CLASS-IX

MATHEMATICS

For

Pre-Foundation Course

CONTENTS

Number System

1

CHAPTER

Number System

- **Natural Numbers**

  Counting numbers are known as natural numbers.
  N = { 1, 2, 3, 4, ... }.

  Even Numbers : All natural numbers which are multiples of 2 are even numbers (i.e.) 2,4,6,8… are even numbers.

  Odd numbers : All natural numbers which are not multiples of 2 are odd numbers.

  Prime and composite Numbers : A natural number which has exactly two factors, 1 and the number itself is called a **prime number**.

  All natural numbers other then 1 which are not prime are **composite numbers**.

  Note : 1 is neither a prime nor a composite number.

  2, 3, 5, 7, 11, 13, 17… are prime numbers. 4, 6, 8, 9, 10, 12 …… are **composite numbers.**

  Co-prime: Any natural numbers a and b are said to be co-prime if HCF(a, b) = 1. For example 4, 9 are co-prime numbers as H.C.F. (4, 9) = 1

- **Whole numbers**

  The natural numbers along with the zero form the set of whole numbers i.e. numbers 0, 1, 2, 3, 4 are whole numbers. W = {0, 1, 2, 3, 4…}

- **Integers**

  The natural numbers, their negatives and zero make up the integers.
  Z = {…–4, –3, –2, –1, 0, 1, 2, 3, 4…}
  The set of integers contains positive numbers, negative numbers and zero.

- **Rational Numbers**

(i) A rational number is a number which can be put in the form p/q, where p and q are both integers and $q \neq 0$.

(ii) A rational number is either a terminating or non-terminating but recurring (repeating) decimal.

(iii) A rational number may be positive, negative or zero.

- **Irrational Number**

  All real numbers are irrational if and only if their decimal representation is non-terminating and non-repeating.
  e.g. $\sqrt{2}$, $\sqrt{3}$, π… etc.

- **Real Numbers**

  Rational number and irrational number taken together form the set of real numbers.

Note: (1) If a and b are two real numbers, then either (i) a > b or (ii) a = b or (iii) a < b

(2) Negative of an irrational number is an irrational number.

(3) The sum of a rational number with an irrational number is always irrational number.

(4) The product of a non-zero rational number with an irrational number is always an irrational number.

(5) The sum of two irrational numbers is not always an irrational number.

(6) The product of two irrational numbers is not always an irrational number.

(7) $\pi = 3.14159265358979\ldots$

while $\frac{22}{7} = 3.1428571428\ldots$

$\therefore\ \pi \neq \frac{22}{7}$ but for calculation we can take $\pi \approx \frac{22}{7}$.

❖ **The Absolute Value (or modulus) of a real Number**

If a is a real number, modulus a is written as |a| ; |a| is always positive or zero. It means positive value of 'a' whether a is positive or negative

$|3| = 3$ and $|0| = 0$, Hence $|a| = a$; if $a = 0$ or $a > 0$ (i.e.) $a \geq 0$

$|-3| = 3 = -(-3)$. Hence $|a| = -a$ when $a < 0$

❖ EXAMPLES ❖

Ex.1 Is zero a rational number? Can you write it in the form of p/q, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number. It can be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc. where denominator $q \neq 0$, it can be negative also.

Ex.2 Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. A rational number between two rational numbers r and s is $\frac{r+s}{2}$.

A rational number between

$\frac{3}{5}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{3}{5} + \frac{4}{5}\right) = \frac{7}{10}$.

And a rational number between

$\frac{3}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{3}{5} + \frac{7}{10}\right) = \frac{13}{20}$

Similarly; $\frac{5}{8}, \frac{27}{40}, \frac{31}{40}$ are between $\frac{3}{5}$ and $\frac{4}{5}$.

So, five rational numbers between

$\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{5}{8}, \frac{13}{20}, \frac{7}{10}, \frac{31}{40}, \frac{27}{40}$

Ex.3 Find six rational numbers between 3 and 4.

Sol. We can solve this problem in two ways.

Method 1 : A rational number between two rational numbers r and s is $\frac{r+s}{2}$.

Therefore, a rational number between 3 and 4 = $\frac{1}{2}(3 + 4) = \frac{7}{2}$

A rational number between 3 and $\frac{7}{2} = \frac{1}{2}\left(\frac{6+7}{2}\right) = \frac{13}{4}$. We can accordingly proceed in this manner to find three more rational numbers between 3 and 4.

Hence, six rational numbers between 3 and 4 are $\frac{25}{8}, \frac{13}{4}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}$.

Method 2 :

Since, we want six numbers, we write 3 and 4 in $\frac{p}{q}$ form with denominator 6 + 1, i.e., $3 = \frac{21}{7}$ and $4 = \frac{28}{7}$.

Then we can check that $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ and $\frac{27}{7}$ are all between 3 and 4.

Hence, the six numbers between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$ and $\frac{27}{7}$.

Ex.4 Are the following statements true or false? Give reasons for your answer.
(i) Every natural number is a whole number. (ii) Every integer is a whole number.
(iii) Every rational number is a whole number.

Sol. (i) True, because N = {1, 2, 3…}
W = {0, 1, 2, 3…}
(ii) False, because negative integers are not whole number.
(iii) False, because rational numbers such as $\frac{1}{4}, \frac{1}{2}$ are not whole numbers.

Ex.5 Find 3 irrational numbers between 3 & 5.

Sol. ∵ 3 and 5 both are rational
The irrational are 3.1010010001…;
3.2020020002…; 3.3030030003…

Ex.6 Find two rational & two irrational numbers between 4 and 5.

Sol. Rational numbers $\frac{4+5}{2} = 4.5$ & $\frac{4.5+4}{2} = \frac{8.5}{2} = 4.25$
Irrational numbers 4.1010010001…
4.2020020002…

Ex.7 Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Sol. We know that, if a and b are two distinct positive irrational numbers, and if $\sqrt{ab}$ is irrational then $\sqrt{ab}$ is an irrational number lying between a and b, if ab is not a perfect square

∴ Irrational number between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2}\times\sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$

Irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2}\times 6^{1/4}} = 2^{1/4} \times 6^{1/8} = 2^{2/8} \times 6^{1/8}$
$= (4 \times 6)^{1/8} = 24^{1/8}$
Hence required irrational numbers are $6^{1/4}$ and $24^{1/8}$

Decimal Representation of Rational Numbers

(A) Terminating

Ex.8 $\frac{6}{5}, \frac{8}{5}, \frac{7}{4}$ … are equal to 1.2, 1.6, 1.75 respectively, so these are terminating

Ex.9 Express $\frac{-17}{8}$ in decimal form by long division method.

Sol. In order to convert $\frac{-17}{8}$ in the decimal form, we first express $\frac{17}{8}$ in the decimal form and the decimal form of $\frac{-17}{8}$ will be negative of the decimal form of $\frac{17}{8}$

We have,

$$\begin{array}{l} 8\overline{)17.000}(2.125 \\ \underline{16} \\ \ \ 10 \\ \ \ \ \underline{8} \\ \ \ \ 20 \\ \ \ \ \underline{16} \\ \ \ \ \ 40 \\ \ \ \ \ \underline{40} \\ \ \ \ \ \ 0 \end{array}$$

$\therefore \ \dfrac{-17}{8} = -2.125$

(B) Non terminating recurring (repeating)

Ex.10 $\dfrac{10}{3} = 3.333\ldots \text{or } 3.\overline{3} \Rightarrow \dfrac{1}{7} = 0.14285714285\ldots \text{or } 0.\overline{142857} \Rightarrow \dfrac{2320}{99} = 23.434343\ldots \text{or } 23.\overline{43}$

These expansions are not terminated but digits are continuously repeated so we use a line on those digits, called bar (¯).

So we can say that rational numbers are of the form either terminating, or non terminating repeating (recurring).

Ex.11 Find the decimal representation of $\dfrac{22}{7}$.

Sol. By long division, we have

$$\begin{array}{l} 7\overline{)22}\ (3.142857142857 \\ \underline{21} \\ \ 10 \\ \ \underline{7} \\ \ \ 30 \\ \ \ \underline{28} \\ \ \ \ 20 \\ \ \ \ \underline{14} \\ \ \ \ \ 60 \\ \ \ \ \ \underline{56} \\ \ \ \ \ \ 40 \\ \ \ \ \ \ \underline{35} \\ \ \ \ \ \ \ 50 \\ \ \ \ \ \ \ \underline{49} \\ \ \ \ \ \ \ \ 10 \\ \ \ \ \ \ \ \ \underline{7} \\ \ \ \ \ \ \ \ \ 30 \\ \ \ \ \ \ \ \ \ \underline{28} \\ \ \ \ \ \ \ \ \ \ 20 \\ \ \ \ \ \ \ \ \ \ \underline{14} \\ \ \ \ \ \ \ \ \ \ \ 60 \\ \ \ \ \ \ \ \ \ \ \ \underline{56} \\ \ \ \ \ \ \ \ \ \ \ \ 40 \\ \ \ \ \ \ \ \ \ \ \ \ \underline{35} \\ \ \ \ \ \ \ \ \ \ \ \ \ 50 \\ \ \ \ \ \ \ \ \ \ \ \ \ \underline{49} \\ \ \ \ \ \ \ \ \ \ \ \ \ \ 1 \end{array}$$

$\therefore \ \dfrac{22}{7} = 3.142857142857\ldots = 3..\overline{142857}$

Conversion of Decimal numbers into Rational number of the form $\frac{m}{n}$

Case I : When the decimal number is of terminating nature.

Step-1 : Obtain the rational number.

Step-2 : Determine the number of digits in its decimal part

Step-3 : Remove decimal point from the numerator. Write 1 in the denominator and put as many zeros on the right side of 1 as the number of digits in the decimal part of the given rational number.

Step-4 : Find a common divisor of the numerator and denominator and express the rational number to lowest terms by dividing its numerator and denominator by the common divisor.

Ex.12 Express each of the following numbers in the form $\frac{p}{q}$.

(i) 0.675 (ii) –25.6875

Sol. (i) $0.675 = \frac{675}{1000}$

$= \frac{675 \div 25}{1000 \div 25} \Rightarrow = \frac{27}{40}$

(ii) $-25.6875 = \frac{-256875}{10000}$

$= \frac{-256875 \div 625}{10000 \div 625} = \frac{-411}{16}$

Case II : When decimal representation is of non-terminating repeating nature.

In a non terminating repeating decimal, there are two types of decimal representations

(i) A decimal in which all the digit after the decimal point are repeated. These type of decimals are known as **pure recurring decimals.**

For example: $0.\overline{6}, 0.\overline{16}, 0.\overline{123}$ are pure recurring decimals.

(ii) A decimal in which at least one of the digits after the decimal point is not repeated and then some digit or digits are repeated. This type of decimals are known as **mixed recurring decimals.**

For example, $2.1\overline{6}, 0.3\overline{5}, 0.7\overline{85}$ are mixed recurring decimals.

- **Conversion of a pure recurring decimal to the form $\frac{p}{q}$**

Algorithm :

Step-1 : Obtain the repeating decimal and put it equal to x (say)

Step-2 : Write the number in decimal form by removing bar from the top of repeating digits and listing repeating digits at least twice. For example, write $x = 0.\overline{8}$ as x = 0.888.... and $x = 0.\overline{14}$ as x= 0.141414......

Step-3 : Determine the number of digits having bar on their heads.

Step-4 : If the repeating decimal has one place repetition, multiply by 10; two place repetition, multiply by 100; three place repetition, multiply by 1000 and so on.

Step-5 : Subtract the number in step 2 from the number obtained in step 4

Step-6 : Divide both sides of the equation by the coefficient of x.

Step-7 : Write the rational number in its simplest form.

❖ EXAMPLES ❖

Ex.13 Express each of the following decimals in the form $\frac{p}{q}$:

(i) $0.\overline{6}$ (ii) $0.\overline{585}$

Sol. (i) Let $x = 0.\overline{6}$

then, x = 0.666… ....(i)

Here, we have only one repeating digit, So, we multiply both sides of (i) by 10 to get

10 x = 6.66… ....(ii)

Subtracting (i) from (ii), we get

10 x – x = (6.66…) – (0.66…)

$\Rightarrow$ 9x = 6 $\Rightarrow$ $x = \frac{6}{9}$ $\Rightarrow$ $x = \frac{2}{3}$ Hence $0.\overline{6} = \frac{2}{3}$

(ii) Let $x = 0.\overline{585}$

$\Rightarrow$ x = 0.585585585... ....(i)

Here, we have three repeating digits after the decimal point. so, we multiply both sides of (i) by 10^3 = 1000 to get

1000 x = 585.585585… ....(ii)

Subtracting (i) from (ii), we get

1000x – x = (585.585585...) – (0.585585585...)

1000x – x = 585

$\Rightarrow$ 999x = 585

$\Rightarrow$ $x = \frac{585}{999} = \frac{65}{111}$

- **Conversion of a mixed recurring decimal to the form $\frac{p}{q}$**

Step-1 : Obtain the mixed recurring decimal and write it equal to x (say)

Step-2 : Determine the number of digits after the decimal point which do not have bar on them. Let there be n digits without bar just after the decimal point

Step-3 : Multiply both sides of x by 10^n so that only the repeating decimal is on the right side of the decimal point.

Step-4 : Use the method of converting pure recurring decimal to the form $\frac{p}{q}$ and obtain the value of x

❖ EXAMPLES ❖

Ex.14 Express the following decimals in the form

(i) $0.3\overline{2}$ (ii) $0.12\overline{3}$

Sol. (i) Let $x = 0.3\overline{2}$

Clearly, there is just one digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by 10 so that only the repeating decimal is left on the right side of the decimal point.

$\therefore$ $10x = 3.\overline{2}$

$\Rightarrow$ $10x = 3 + 0.\overline{2}$ $\left[\because 0.\overline{2} = \frac{2}{9}\right]$

$\Rightarrow$ $10x = 3 + \frac{2}{9}$ $\Rightarrow$ $10x = \frac{9 \times 3 + 2}{9}$ $\Rightarrow$ $10x = \frac{29}{9}$ $\Rightarrow$ $x = \frac{29}{90}$

(ii) Let x = $0.12\overline{3}$

Clearly, there are two digits on the right side of the decimal point which are without bar. So, we multiply both sides of x by $10^2 = 100$ so that only the repeating decimal is left on the right side of the decimal point.

$\therefore$ $100x = 12.\overline{3}$

$\Rightarrow$ $100x = 12 + 0.\overline{3}$ $\Rightarrow$ $100x = 12 + \frac{3}{9}$ $\Rightarrow$ $100x = \frac{12 \times 9 + 3}{9}$

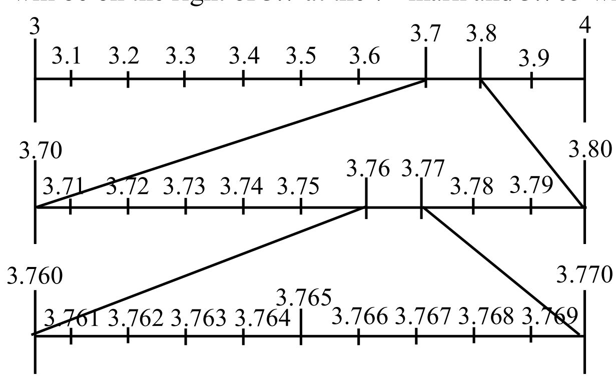
$\Rightarrow$ $100x = \frac{108 + 3}{9} = \frac{111}{9}$ $\Rightarrow$ $x = \frac{111}{900} = \frac{37}{300}$

Representing Rational Numbers on the number line

Number line is a geometrical straight line with arbitrarily defined zero.
We understand the concept with the help of following examples.

Ex.15 Represent 3.765 on the number line.

Sol. This number lies between 3 and 4. The distance 3 and 4 is divided into 10 equal parts. Then the first mark to the right of 3 will represent 3.1 and second 3.2 and so on. Now, 3.765 lies between 3.7 and 3.8. We divide the distance between 3.7 and 3.8 into 10 equal parts 3.76 will be on the right of 3.7 at the sixth mark, and 3.77 will be on the right of 3.7 at the 7th mark and 3.765 will lie between 3.76 and 3.77 and so on.

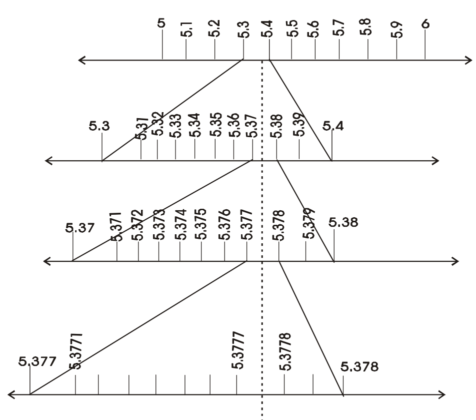


To mark 3.765 we have to use magnifying glass method

Ex.16 Visualize the representation of $5.3\overline{7}$ on the number line upto 5 decimal places.

Sol. We have, $5.3\overline{7}$ = 5.3777….

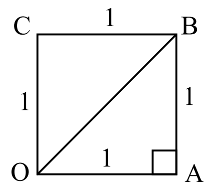
This number lies between 5.3 and 5.4. The distance between 5.3 and 5.4 is divided into 10 equal parts. Then the first mark to the right of 5.3 will represent 5.31 and second 5.32 and so on. Now, 5.3777 lies between 5.37 and 5.38. We divide the distance between 5.37 and 5.38 into 10 equal parts and so on.



Representing Irrational numbers on the number line

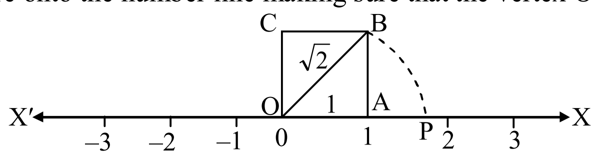
❖ **Represent $\sqrt{2}$ & $\sqrt{3}$ on the number line**

Greeks discovered this method. Consider a unit square OABC, with each side 1 unit in length. Then by using pythagoras theorem



$OB = \sqrt{1+1} = \sqrt{2}$

Now, transfer this square onto the number line making sure that the vertex O coincides with zero

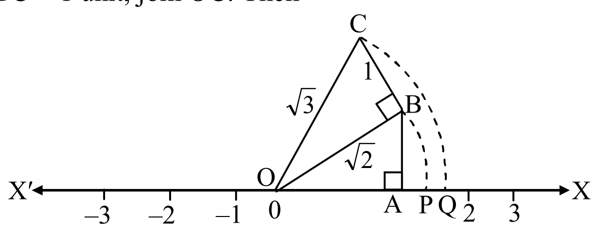


With O as centre & OB as radius, draw an arc, meeting OX at P. Then

$OB = OP = \sqrt{2}$ units

Then, the point P represents $\sqrt{2}$ on the number line.

Now draw, $BC \perp OB$ such that BC = 1 unit, join OC. Then



$OC = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{3}$ units

With O as centre & OC as radius, draw an arc, meeting OX at Q. Then

$OQ = OC = \sqrt{3}$ units

Then, the point Q represents $\sqrt{3}$ on the number line.

Remark : In the same way, we can locate $\sqrt{n}$ for any positive integer n, after $\sqrt{n-1}$ has been located.

❖ **Existence of $\sqrt{n}$ for a positive real number**

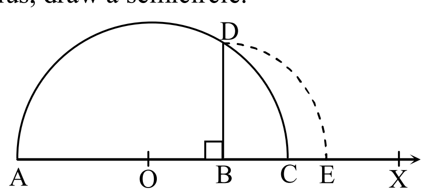
(Geometrical Representation)

Represent the value of $\sqrt{4.3}$ geometrically :

Draw a line segment AB = 4.3 units and extend it to C such that BC = 1 unit.

Find the midpoint O of AC by drawing the perpendicular bisector of AC.

With O as centre and OA a radius, draw a semicircle.



Now, draw $BD \perp AC$, intersecting the semicircle at D. Then, $BD = \sqrt{4.3}$ units.

With B as centre and BD as radius, draw an arc, meeting AC produced at E.

Then, $BE = BD = \sqrt{4.3}$ units.