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Complete MATH

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CLASS 11&12th

CLASS 12th

Vector Algebra

01. Representation of Vectors

Vectors are represented by directed line segments such that the length of the line segment is the magnitude of the vector and the direction of arrow marked at one end emphasizes the direction of the vector. A vector, denoted by $\overrightarrow{PQ}$, is determined by two points P, Q such that the magnitude of the vector is the length of the straight line PQ and its direction is that from P to Q. The point P is called the *initial point* of vector $\overrightarrow{PQ}$ and Q is called the *terminal point or tip*. Vectors are generally denoted by $\vec{a}, \vec{b}, \vec{c}$ etc.

Figure

NOTE ☞ *Every vector $\overrightarrow{PQ}$ has the following three characteristics:*

Length *The length of* $\overrightarrow{PQ}$, will be denoted by $|\overrightarrow{PQ}|$ or PQ. *Known as modules of* $\overrightarrow{PQ}$.

Support *The line of unlimited length of which PQ is segment is called the support of the vector* $\overrightarrow{PQ}$.

Sense *The sense of* $\overrightarrow{PQ}$ is form P to Q *and that of* $\overrightarrow{QP}$ *is form* Q *to* P. *Thus, the sense of a directed line segment is form its intial point to the terminal point.*

Types of Vectors

Equal Vectors *Two vectors* $\vec{a}$ *and* $\vec{b}$ *are said to be equal, written as* $\vec{a} = \vec{b}$, *if they have (i) the same length (ii) the same or parallel support, and (iii) the same sense.*

Zero or Null Vector *A vectors whose initial and terminal points are coincident is called the zero or the null vector.*

Vectors other than the null vector are called *proper vectors*.

Unit Vector *A vector modulus is unity, is called a unit vector. The unit vector in the direction of a vector* $\vec{a}$ *is denoted by* $\hat{a}$, *read as 'a cap'. Thus,* $|\hat{a}| = 1$.

Like and Unlike Vectors *Vectors are said to be like when they have the same sense of direction and unlike when they have opposite directions.*

Co-Initial Vectors *Vectors having the same initial point are called co-initial vectors.*

Coplanar Vectors *A system of vectors is said to be coplanar, if their supports are parallel to the same plane.*

Note that two vectors are always coplanar.

Coterminous Vectors *Vectors having the same terminal point are called coterminous vectors.*

Negative of a Vector *The vector which has the same magnitude as the vector* $\vec{a}$ *but opposite direction, is called the negative of* $\vec{a}$ *and is denoted by* $-\vec{a}$. *Thus, if* $\overrightarrow{PQ} = \vec{a}$, *then* $\overrightarrow{QP} = -\vec{a}$.

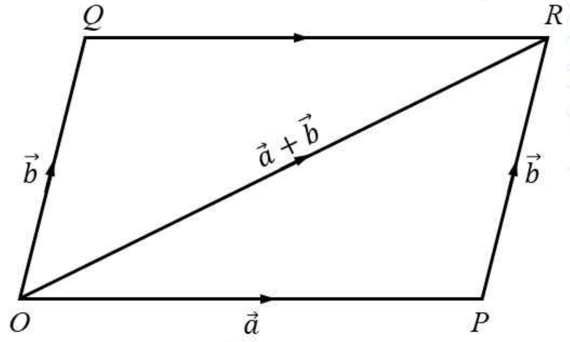
Reciprocal of A Vector *A vector having the same direction as that of a given vector* $\vec{a}$ *but magnitude equal to the reciprocal of the given vector is known as the reciprocal of* $\vec{a}$ *and is denoted by* $\vec{a}^{-1}$. *Thus, if* $|\vec{a}| = a$, $|\vec{a}^{-1}| = 1/a$.

Localized and Free Vectors *If the value of a vector depends only on its length and direction and is independent of its position in the space, it is called a free vector and when the value of a vector depends on its position in space then its called a localized vector.*

Algebra Vectors

If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their sum $\vec{c}$ is represented by the diagonal of the parallelogram which is coinitial with the given vectors.
Symbolically, we have

$$\overrightarrow{OP} + \overrightarrow{OQ} = \overrightarrow{OR} \Rightarrow \vec{a} + \vec{b} = \vec{c}$$



Figure

Therefore, in triangle OPR, we have

$$\overrightarrow{OP} + \overrightarrow{PR} = \overrightarrow{OR}.$$

Thus, it follows that if two vector are represented in magnitude and direction by the two sides of a triangle taken in the same order, then their sum is represented by the third side taken in the reverse order. This is called the triangle law of addition of vectors.

Properties of Addition of Vectors

(i) *Vector addition is communitive i.e.,*
$\vec{a} + \vec{b} = \vec{b} + \vec{a}$ *for any two vectors* $\vec{a}$ and $\vec{b}$.

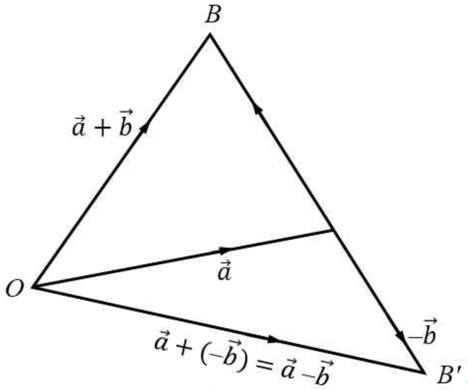
(ii) *Vector addition is associative i.e.,*
$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ *for any three vectors* $\vec{a}, \vec{b}, \vec{c}$.

(iii) *(Existence of additive identity) For every vector* $\vec{a}$, *we have*
$\vec{a} + \vec{0} = \vec{a} = \vec{0} + \vec{a}$, *where* $\vec{0}$ *is the null vector.*

(iv) (Existence of additive inverse) For every vector $\vec{a}$, *there corresponds a vector* $-\vec{a}$ *such that* $\vec{a} + (-\vec{a}) = \vec{0} = (-\vec{a}) + \vec{a}$.

Subtraction of Vectors

If $\vec{a}$ and $\vec{b}$ are two vectors, then the subtraction of $\vec{b}$ from $\vec{a}$ is defined as the vector sum of $\vec{a}$ and $-\vec{b}$ and is denoted by $\vec{a} - \vec{b}$ *i.e.*, $\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$.
Thus, to subtract $\vec{b}$ from $\vec{a}$, reverse the direction of $\vec{b}$ and add to $\vec{a}$ as shown in Figure.



Figure

Multiplication of a Vector by a Scalar

Let m be a scalar and $\vec{a}$ be a vector, then $m\vec{a}$ is defined as a vector having the same support as that of $\vec{a}$ such that its magnitude is $|m|$ times the magnitude of $\vec{a}$ and its direction is same as or opposite to the direction of $\vec{a}$ according as m is positive or negative. i.e. $\vec{a} = |\vec{a}|\,\hat{a}$ or, $\hat{a} = \frac{1}{|\vec{a}|}\vec{a}$

For any vector $\vec{a}$, we also define

$$1\vec{a} = \vec{a},\ (-1)\,\vec{a} = -\vec{a} \text{ and } \overrightarrow{0a} = \vec{0}$$

The following are properties of multiplication of vectors by scalars:

For vectors $\vec{a}, \vec{b}$ and scalars *m, n* we have

(i) $m(-\vec{a}) = (-m)\vec{a} = -(m\vec{a})$

(ii) $(-m)(-\vec{a}) = m\vec{a}$

(iii) $m(n\vec{a}) = (mn)\vec{a} = n(m\vec{a})$

(iv) $(m+n)\vec{a} = m\vec{a} + n\vec{a}$

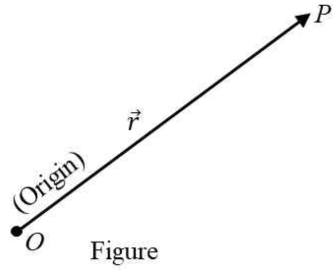
(v) $m(\vec{a}+\vec{b}) = m\vec{a} + m\vec{b}$

Linear Combination of Vectors

A vector $\vec{r}$ is said to be a linear combination of vectors $\vec{a}, \vec{b}, \vec{c}$... etc. if there exist scalars *x, y, z* etc., such that $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c} + \ldots$

Position Vector

If a point *O* is fixed as the origin in space (or plane) and *P* is any point, then $\overrightarrow{OP}$ is called the position vector of *P* with respect to *O*. If we say that *P* is the point $\vec{r}$ with respect to some origin *O*.



Figure