MATHEMATICS

CLASS NOTES FOR CBSE

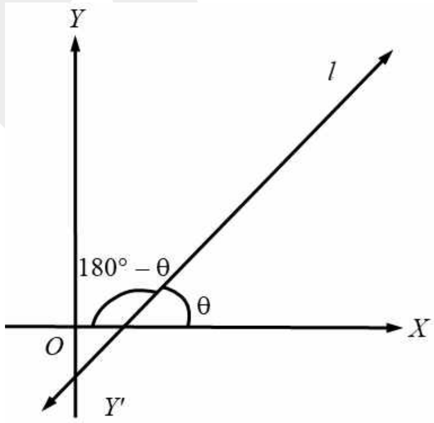
Chapter 10. Straight Lines

01. Straight Lines

Every first degree equation in x, y represent a straight line. so, $ax + by + c = 0$ is the general equation of a line.

(i) **Slope (Gradient) of a Line**

A line in a coordinate plane forms two angles with the x-axis, which are supplementary. The angle (say) θ made by the line l with positive direction of x-axis and measure anti clockwise is called the ***inclination of the line***. Obeviously $0° \leq \theta$ θ $180°$. (Figure)



Figure

NOTE 1☞ Lines parallel *to* x-axis, or coinciding with x-axis, have inclination of 0°.

NOTE 2☞ Inclination of a vertical line (parallel to or coinciding with y-axis) is 90°.

The trigonometrical tangent of the inclination of line l is called the slope or gradient of the line l.

The slope of a line is generally denoted by m.

NOTE 1☞ The slope of a line whose inclination is 90° is not defined.

NOTE 2☞ The slope of a line whose inclination is 0° is $\tan 0° = 0$.

NOTE 3☞ The slope of x-axis is zero and slope of y-axis is not defined.

(ii) ***Slope of a line when coordinates of any two points on the line are given***

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on non-vertical line l whose inclination is θ. Obviously, $x_1 \neq x_2$, otherwise the line will become perpendicular to x-axis and its slope will not be defined. The inclination of the line l may be acute or obtuse. Let us take these tow cases.

Draw perpendicular QR to x-axis and PM perpendicular to RQ as shown in Figure (i) (ii).

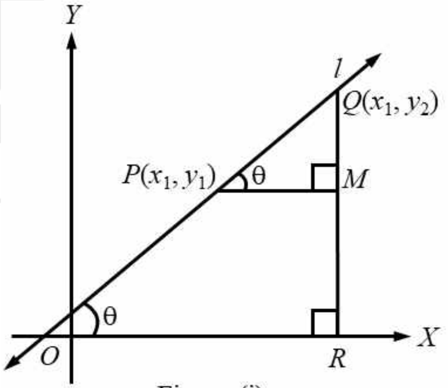


Figure (i)

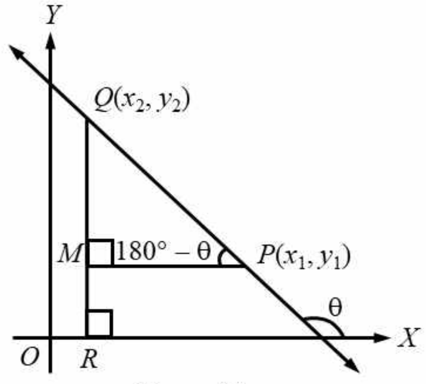


Figure (ii)

Case I When angle θ is acute :

in Figure (i), ∠MPQ = θ.

Therefore, slope of line $l = m = \tan\theta$. ...(i)

But in ΔMPQ, we have $\tan\theta = \dfrac{MQ}{MP} = \dfrac{y_2 - y_1}{x_2 - x_1}$ ...(ii)

From equations (i) and (ii), we have $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Case II When angle θ is obtuse :

In Figure (ii), we have

∠MPQ = 180° – θ.

Therefore, θ = 180° – ∠MPQ.

NOw, slope of the line l

$$m = \tan\theta$$
$$= \tan(180° - \angle MPQ) = -\tan\angle MPQ$$
$$= -\frac{MQ}{MP} = -\frac{y_2 - y_1}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Consequently, we see that in both the cases the slope m of the line through the points (x_1, y_1) and (x_2, y_2) is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

(iii) ***Conditions for parallelism and perpendicularity of lines in terms of their slopes***

In a coordinate plane, suppose that non-vertical lines l_1 and l_2 have slopes m_1 and m_2, respectively. Let their inclinations be α and β, respectively.

If the line l_1 is parallel to l_2 (Figure), then their inclinations are equal, i.e.,

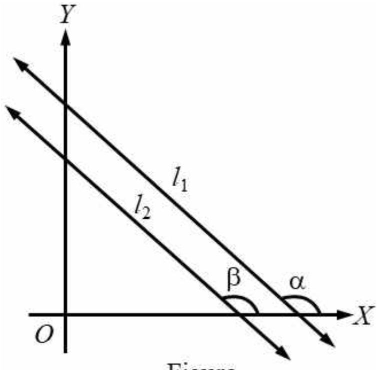
α = β, and hence, tan α = tan β

Therefore m_1 and m_2, i.e., their slopes are equal.

Conversely, if the slope of two lines l_1 and l_2 is same, i.e.,

$$m_1 = m_2.$$

Then $\tan \alpha = \tan \beta.$



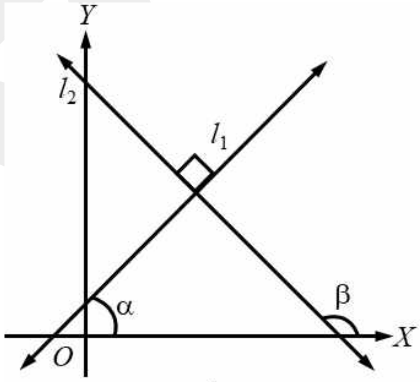
Figure

By the property of tangent function (between 0° and 180°), $\alpha = \beta$.

Therefore, the lines are parallel.

Hence, two non vertical lines l_1 and l_2 are parallel if and only if their slopes are equal.

If the lines, l_1 and l_2 are perpendicular (Figure), then $\beta = \alpha + 90°$.



Figure

Therefore, $\tan \beta = \tan (\alpha + 90°)$

$$= -\cot \alpha = -\frac{1}{\tan \alpha}$$

i.e., $m_2 = -\dfrac{1}{m_1}$ or $m_1 m_2 = -1$

Conversely, if $m_1 m_2 = -1$, i.e., $\tan \alpha \tan \beta = -1$.

Then $\tan \alpha = -\cot \beta = \tan (\beta + 90°)$ or $\tan (\beta - 90°)$

Therefore, α and β differ by 90°.

Thus, lines l_1 and l_2 are perpendicular to each other.

Hence, two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other,

i.e., $m_2 = -\dfrac{1}{m_1}$ or $m_1 m_2 = -1$