.,} \qquad (1+x)^n = \sum_{r=0}^{n} {}^nC_r x^r$$

This is the expansion of $(1+x)^n$ in ascending powers of x.