MATHEMATICS

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01. Joint Equation of A Pair of Straight Lines

The joint equation of the straight lines $a_1 x + b_1 y = 0$ and $a_2 x + b_2 y = 0$ is of the form $ax^2 + 2hxy + by^2 = 0$ which is known as the homogeneous equation of second degree. Thus, the combined equation of a pair of straight lines passing through the origin is a homogeneous equation of second degree.

Consider the equation $ax^2 + 2hxy + by^2 = 0$. This equation can be re-written as

$$b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0 \qquad \text{[Dividing throughout by } x^2\text{]}$$

$$\Rightarrow \quad \frac{y}{x} = \frac{-h \pm \sqrt{h^2 - ab}}{b}$$

$$\Rightarrow \quad y = \left(\frac{-h \pm \sqrt{h^2 - ab}}{b}\right)x$$

$\Rightarrow \quad y = m_1, x, y = m_2 x$, where

$$m_1 = \frac{-h + \sqrt{h^2 - ab}}{b} \text{ and } m_2 = \frac{-h - \sqrt{h^2 - ab}}{b}$$

The value of m_1 and m_2 are (i) real and distinct if $h^2 > ab$ (ii) real and equal if $h^2 > ab$, and (iii) imaginary if $h^2 = ab$.

Also, $m_1 + m_2 = -\frac{2h}{b}$ and, $m_1 m_2 = \frac{a}{b}$

Thus, $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines $y = m_1 x$ and $y = m_2 x$ which are real and distinct if $h^2 > ab$, Also,

$$m_1 + m_2 = -\frac{2h}{b} = -\frac{\text{Coefficient of } xy}{\text{Coefficient of } y^2}$$

and, $$m_1 m_2 = \frac{a}{b} = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } y^2}$$

02. Angle Between The Pair of Lines

The acute angle between the two lines represented by $ax^2 + 2hxy + by^2 = 0$ & are given as $y = m_1 x$ (slope $= m_1$) & $y = m_2 x$ (slope $= m_2$) is

$$\text{Tan } \theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$\text{Tan } \theta = \frac{|m_2 - m_1|}{|1 + m_1 m_2|} = \frac{\sqrt{(m_2 - m_1)^2}}{|1 + m_1 m_2|} = \frac{\sqrt{m_2^2 + m_1^2 - 2m_1 m_2}}{|1 + m_1 m_2|}$$

$$= \frac{\sqrt{m_1^2 + m_2^2 + 2m_1 m_2 - 4m_1 m_2}}{|1 + m_1 m_2|}$$

$$= \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{|1 + m_1 m_2|}$$

$$= \frac{\sqrt{\left(-\frac{2h}{b}\right)^2 - 4\frac{a}{b}}}{\left|1 + \frac{a}{b}\right|} = \frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{\frac{|a+b|}{|b|}}$$

$$= \frac{\frac{\sqrt{4h^2 - 4ab}}{|b|}}{\frac{|a+b|}{|b|}} = \frac{2\sqrt{h^2 - ab}}{|a+b|} = \frac{2\sqrt{h^2 - ab}}{|a+b|}$$

Lines are parallel iff they are coincident

iff $h^2 = ab$

($\tan\theta = 0 \Rightarrow \theta = 0$)

Lines are perpendicular iff $m_1 m_2 = -1$

iff $\frac{a}{b} = -1$

or $a = -b$ or $a + b = 0$

$a + b = 0$