MATHS

Live eBook

01. Rank of a Word in Dictionary

Rank of a word is the position of that word when we arrange the words formed by alphabets of that given word in dictionary order.

For example : Suppose you are given a work in which none of the letters are repeated & are asked to find the rank of the word in a dictionary if the word given is CAT it will be very easy to find its rank. You first write down all possible combinations of the letters which are CAT, CTA, ATC, TCA, ACT, TAC. Now you arrange them in alphabetical order which gives ACT, ATC, CAT, CTA, TAC, TCA.

CAT is the 3rd in the above list so the rank of the word CAT is 3.

But how to compute the rank when the difficulty level is slightly high?

Suppose we need to **Find the rank of the word SOURAV.**

Then, **Shortcut method**:-

SOURAV

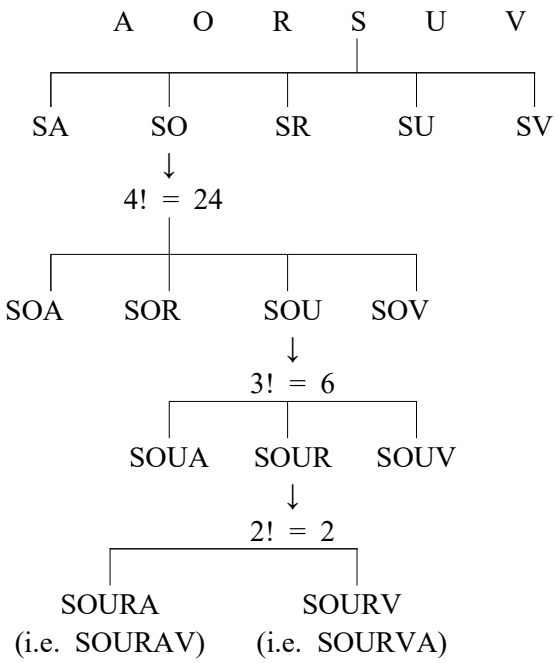
Total 6 letters

Alphabetically increasing order is

A, O, R, S, U, V

Different Sections in Dictionary

A	O	R	S	U	V

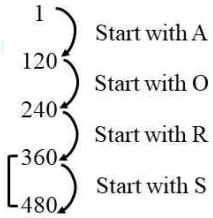


(i) 6 letter words beginning with A = 5! = 120
(A _ _ _ _ _
5 Positions &
5 distinct letters S, O, U, R, V to arrange
∴ ways = 5! = 120)

(ii) (6 letter words beginning with O = 5! = 120
O _ _ _ _ _
5 Positions &
5 distinct letters S, U, R, A, V to arrange
∴ ways = 5! = 120)

(iii) (6 letter words starting from R = 5! = 120
R _ _ _ _ _
5 Positions &
5 distinct letters S, O, U, A, V to arrange
∴ ways = 5! = 120)

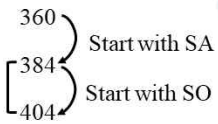
(iv) (6 Letter words beginning from S
S _ _ _ _ _ = 120 = 5!
5 Positions &
5 distinct letters O, U, R, A, V
to arrange
∴ ways = 120 = 5!)



Now,

Words Starting with SA = 24 = 4!

(**S** A _ _ _ _
4 positions &
4 distinct letters U, R, O, V
∴ ways = 4!)

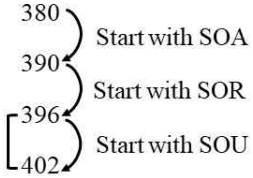


words starting with SO = 24 = 4!
(**S** **O** _ _ _ _
4 positions &
4 distinct letters U, R, V, A to arrange
∴ ways = 4!)

Now,
Words starting with SOA = 6 = 3!

(**SO** A _ _ _ = 3! = 6
3 places &
3 distinct letters U, R, V to arrange)

Words starting with SOR = 3! = 6
(**SO** R _ _ _
3 places &
3 distinct letters U, V, A to arrange
∴ ways = 3! = 6)



words starting with SOU = 6 = 3!
SO U _ _ _
3 places &
3 distinct letters R, V, A to arrange
∴ ways = 3! = 6

Now,
words starting with SOUA = 2! = 2
SOU A _ _
2 places &
2 distinct letters to arrange R, V
∴ ways = 2!

Words starting with SOUR = 2! = 2
SOU R _ _
2 places &
2 distinct letters to arrange V, A to arrange
∴ ways = 2!
i.e. SOURAV → 399th word
SOURVA → 400th word.
∴ Rank of SOURAV = 399 from the beginning & rank of SOURAV from the end
= 720 – 399 + 1
= 322

(JEE Advanced – 2007)

Q1. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in an English dictionary. The number of words that appear before the word COCHIN is

(a) 360
(b) 192
(c) 96
(d) 48

Ans. (c)

Solution : Shortcut method:-
COCHIN

1	5	1	2	3	4
C	O	C	H	I	N

Trick :

$$\underbrace{(I)\times(i)}_{\text{for C}} + \underbrace{(II)\times(ii)}_{\text{for O}} + \underbrace{(III)\times(iii)}_{\text{for C}} + \underbrace{(IV)\times(iv)}_{\text{for H}} + \underbrace{(V)\times(v)}_{\text{for I}} + \underbrace{(VI)\times(vi)}_{\text{for N}} + 1 = \text{Rank}$$

(I) = 5!, (II) = 4!, (III) = 3!, (IV) = 2!, (V) = 1!, (VI) = 0! &

$$(i) = \frac{\text{No. of smaller ranks from 1 on RHS of C}}{(\text{No. of repeating letters C onwards towards right of C})!} = \frac{0}{2!} = 0$$

(ii) $= \dfrac{\text{No. of smaller ranks from 5 on RHS of O}}{\text{(No. of repeating letters O onwards towards right of O)!}} = \dfrac{4}{1!} = 4$

(iii) $= \dfrac{\text{No. of smaller ranks from 1 on RHS of C}}{\text{(No. of repeating letters C onwards towards right of C)!}} = \dfrac{0}{1!}$

(iv) $= \dfrac{\text{No. of smaller ranks from 2 on RHS of H}}{\text{(No. of repeating letters H onwards towards right of H)!}} = \dfrac{0}{1!}$

(v) $= \dfrac{\text{No. of smaller ranks from 3 on RHS of I}}{\text{(No. of repeating letters I onwards towards right of I)!}} = \dfrac{0}{1!}$

(vi) $= \dfrac{\text{No. of smaller ranks from 4 on RHS of N}}{\text{(No. of repeating letters N onwards towards right of N)!}} = \dfrac{0}{1!}$

$\therefore$ Rank $= \left(5! \times \dfrac{0}{2!}\right) + \left(4! \times \dfrac{4}{1!}\right) + \left(3! \times \dfrac{0}{1!}\right) + \left(2! \times \dfrac{0}{1!}\right) + \left(1! \times \dfrac{0}{1!}\right) + \left(0! \times \dfrac{0}{1!}\right) + 1 = 96 + 1 = 97$

$\therefore$ Rank of COCHIN = 97.

$\therefore$ 96 words lies before COCHIN.

(JEE Main – 2016)

Q2. If all the words (with or without meaning) having 5 letters, formed using the letters of the word SMALL & arranged as in a dictionary then the position of the word SMALL is:-

(a) 58^{th}

(b) 46^{th}

(c) 59^{th}

(d) 52^{nd}

Ans. (a)

Solution : Shortcut method:-

SMALL

Give chronological order to letters of the word SMALL i.e. A,L,L,M,S

i.e. 4^{th} 3^{rd} 1^{st} 2^{nd} 2^{nd}

S M A L L

(Write the order above the alphabets).

Trick :

Rank $\underbrace{\text{(I)}\times\text{(i)}}_{\text{for S}} + \underbrace{\text{(II)}\times\text{(ii)}}_{\text{for M}} + \underbrace{\text{(III)}\times\text{(iii)}}_{\text{for A}} + \underbrace{\text{(IV)}\times\text{(iv)}}_{\text{for L}} + \underbrace{\text{(V)}\times\text{(v)}}_{\text{for L}} + 1$

(I) = 4!, (II) = 3!, (III) = 2!, (IV) = 1!, (V) = 0!

$\therefore$ Rank = 4!(i) + 3!(ii) + 2!(iii) + 1!(iv) + 0!(v) + 1

Now using : $\overset{4^{th}}{S}\ \overset{3^{rd}}{M}\ \overset{1^{st}}{A}\ \overset{2^{nd}}{L}\ \overset{2^{nd}}{L}$

(i) $\dfrac{\text{No. of smaller ranks from 4 on RHS of S}}{\text{(No. of repeating letters S onwards towards right of S)!}} = \dfrac{4}{2!} = 2$

(ii) $= \dfrac{\text{No. of smaller ranks from 3 on RHS of M}}{\text{(No. of repeating letters M onwards towards right of M)!}} = \dfrac{3}{2!} = \dfrac{3}{2}$

(iii) $= \dfrac{\text{No. of smaller ranks from 1 on RHS of A}}{\text{(No. of repeating letters A onwards towards right of A)!}} = \dfrac{0}{2!} = 0$

(iv) $= \dfrac{\text{No. of smaller ranks from 2 on RHS of L}}{\text{(No. of repeating letters L onwards towards right of L)!}} = \dfrac{0}{2!} = 0$

(v) $= \dfrac{\text{No. of smaller ranks from 2 on RHS of L}}{\text{(No. of repeating letters L onwards towards right of L)!}} = \dfrac{0}{1} = 0$

$\therefore$ Rank is $= (4! \times 2) + \left(3! \times \dfrac{3}{2}\right) + (2! \times 0) + (1! \times 0) + (0! \times 0) + 1$

$= 48 + 9 + 1 = 58$

$\therefore$ Rank is 58^{th}