

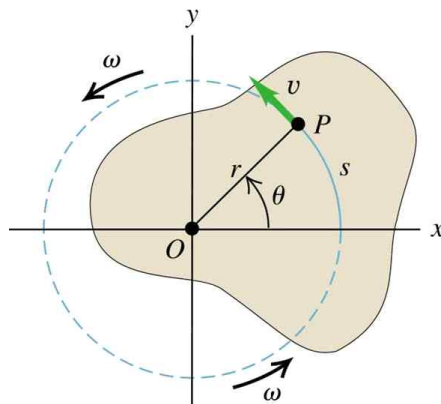
PHYSICS

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01. Kinematics

Consider a rigid body rotating about a given fixed line.



The average angular velocity during the time interval Δt is $\omega = \frac{\Delta\theta}{\Delta t}$.

The instantaneous angular velocity at time t is $\omega = \frac{d\theta}{dt}$.

If the body rotates through equal angles in equal time intervals (irrespective of the smallness of the intervals), we say that it rotates with uniform angular velocity. In this case $\omega = d\theta/dt = \text{constant}$ and thus $\theta = \omega t$. If it is not the case, the body is said to be rotationally “accelerated”. The angular acceleration is defined as

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt} \left(\frac{d\theta}{dt} \right) = \frac{d^2\theta}{dt^2}.$$

If the angular acceleration α is constant, we have

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\text{and } \omega^2 = \omega_0^2 + 2 \alpha \theta$$

02. Relation between the Linear Motion of a Particle of a Rigid Body and its Rotation

The linear speed along the tangent is

$$v = \frac{ds}{dt} = r \cdot \frac{d\theta}{dt}$$

$$\therefore v = r\omega$$

and the linear acceleration along the tangent, i.e., the tangential acceleration, is

$$a = \frac{dv}{dt} = r \cdot \frac{d\omega}{dt}$$

$$\therefore a = r\alpha.$$

03. Moment of Inertia

Moment of inertia of a body about a given axis is the sum of the products of masses of all the particles of the body and squares of their respective perpendicular distances from the axis of rotation.

$$I = \sum_{i=1}^{i=n} m_i r_i^2$$

moment of inertia of a body about a given axis is numerically equal to torque acting on the body rotating with unit angular acceleration about it.

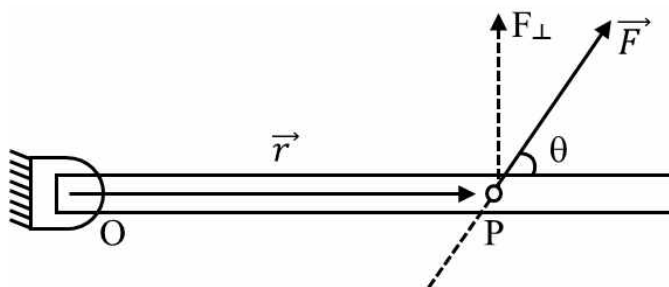
$$\vec{\tau} = I\vec{\alpha}$$

Moment of Inertia of Some bodies of regular shape

S.No.	Body	Axis	Moment of Inertia
i.	Thin circular ring, radius R	Perpendicular to plane, at centre	MR^2
ii.	Thin circular ring, radius R	Diameter	$M R^2/2$
iii.	Thin rod, length L	Perpendicular to rod, at mid point	$M L^2/12$
iv.	Circular disc, radius R	Perpendicular to disc at centre	$M R^2/2$
v.	Circular disc, radius R	Diameter	$M R^2/4$
vi.	Hollow cylinder, radius R	Axis of cylinder	$M R^2$
vii.	Solid cylinder, Radius R	Axis of cylinder	$M R^2/2$
viii.	Solid sphere, Radius R	Diameter	$2 M R^2/5$

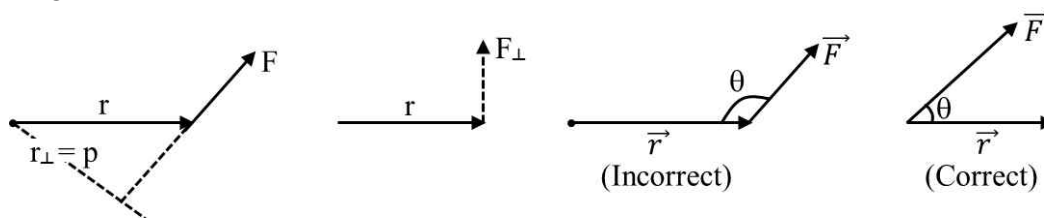
04. Torque of a Force about a Point

A pair of forces of equal magnitude but acting in opposite directions with different lines of action is known as a **couple** or **torque**. If you want to rotate the rod, you have to push any point P of the rod by a force F . The perpendicular component of the force, that is, F_{\perp} ($= F \sin \theta$) is responsible for rotating rod. This turning or twisting or rotating effect of the force is called torque denoted by τ .



The rod rotates by the rotational effect (torque τ) of the force \vec{F} which is given as $\vec{\tau} = \vec{r} \times \vec{F}$

- i. $\tau = r F \sin\theta = pF = rF_{\perp}$, where $\theta =$ Angle between \vec{r} and \vec{F} .
 ii. Change in direction of \vec{F} reverses the direction of $\vec{\tau}$.



Figure

05. Angular Momentum

Angular momentum \vec{l} of a particle about an inertial reference point is defined as the vector product of the position vector \vec{r} and linear momentum \vec{p} of the particle relative to the reference point.

$$\vec{l} = \vec{r} \times \vec{p} (= \vec{r} \times m\vec{v})$$

where \vec{p} is the linear momentum and \vec{r} is the position vector of the particle from the given point. The angular momentum of a system of particles is the vector sum of the angular momenta of the particles of the system. Thus,

$$\vec{L} = \sum_i \vec{l}_i = \sum_i (\vec{r}_i \times \vec{p}_i)$$

06. Conservation of Angular Momentum

According to this principle, when no external torque acts on a system of particles, then the total angular momentum of the system remains always a constant.

For a system of n particles making up a rigid body, total torque acting on the body is due to external forces only. The internal forces between the particles of the body do not contribute to the torque.

As
$$\vec{\tau}_{total} = \frac{d}{dt} (\vec{L}_{total})$$

\therefore When no external torque acts on the system, $\vec{\tau}_{total} = 0$

\therefore
$$\frac{d}{dt} (\vec{L}_{total}) = 0 \quad \text{or} \quad \vec{L}_{total} = \text{constant}$$

i.e.,
$$\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots + \vec{L}_n = \text{constant} \quad (\text{vector})$$

This is the principle of conservation of angular momentum.