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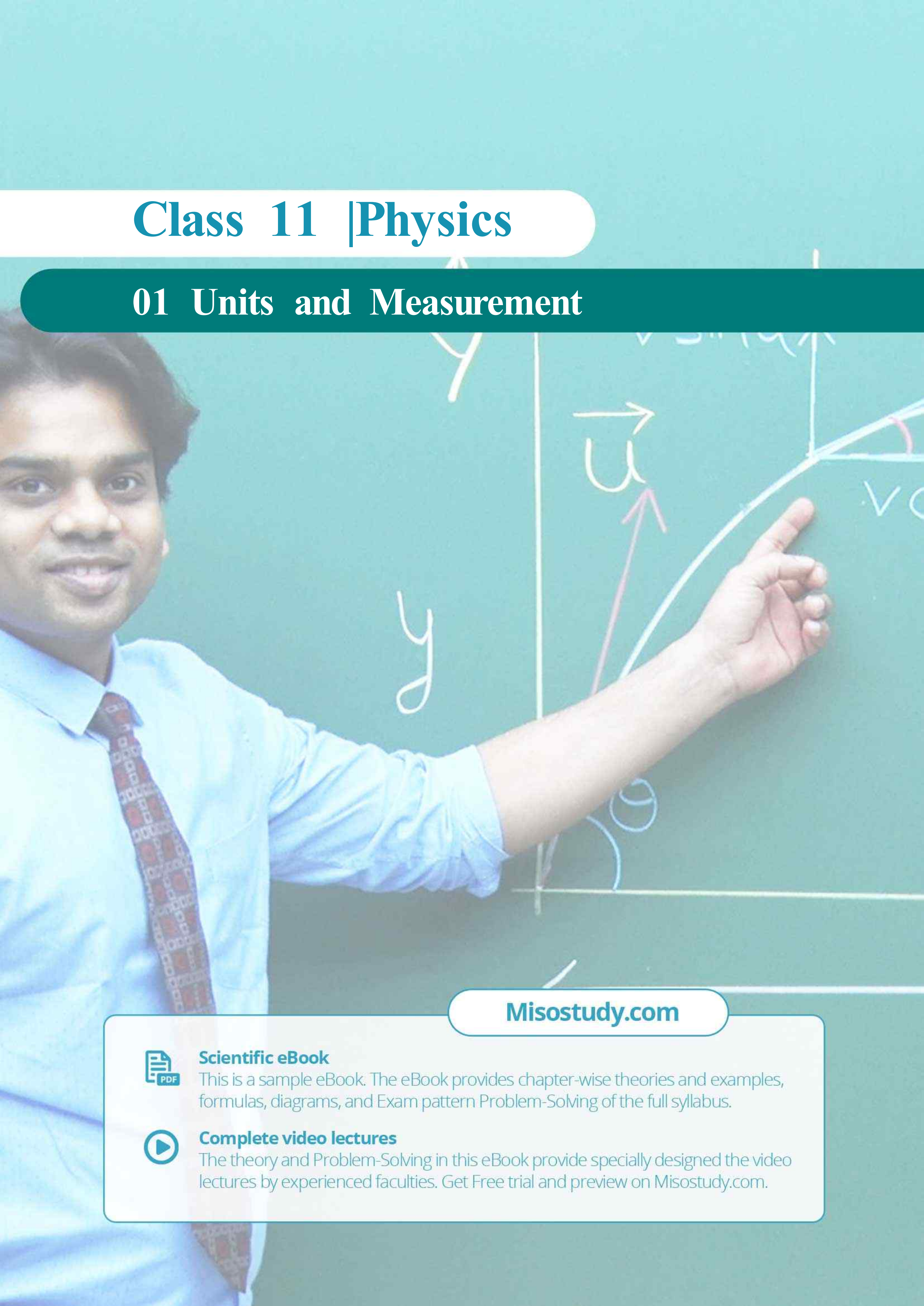

Class 11 |Physics

01 Units and Measurement

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01. Introduction

- Biology; Bios = life, Logos = Study, means study of life is biology.
- **"Biology** is the science of life forms and living processes"
- **Systematic** and **monumental** description of life forms made human to make a detailed system of identification, nomenclature and classification of organisms i.e. **Taxonomy.**
- Hence the study of identification, nomenclature and identification is called taxonomy.
- All the organisms have been evolved by a very long and complex process of **evolution**, so they all are related to each other by sharing of some **common genetic material** but up to varying degrees. This sharing may be less or more among different cases..
- When human came to know this fact then he humbled and led to cultural movements for conservation of **biodiversity.**
- Sharing of common characters was also proved when human studied the similarities among living organisms both **horizontally** and **vertically.**
- The living world is full of amazing diversity of living organisms.
- The diversity of habitats of organisms is also very vast and amazing.
- This diversity makes us deeply reflect on "What indeed is life" ? This question actually asks to solve two problems.
  - **(i)** First is a technical → What living is as opposed to the non living means **Living v/s Non living.**
  - **(ii)** Second is a philosophical one → what the **purpose of life** is?
    As scientists we will try to solve the first question, because the second question is more related to philosophy rather science.

02. Characters of Living Organism

Following are the main characters of "living"-

GROWTH
REPRODUCTION } Not the defining properties/characters/features

METABOLISM
CELLULAR ORGANISATION } Defining properties/characters/features
CONSCIOUSNESS

The character which has no exception is called as **defining property** of life.

Growth

- Increase in mass or overall size of a tissue or organism or its parts is called growth.
- Increase in mass and increase in number of individuals are twin characters of growth.
- Growth is an irreversible permanent increase in size of an organ or its parts or even of an individual cell.
- Growth is of two types :
  - **(a) Intrinsic growth :-** This growth is from inside of the body of living organisms.
  - **(b) Extrinsic growth :-** This growth is from outside i.e. accumulation of material on any body surface Non living exhibts this type of growth.
- Growth is of two types :
  - **(a) Indeterminate growth = Unlimited growth** → Growth which occurs continuously throughout their life span is indeterminate growth or unlimited growth. It occurs in plants and not in animals.
  - **(b) Determinate growth = Limited growth** → Growth which occurs only up to a certain age is determinate growth or Limited growth. It occurs in animals. However, cell division occurs in certain tissues to replace lost cells.
- In majority of higher plants and animals, growth and reproduction are mutually **exclusive events.**
- Because both living and nonliving exhibit growth so it can not be taken as defining property.
- Growth from inside (intrinsic growth) can be taken as **defining property.**

Reproduction

Reproduction is one of the fundamental characteristics of living organisms. It can be defined as the production of new individuals of same kind by the grown up individuals. It is characteristic exhibited by living organisms which can produce new young ones of their own kind. There are two modes of reproduction — **asexual** and **sexual.**

- In lower organisms like yeasts and Hydra, budding takes place in which new individuals are produced by the formation of an outgrowth known as ‘bud’.
- Fragmentation is another mode of asexual reproduction, as in this, body of an organism (parent body) breaks up into two or more parts (known as fragments) each of which grows into a new individual. It is also quite common in filamentous algae, fungus, bryophytes (at protonema stage which occurs during life cycle in mosses).
- Planaria (flat worms) exhibit an extraordinary ability to regenerate its lost body parts completely (which is known as true regeneration).

Metabolism :

- The sum total all the chemical reactions occuring in our body is metabolism.
- All living organisms, both unicellular and multicellular exhibit metabolism.
- No non-living object shows metabolism.
- In this way metabolism is a defining character of living organisms because it has no exceptions.
- Now we have known most of the chemical or metabolic reactions occuring in our body so we can demonstrate many of them in a cell free medium or in a test tube in lab..
- The isolated metabolic reaction outside the body of an organism, performed in a test tube (in-vitro) is neither living nor nonliving.
- These isolated reaction can be regarded as living things, but they are definitely living reactions because they are similar to the reactions performing in our body.

- Here we should not forget the fact that metabolism is the total sum of all the chemical reactions performing in our body, it is not the sum of few or more living reactions.
- All organisms are made of small or big chemicals perform thousands of reactions and form some other chemicals also in the bodies of living organisms.

Consciousness

- Most obvious and technically complicated feature of all living organism. All living organisms are able to detect changes, i.e., sense their surroundings and can also respond to them. This is known as sensitivity which is defined as the ability to detect changes in the environment and to give response towards it accordingly. Any change that can be detected by an organism is called stimulus. This can be physical (like intensity, duration, direction of light, sound, change in temperature, duration of day length, i.e., photoperiod, etc.), chemical (like acids, pollutants, etc.) or biological (like other organisms).

03. Diversity In The Living World

- We consider vast areas like forest, desert, plateau etc. we find that number and kinds of living organisms increase many folds. These different kinds of plants, animals and other organisms are referred to as '**Biodiversity**' of this earth.
- **Biodiversity** is the number and various kinds of organisms found on earth. It stands for the variability found among living organisms inhabiting this world.
- **Biodiversity (Greek word bios = life; diversity = forms) or biological diversity** can be defined as the vast array of species of microorganisms, algae, fungi, plants, animals occuring on the earth either in the terrestrial or aquatic habitats and the ecological complexes of which they are a part.
- According to **IUCN** (International Union of Conservation of Nature and Natural resources), currently known and described species of all organisms are between **1.7-1.8 million**. These millions of plants, animals and other organisms in the world cannot be recognised, studied or described by an individual on its own.
- There is need to standardize the names of all living organisms after proper identification, in order to study such diverse life forms. Therefore, for better understanding of biodiversity scientists have established a definite system of principles, procedures and terms which identifies, categories and assigns specific name to each and every organism known to us. Such systems are acceptable to all biologists all over the world.
- The scientific need for simple, stable and internationally accepted systems for naming the living organisms of the world has generated, a process called **"Nomenclature"**. And, before assigning a specific name to an organism, one should determine or know its kind or features correctly, so that one can identify it in each every part of the world. This is known as **"Identification"**

04. Taxonomy

- This word was proposed by **A.P. De Candolle** in his book **"Theories elementaire de la botanique"** (Theory of elementary botany)

- **Taxonomy includes study of following 4 points**

Characterization - Identification of character
Identification - Identification of living organisms
Nomenclature - Nomenclature of living organisms
Classification - Classification of living organisms in groups

05. Systematics

(i) The term **"Systematics"** was pro posed by **Linnaeus**. The word systematics is derived from the latin word "systema" which means systematic arrangement of organisms.
(ii) **According to G. Simpson** : Systematics is the study of diversity of organisms and all their comparative and evolutionary relationship.

NOTE ☞ It includes description of external morphological characters of plants of living organisms.
eg. Morphological characters of Root, Stem, Leaves, Flowers.

06. New Systematics or Neosystematics or Biosystematics

- **Neo-systematics** – A new branch – Name given by **Julian Huxley (1940)**
- It includes description of all the characters **(external + internal)**
  e.g. Anatomical characters, Histological characters, Embryological characters, Identical characters
- It is used to know the **inter-relationship** between living organisms.

NOTE ☞ New systematics is mainly based on **evolutionary** as well as **genetic relationship** as compared to morphological characters.

07. Types of Taxonomy

- **Alpha taxonomy or classical taxonomy :** It is based on **external morphology** of plants.
- **Bete taxonomy or Explorative taxonomy :** Besides external **morphology** it also includes **internal** characters like embryological, cryological, anatomical characters etc.
- **Omega taxonomy or Encyclopaedic taxonomy :** Omega taxonomy have widest scope. It is based on **all the information** or data available about plants.
- **Chemotaxonomy :** The uses of chemical characters of plants in classification or in solving taxonomic problems is called chemotaxonomy or chemical taxonomy. It is based on the **chemical constituents** of plants. The fragrance and taste vary from species to species. The basic chemical compounds used in chemotaxonomy are **alkaloids, carotenoides, tannins,** polysaccharide, nucleic acids, fatty acids, amino acids, **aromatic compounds** etc.

- **Cytotaxonomy :** The use of cytological characters of plants in classification or in solving taxonomic problems is called cytotaxonomy. Cytological data have been used in many cases to find out the affinities among genera.
- **Karyotaxonomy :** Based on characters of **nucleus and chromosomes.** Pattern of **chromosomal bands** (dark bands and light bands) is most specific character.
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- **Adansonian system or Phenetic Classification or Numerical Classification.**
  - ❖ Proposed by **"Sokel** and **Sneath"**
  - ❖ In it plants are classified on the basis of number of similarities and dissimilarities.
  - ❖ In this, importance to any one character is not given, **all characters have same importance.**

    While in **natural** classification **floral characters** have importance than morphological characters.

08. Significance of Taxonomy

- Most significant feature is **identification** of living organism.
- With the help of taxonomy **diversity** of living being can be studied easily.
- Maximum diversity of living beings are found in **tropical rain forests.** These forest have heavy rain fall through out the year. In **India** maximum tropical rain forest are found in **Andaman & Nicobar** and in all **eastern states** (Assam, Meghalaya, West Bengal etc.)
- At present, 300 lakh **(30 million)** type of living organism are found on our earth.
- 17 lakh **(1.7 million)** type of living organism have been **discovered till now.**
- Out of them **12 lakh** types are **animal** and **5 lakh** types are **plants.**

09. Taxonomic Category

Classification is not a single step process but involves heirarchy of steps in which each step represent a rank or category.

- **Species :**
  Taxonomic studies consider a group of individual organism with fundamental similarities as a species. One should be able to distinguish one species from the other closely related species based on the distinct morphological difference.
- **Genus :**
  Genus comperises a group of related species which has more characters in common in comparison to species of other genera.
- **Family :**
  Family has a group of related genera with still less number of similarities as compared to genus and species. Families are characterized on the basis of both vegetative and reproductive feature of plant species.
  For example :Three different genera **Solanum, Petunia** and **Datura** are included in family solanaceae.
- **Order :**
  Order being a higher category is the assemblage of families which exhibit a few similar character.
  For example : Plant families like **convolvulaceae, Solanaceae** are included in the order **polymoniales mainly based on the floral characters.**
- **Class :**
  Class includes organism of related orders having less similarities than orders.
- **Division :**
  Division includes all organisms belonging to different classes having a few common characters. There 7 main taxonomic categories. They are obligate or essential or broad categories i.e. they are strictly used at the time of any plant classification. There are some extra or sub categories, like sub division, sub order, sub family, etc. They are used only when they are needed.

Biggest group →
- Kingdom
- Division/Phylum
- Class
- Order - Cohort
- Family
- Genus
- Species

❖ The classification of any plant or animal is written in **descending** or ascending order.

❖ Hierarchy – **Descending** or ascending arrangement of taxonomic categories is known as hierarchy.

❖ **Species :** Smallest taxonomic category → It is basic unit of classification.

10. Nomenclature

- **Polynomical system :**
  - ❖ According to this system name of any plant consists of many words.
  - ❖ For eg. Caryophyllum → Caryophyllum saxatilis folis gramineus umbellatis corymbis
  - ❖ Based of morphology mainly

- **Trinomical system :**
  - ❖ Proposed by **Lamarck.**
  - ❖ According to this system name of any plant or species is composed of three names-
    - **Generic name**
    - **Specific epithet**
    - **Subspecific name (Name of variety)**
  - ❖ When members of any species have large variations then trinomial system is used.
  - ❖ On the basis of dissimilarities this species is classified into sub species. eg.
    Brassica oleracea var. botrytis (Cauliflower)
    Brassica oleracea var. capitata (Cabbage)
    Brassica oleracea var. caulorapa (Knol-Khol)

- **Binomial system :**
  - ❖ Binomial system was first proposed by **Gaspard Bauhin** in his book -**"Pinax Theatre Botanica"**
  - ❖ **Carolus Linnaeus :** Linnaeus used this nomenclature system for the first time on large scale and proposed scientific name of all the plants and animals.
  - ❖ **Linnaeus** is the **founder** of binomial system.
  - ❖ Linnaeus proposed scientific name of plants in his book **"Species Plantarum"**
  - ❖ It was published on **1 May, 1753.** So this was the initiation of binomial system for plants.
  - ❖ So any name proposed (for plants) before this date is **not** accepted today.
  - ❖ **Linnaeus** proposed scientific name of animals in his book **"Systema Naturae"** (10th edition).
  - ❖ This 10th edition of **Systema Naturae** was first published on **1 August, 1758.**
  - ❖ So initiation of binomial system **for animals** is believed to be started on **1 August, 1758.**

11. ICBN

- **Main rules of ICBN :**
  - ❖ According to binomial system name of any species consists of two components or words.
    - Generic name Name of genus
    - Specific epithet

e.g. ***Solanum tuberosum*** (Potato) ***Mangifera indica*** (Mango)

↓ ↓ ↓ ↓

Generic name Specific epithet Generic name Specific epithet

- ❖ In plant nomenclature (ICBN) tautonyms are not valid i.e. generic name and specific epithet should not be same in plants.

  eg. *Mangifera mangifera*

  But tautonyms are valid in animal nomenclature (ICZN-International Code of Zoological Nomenclature)

  eg. *Naja naja* (Indian cobra), *Rattus rattus* (Rat)
- ❖ First letter of generic name should be in capital letter and first letter of specific epithet should be in small letter.

  eg. *Mangifera indica*
- ❖ But if specific epithet is based on the name of any person, its first letter should be in capital letter.

  eg. *Isoetes pantii*
- ❖ When written with free hand or typed, then generic name and specific epithet should be separately underlined.

  But during printing name should be in italics to indicate their latin orgin.
- ❖ Name of scientist (who proposed nomenclature) should be written in short after the specific epithet.

  eg. *Mangifera indica* Linn.
- ❖ Name of scientist should be neither underlined nor in italics, but written in Roman letters (simple alphabets)
- ❖ Sciectific names should be derived from Latin (usually) or Greek languages because they are dead languages.
- ❖ Type specimen (Herbarium Sheet) of newly discovered plant should be placed in herbarium (Dry garden).

12. Classification

According to **"A.P. de Candolle",** Classification is of two types

(i) Empirical Classification **(ii) Rational Classification**

- **Empirical Classification :**
  - ❖ In this type, the actual nature or character of plants is not considered.
  - ❖ Plants are classified on the basis of their alphabetical order.
  - ❖ In this way plants are classified into 26 groups.

- **Rational Classification**

  In this classification, plants are classified on the basis of their actual character or nature i.e. by viewing the characters.

13. Taxonomical Aids

- Accurate classification and identification of organisms is required which needs field studies and intensive laboratory work. This is done after collection of actual specimens of plants and animal species which is the primary source of all taxonomic studies.
- Hence, these taxonomical studies help in
  Fundamental study of different living organisms.
  Also aid in their systematic study.
  Information gathered is stored with specimens for future studies.

Herbarium

- It is defined as "store house of collected plant specimens that are dried, pressed and preserved on sheets". Further, these sheets are arranged in the sequence of a universally accepted system of classification.
- Such herbaria serve as quick source of reference in taxonomical studies. It also provides information about the local flora as well as flora of distant areas. This information is also useful in locating wild varieties and relatives of economically important plants.
  List of some Herbaria of the world :
  (i) Royal Botanical Gardens, Kew (England)
  (ii) Central National Herbarium, Calcutta

14. Botanical Gardens

Botanical garden is an institution located in an enclosed piece of land which grows numerous kinds of plants obtained from different places for botanical studies. Each plant is first identified and then labelled indicating its botanical/scientific name and its family.

List of some Botanical Gardens

(i) Royal Botanical Garden, Kew (England)
(ii) National Botanica Garden, Lucknow
(iii) Indian Botanical Garden, Howrah
(iv) Lloyd Botanical Garden, Darjeeling

15. Museum

- Museum is a place used for storing, preservation and exhibition of both plants and animals. All educational institutes and universities maintain museums in their Botany and Zoology departments.
- A commonly used preservative solutions is "Formalin". Plants and animals specimen may also be preserved as dry specimens. For instances, insects are collected, killed and pinned before preserving them in special insect boxes while larger animals like reptiles, birds and mammals are usually stuffed and then preserved.
- National Museum of Natural History (NMNH) in Delhi is important from natural science point of view.

16. Zoological Parks

- Zoological parks commonly known as zoos are the places where live wild animals are kept in protected environment which is made similar to their natural habitats as much as possible. Here, they are provided with protection and care by human beings.
- These parks serve as ideal means to study and learn different food habits and behaviour of variety of animals. So, students should visit nearby zoos for knowledge and entertainment both.
- National Zoological Park (Delhi) is one of the finest zoos of Asia.

17. Key

- Key is an important taxonomic aid used for identification of plants and animals based on the **similarities** and **dissimilarities.** Actually, it is a set of alternate characters of different types arranged sequence wise in such a fashion that by selection and elimination one can quickly find out the name of the organism.
- The keys are based on the set of contrasting characters generally in a pair known as **"couplet".** Each character of the couplet or statement in the key is called as **"lead".**
- One has to choose correct option between two statements of characters of definite species so that the animal or plant is identified accurately. **Keys are generally analytical in nature.**

18. Flora, Manuals, Monographs and Catalogues

- These flora, manuals, monographs, etc. are recorded descriptions of plants, animals and other organisms. They provide correct identification and description of variety of living organisms.
  - ❖ **Flora :** It is a book containing information about plants found in a particular area. It gives the actual account of habitat and distribution of various plants of a given area. These provide the index to the plant species found in a particular area. For example, Flora of Delhi by J.K. Maheshwari.
  - ❖ **Manual :** It is a book containing complete listing and description of the plants growing in a particular area. They provide useful information for identification of names of various species found in an area.
  - ❖ **Monograph :** It contains information of any one taxon.
  - ❖ **Catalogue :** It include the alphabetical arrangement of species of a particular place describing their features.

CBSE Exam Pattern Exercise
Subjective Questions (1)

(Q1 to 3) One Mark

1. Write the correct order of sequence of taxonomical categories.

2. What do you mean by chemotaxonomy?

3. Define genus

(Q4 to 6) Two Marks

4. Elaborate about keys as one of taxonomical aids?

5. What is classical taxonomy and modern taxonomy?

6. What is meant by living ? Give any four defining features of all life forms.

(Q7 to 8) Three Marks

7. What are taxonomical aids? Give importance of herbaria & mueseums . How are botanical gardens and zoological parks useful in conserving biodiversity?

8. Metabolism is a defining feature of all living organisms without exception. Isolated metabolic reactions in vitro are not living things but surely living reactions. Comment

(Q9 to 10) Five Marks

9. ICBN has provided a code for classification of plants. Give hierarchy of units of classification botanists follow while classifying plants.

10. What is binomial system of nomenclature? Who proposed this system? Why is binomial nomenclature the most acceptable mode of naming organism?

Answer & Solution

Q1
Species→genus → family → order → class → Phylum → kingdom

Q2
Chemotaxonomy is based on the presence or absence of certain chemicals in cells or tissues.

Q3
Terms 'Genus was coined by Tournefort. Genera are the aggregates of closely related species. For example, Potato (*Solanum tuberosum*), tomato (Solanum lycopersicum) and brinjal (Solanum nigrum) are three diff

Q4
It is a taxonomical aid used for the identification of plants and animals. The keys are based on the paired contrasting characters called as couplet. It represent two alternate forms of a given character. Keys are generally of two types
(i) **Indented or yoked keys** It has a sequence of forms of different characters.
(ii) **Bracketed keys** It includes one pair of contrasting statements for a given characters.
Separate taxonomic keys are required for each taxonomic category such as family, genus and species for identification purposes.

Q5
1. Classical Taxonomy deals with morphospecies. Modern Taxonomy deals with biological species.
2. Classical Taxonomy has typological concept. Modern Taxonomy has a population or biosystematic concept.
3. In Classical taxonomy, species is considered to be static. In moder taxonomy, species is considered to be dynamic.
4. Classical taxonomy does not study evolution and inter-relationships of species. Modern Taxonomy studies premitiveness, advancement and inter-relationships of specoes.

Q6
The living organism exhibit distinctive characteristics, which are as follows (i) Growth All living organisms grow in size as well as in number. Plants show growth all through their life whereas animals up to a certain growth period after which growth ceases. (ii) Reproduction The process of

reproduction is essential for the continuity of life on earth. Every organism whether unicellular or multicellular gives rise to an individual of its own kind. Lower organisms usually reproduce asexually, e.g.. Hydra, fungi, yeast, etc. Sexual reproduction is found in advanced group of organisms, which involves two parents. In Amoeba growth and reproduction however is synonymous. (iii) Metabolism There are thousands of enzymatic ally controlled reactions occurring in all living cells. These are synthesizing (anabolic) and breaking down (catabolic) reactions. Hence, it is the most important characteristic of living organisms. (iv) Response to stimuli Consciousness and response to stimulus is the defining property of all living organisms. Plants respond to light, water temperature. Unicellular organisms also sense their environment and respond accordingly. (v) Cellular organization The cellular organization of the body is the defining feature of life forms. Cells work together in hierarchical manner group of cells make tissues, tissues make organs, organs make systems, systems when work in co-ordination form an individual. So, we can say living organisms are self replicating, evolving and capable of responding to external stimuli.

Q7
The taxonomic aids are the aids which help in identification, classification and naming of a newly discovered organisms (plant or animal). It could be in the form of preserved document like herbaria or specimen kept at museums or scientific institutions. Other aids can be in the form of written document like monograph ,taxonomic keys, couplets, etc. A new organism found can be studied while comparing it with living plants and animals living in protected areas like Botanical gardens, Zoological parks, etc. Botanical gardens helps in conservation of plants by (i) Growing important local plant species and keeping record of them. (ii) Growing and maintaining rare and endangered species. (iii) Supplying seeds for different, aspects of botanical research. Whereas zoological parks also contribute in conserving biodiversity by (i) Providing natural environment and open space to animals, i.e., wild life species. (ii) Keeping them safe from their predators ensuring protection, food and shelter. (iii) Providing home to different native and exotic wild animals. (iv) Involving in the rescue of endangered species. (v) Facilitating breeding of animals and releasing them free. Thus, both botanical gardens and zoological parks play an important role in conservation ofbiodiversity.

Q8
All living organisms are made of chemicals which belong to various classes and have different sizes and functions. In the cells, these chemicals are constantly being made and changed into some other biomolecules. These conversions or chemical reactions taking place in the body are collectively known as metabolism. All plants, animals, fungi and microbes exhibit metabolism. Non-living objects do not exhibit metabolism. So we can say metabolism is a defining feature of all living organisms without exception.
Metabolic reactions can be demonstrated outside the body in a test-tube. A metabolic reaction preformed in a test-tube (in vitro) is neither living nor non-living but surely living reaction.

Q9
The study of taxonomy has led to the taxonomic categories – Kingdom, phylum, class, order, family, genus, and species. Now let us see how all the organisms are classified into the hierarchy.

Species

It is the lowest level of classification and shows the high level of similarities among the organisms. One species can be distinguished from other closely related species based on distinct differences in morphology. Let us look at an example; Plasmodium vivax and Plasmodium falciparum – Both are malaria-causing parasites but have different effects on a patient. Plasmodium is the name of the genus and has a number of species which show distinct morphological characteristics.

Genus

This taxonomic group comprises several species which have similar characteristics but different from that of species from another genus. Considering the same example of Plasmodium, it is a genius with multiple species which are similar to each other and differs from the species of another genus.

Family

This category of taxonomy includes various genus which shares some resemblance among themselves. However, the number of similarities decrease compared to species and genus. For example, the genus of tiger, leopard, lion, i.e., Panthera and the genus of cats i.e., Felis are grouped together in the family Felidae.

Order

The classification which begins with the order has fewer comparisons as an effect, they are grouped based on aggregates of characteristics. A group of families showing somewhat few similarities forms an order. For example, the order of carnivores i.e., Carnivore includes families like Felidae and Canidae.

Class

A group of Orders which share a few similarities forms a Class. For example- Orders of primates and carnivores are grouped together in the Class of mammals.

Phylum

This is the next level of classification was along a number of Classes are clubbed up to form one Phylum. Example – aves, fishes, reptiles, mammals, and amphibians combined to form the phylum of or Chordata.

Kingdom

This is the highest level of classification. We have Kingdoms like Plantae, Animalia, Fungi, Protista, and Monera.

Q10

Binomial nomenclature, introduced by Carolus Linnaeus is the method of naming an organism with the genus name first and species name later.

Conventions followed while naming

*Name of the genus begins with a capital letter.

*Name of the species should begin with a small letter.

*Scientific name should be in Italics when printed.

*Genus name and the species name should be underlined separately while handwritten.

The advantages of scientific over common names are that they are accepted by speakers of all languages, that each name applies only to one species, and that each species has only one name.

CLASS 11

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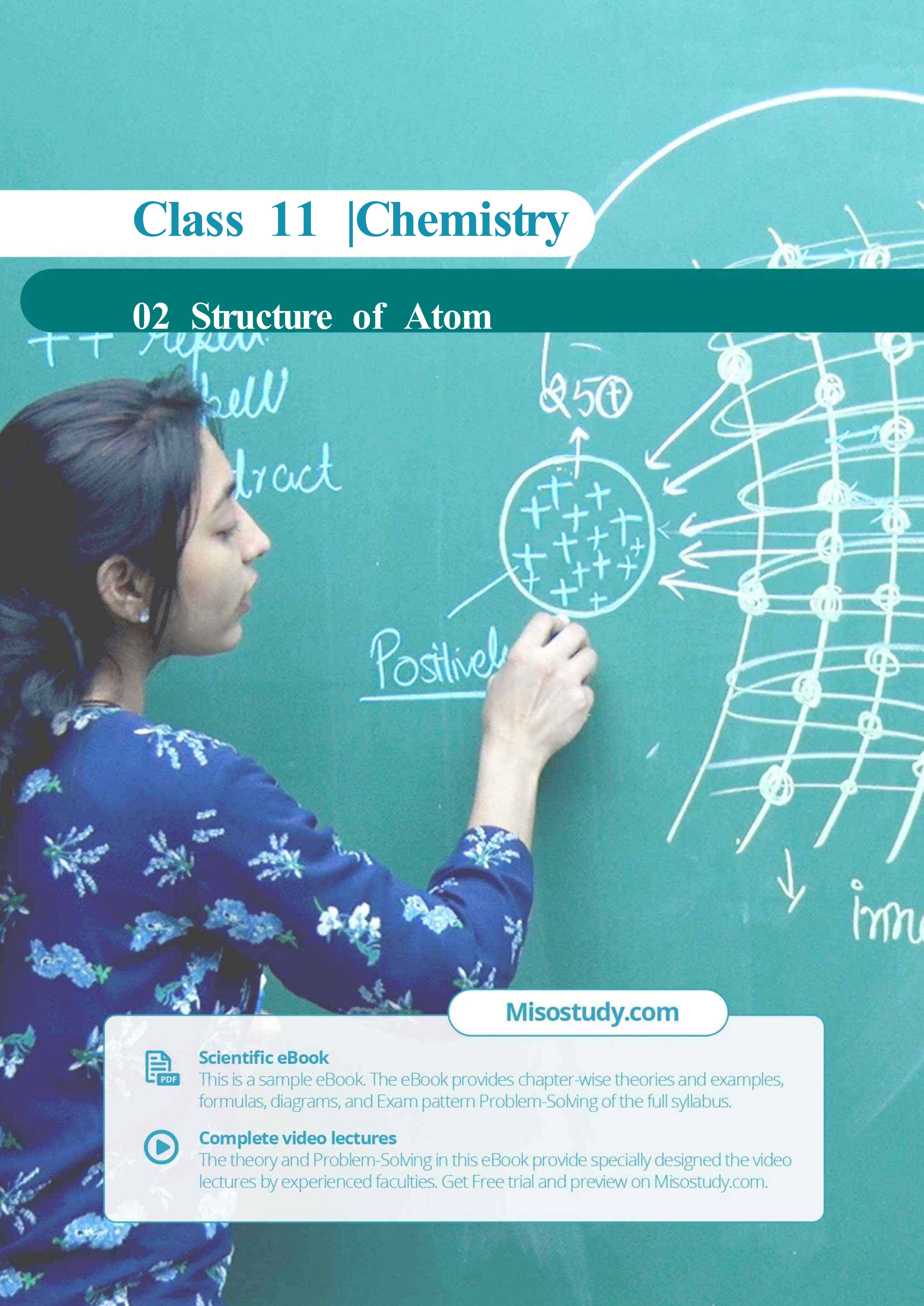
Class 11 |Chemistry

02 Structure of Atom



01. Introduction

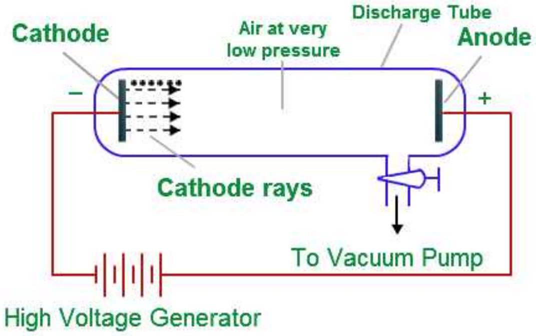
John Dalton 1808, believed that matter is made up of extremely minute indivisible particles, called atom which takes part in chemical reactions. These particle can neither be created nor be destroyed. However, modern researches have conclusively proves that atom is no longer an indivisible particle. Modern structure of atom is based on Rutherford's scattering experiment, quantization of energy and wave mechanical model.

Composition of Atom

The works of J.J. Thomson and Ernst Rutherford actually laid the foundation of the modern picture of the atom. If is now verified that the atom consists of several sub-atomic particles like electron, proton, neutron, positron, neutrino, meson etc. Out of these particles the electron, proton and the neutron are called fundamental subatomic particles.

02. ELECTRON ($_{-1}e^0$, e)

Electron was discovered by J.J. Thomson(1897) and it is a negatively charged particle.
Cathode rays were discovered by William Crooke & J.J. Thomson using a cylindrical hard glass tube fitted with two metallic electrodes. This tube was known as discharge tube. They passed electricity (10,000V) through a discharge tube at very low pressure. Blue rays emerged from the cathode. These reys were termed as Cathode rays.



Properties of Cathode rays

(i) Cathode rays travel in straight line.

(ii) Cathode rays produce mechanical effect, as they can rotate the wheel placed in their path.

(iii) Cathode rays consist of negatively charged particles known as electron.

(iv) Cathode rays travel with high speed.
(v) Cathode rays can cause fluorescence.
(vi) Cathode rays heat the object on which they fall due to transfer of kinetic energy to the object.
(vii) When cathode rays fall on heavy metals, X-rays are produced.
(viii) Cathode rays possess ionizing power i.e., they ionize the gas through which they pass.
(ix) The cathode rays produce scintillation on the photographic plates.
(x) They can penetrate through thin metallic sheets.
(xi) The nature of these rays does not

03. Thomson's Model of Atom [1904]

- Thomson was the first to propose a detailed model of the atom.
- Thomson proposed that an atom consists of a uniform sphere of positive charge in which the electrons are distributed more or less uniformly.
- This model of atom is known as "Plum-Pudding model" or "Raisin Pudding Model" or "Water Melon Model".

Drawbacks:

- An important drawback of this model is that the mass of the atoms is considered to be evenly spread over that atom.
- It is a static model. It does not reflect the movement of electron.
- It could not explain the stability of an atom.

04. Rutherford's Scattering Experiment

Rutherford observed that

(i) Most of the α-particles (nearly 99.9%) went straight without suffering any deflection.
(ii) A few of them got deflected through small angles.
(iii) A very few (about one in 20,000) did not pass through the foil at all but suffered large deflections (more than 90°) or even came back in the direction from which they have come i.e. a deflection of 180°.

Following conclusions were drawn from the above observations-

(i) Since most of the α-particle went straight through the metal foil undeflected, it means that there must be very large empty space within the atom.
(ii) Since few of the α-particles were deflected from their original paths through moderate angles; it was concluded that whole of the +ve charge is concentrated and the space occupied by this positive charge is very small in the atom.
  - When α-particles come closer to this point, they suffer a force of repulsion and deviate from their paths.
  - The positively charged heavy mass which occupies only a small volume in an atom is called nucleus. It is supposed to be present at the centre of the atom.

(iii) A very few of the α-particles suffered strong deflections on even returned on their path indicating that the nucleus is rigid and α-particles recoil due to direct collision with the heavy positively charged mass.

Drawbacks of rutherford model-

(i) This theory could not explain stability of atom. According to Maxwell electron loose its energy continuously in the form of electromagnetic radiations. As a result of this, the e- should loose energy at every turn and move close and closer to the nucleus following a spiral path. The unlimited result will be that it will fall into the nucleus, thereby making the atom unstable.

(ii) If the electrons loose energy continuously, the observed spectrum should be continuous but the actual observed spectrum consists of well defined lines of definite frequencies. Hence the loss of energy by electron is not continuous in an atom.

05. Distance of closest approach :

When the α-particles approaches the nucleus to made a head-on collision with a the nucleus, the α-particle approaches the nucleus until coulombic potential energy of repulsion, $k\frac{Z_1Z_2e^2}{r}$, becomes equal to its initial K.E., $\frac{1}{2}$ m.v^2.

Thus $\frac{1}{2}\ mv^2 = k\frac{Z_1Z_2e^2}{r}$

Hence, the distance of closest approach, $r = \frac{k\, z_1\, z_2\, e^2}{\left(\frac{1}{2}mv^2\right)}$.

The nucleus must be further smaller than the distance of closest approach.

06. Moseley Experiment (Discovery of Atomic Number)

Moseley (1912-1913), investigated the X-rays spectra of 38 different elements, starting from aluminium and ending in gold. He measured the frequency of principal lines of a particular series (the α-lines in the K series) of the spectra. It was observed that the frequency of a particular spectral line gradually increased with the increase of atomic mass of the element. But, it was soon realised that the frequency of the particular spectral line was more precisely related with the serial number of the element in the periodic table which he termed as atomic number (Z). He presented the following relationship:

$$\sqrt{v} = a(Z-b)$$

where, v=frequency of X-rays, Z=atomic number, ‘a’ and ‘b’ are constants. When the values of square root of the frequency were plotted against atomic number of the elements producing X-rays, a straight line was obtained.

07. Some Atomic Terms

Atomic number = Number of unit positive charge on nucleus = Number of protons in nucleus = Number of electrons in neutral atom.
Two different elements can never have identical atomic number.

Mass number(A) = Number of protons + Number of neutrons or Atomic number (Z)
Hence Number of neutrons = A – Z.
The atom of an element X having mass number (A) and atomic number (Z) may be represented by a symbol,

Isotopes : Atoms of an element with the same atomic number but different mass number.

eg.	$_1H^1$	$_1H^2$	$_1H^3$
	Protonium	deuterium	Tritium

Isodiapheres : The elements which have same value of (n – p) is called Isodiapheres.

eg.	$_7N^{14}$	$_8O^{16}$
Values of (n - p)	0	0

Isotone : Elements which contain same no. of neutron is called **Isotone**.

eg.	$_{14}Si^{30}$	$_{15}P^{31}$	$_{16}S^{32}$
number of neutrons	16	16	16

Isobar : The two different atoms which have same mass number but different atomic number is called Isobar.

eg.	$_{18}Ar^{40}$	$_{19}K^{40}$	$_{20}Ca^{40}$

Isoelectronic : Ion or atom or molecule which have the same mass number of electron is called Isoelectronic species.

eg.	$_{17}Cl^-$	$_{18}Ar$	$_{19}K^+$	$_{19}Ca^{+2}$
No. of electrons	18	18	18	18

08. Bohr's Atomic Model

This model was based on the quantum theory of radiation and the classical law of physics. It gave new idea of atomic structure in order to explain the stability of the atom and emission of sharp spectral lines.

Postulates

(i) The atom has a central massive core nucleus where all the protons and neutrons are present. The size of the nucleus is very small.

(ii) The electron in an atom revolve around the nucleus in certain discrete orbits. Such orbits are known as stable orbits or non – radiating or stationary orbits.

(iii) An electron can move only in those permissive orbits in which the angular momentum (mvr) of the electron is an integral multiple of h/2π Thus,

$$mvr = n\frac{h}{2\pi}$$

Where, m = mass of the electron, r = radius of the electronic orbit, v = velocity of the electron in its orbit.

(iv) The angular momentum can be $\frac{h}{2\pi}, \frac{2h}{2\pi}, \frac{3h}{2\pi}, ...\frac{nh}{2\pi}$. This principal is known as quantization of angular momentum. In the above equation 'n' is positive integer which has been called as principal quantum number. It can have the values n=1,2,3,...... (form the nucleus). Various energy levels are designed as K(n=1), L(n=2), M(n=3),......etc. Since the electron present in these orbits is associated with some energy, these orbits are called energy levels.

(v) The emission or absorption of radiation by the atom takes place when an electron jumps from one stationary orbit to another.

09. Calculation of the radius of the Bohr's orbit :

Suppose that an electron having mass 'm' and charge 'e' revolving around the nucleus of charge 'Ze' (Z is atomic number & e is charge) with a tangential/linear velocity of 'v'. Further consider that 'r' is the radius of the orbit in which electron is revolving.

According to Coulomb's law, the electrostatic force of attraction (F) between the moving electron and nucleus is -

$$F = \frac{KZe^2}{r^2}$$

Where : $K = \text{constant} = \frac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{Nm}^2/\text{C}^2$

and the centripetal force $F = \frac{mv^2}{r}$

Hence $\frac{mv^2}{r} = \frac{KZe^2}{r^2}$ or, $v^2 = \frac{KZe^2}{mr}$ ...(1)

From the postulate of Bohr, $mvr = \frac{nh}{2\pi}$ or, $v^2 = \frac{n^2h^2}{4\pi^2m^2r^2}$ ...(2)

From equation (1) and (2) :

$\therefore\ r = \frac{n^2h^2}{4\pi^2mKZe^2}$ On putting the value or e, h, m,

$r = 0.529\times\frac{n^2}{Z}\text{A}$

10. Calculation of velocity of an electron in Bohr's orbit

Velocity of the revolving electron in n^{th} orbit is given by

$mvr = \frac{nh}{2\pi}$ $v = \frac{nhy}{2\pi mr}$

Putting the value of r in above equation then $v = \dfrac{nh \times 4\pi^2 mZe^2}{2\pi mn^2h^2}$, $v = \dfrac{2\pi Ze^2}{nh}$

on putting the values of e and h,

$v = 2.188 \times 10^6 \times \dfrac{Z}{n} m/sec$

11. Calculation of energy of an electron :

The total energy of an electron revolving in a particular orbit is -
T. E. = K. E. + P. E.

The K.E. of an electron $= \dfrac{1}{2}mv^2$

and the P.E. of an electron $= -\dfrac{KZe^2}{r}$

Hence, $\text{T.E.} = \dfrac{1}{2}mv^2 - \dfrac{KZe^2}{r}$ .....(3)

But $\dfrac{mv^2}{r} = \dfrac{KZe^2}{r^2}$ or $mv^2 = \dfrac{KZe^2}{r}$

Substituting the value of mv^2 in the equation (3)

$$\text{T.E.} = \frac{KZe^2}{2r} - \frac{KZe^2}{r} = -\frac{KZe^2}{2r}$$

So, $\text{T.E.} = -\dfrac{KZe^2}{2r}$

Substituting the value of 'r' in the equation of T.E. .

$$E = -\frac{kZe^2}{2} \times \frac{4\pi^2 Ze^2 mk}{n^2h^2} = -\frac{2\pi^2 Z^2 e^4 mk^2}{n^2h^2}$$

Thus, the total energy of an electron in n^{th} orbit is given by

$$E_n = \frac{2\pi^2 Z^2 e^4 mk^2}{n^2h^2}$$

$$= -13.6 \times \frac{Z^2}{n^2} \text{ eV/atom}$$

$$= -21.8 \times 10^{-19} \times \frac{Z^2}{n^2} \text{ J/atom}$$

$$= -313.6 \times \frac{Z^2}{n^2} \text{ Kcal/mole}$$

12. Relation between P. E., K. E. & T. E. :

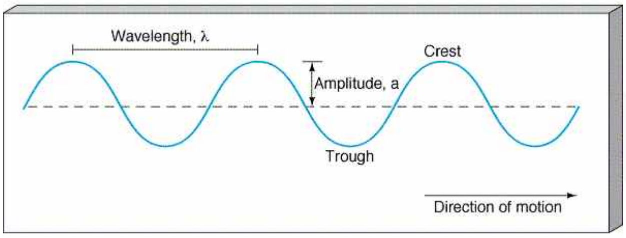
$\text{P.E.} = -\dfrac{kZe^2}{r}$, $\text{K.E.} = \dfrac{1}{2}\dfrac{kZe^2}{r}$, $\text{T.E.} = -\dfrac{1}{2}\dfrac{kZe^2}{r}$,

$$\text{T.E.} = \frac{\text{P.E.}}{2} = -\text{K.E.}$$

13. Details About Waves

A wave motion is a means of transferring energy from one point to another point without any actual transportation of matter between these points. When we throw a piece of stone on the surface of water in a pond, we observe circles of ever increasing radius, till they strike the wall of the pond.

14. Some important terms related with wave motion.



(i) **Wave length (λ):** The distance between two adjacent crest or trough of the wave (or the distance between two similar neighbouring points)

(ii) **Time period (T):** Time for one complete oscillation of wave is called the period (T). Time taken by the wave to travel a distance equal to one wavelength. If C is the speed of wave, then $C\frac{\lambda}{T}$.

(iii) **Frequency (v):** Number of oscillations per unit time is called frequency. $v\frac{C}{\lambda}$

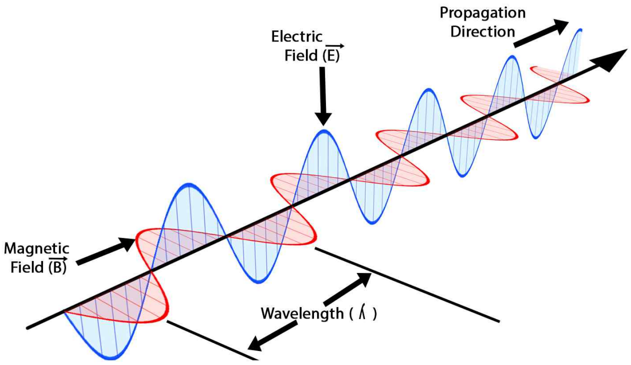
(iv) **Wave number $(\bar{v})$:** Number of wavelength per unit length. $(\bar{v}) = \frac{1}{\lambda}$

(v) **Amplitude (A):** It is the height of crest or depth of a trough of a wave.

15. Characteristics of electromagnetic radiations:

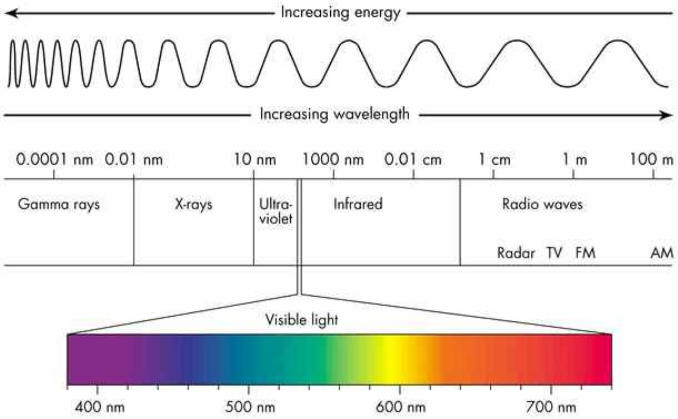
(i) All electromagnetic waves move or travel with the same speed equal to that of light.
(ii) They do not require any medium to propagate.
(iii) These consist of electric and magnetic field that oscillate in the direction perpendicular to each other and to the direction in which the wave propagates (as shown in above diagram)

Electromagnetic Wave



16. Electromagnetic Spectrum

Arrangement of various types of electromagnetic radiations in order of their increasing (or decreasing) wavelengths or frequencies is known as electromagnetic spectrum.



17. Planck's Quantum Theory

According to this theory atoms or molecules could emit or absorb energy only in discrete quantities (small packets) and not in any arbitary amount. Planck gave the name quantum to the smallest quantity of energy that can be emitted in the form of E.M. radiation.
The energy of a photon is proportional to its frequency and is given by
$E = h\upsilon$ where $h = 6.626\times10^{-34}$ J sec
A body can emit or absorb energy only in terms of the integral multiples of quantum, i.e.
$E = n.\ h\upsilon$, where n = 1,2,3,.....

i.e. a body can emit or absorb energy as $h\upsilon$, $2h\upsilon$ ..... but it can not emit or absorb energy in fractional values of $h\upsilon$ such as 1.5 $h\upsilon$, 2.5 $h\upsilon$.
Einstein supported the planck's theory and explained the photoelectric effect considering that electromagnetic radiations also propagate in the form of photon. Energy of each photon depends on frequency of light $\left(E = h\upsilon = \frac{hc}{\lambda}\right)$.

Since wave character of light explains the interference and diffraction phenomenon while the particle character explains, black body radiations and photoelectric effect, the light was considered to have wave as well as particle character [Dual character of light.]

(i) Wave nature : diffraction, interference, polarisation.
(ii) Particle nature : photoelectric effect.

18. Hydrogen Spectrum

Hydrogen spectrum is an example of line or atomic emission spectrum. When an electric discharge is passed through hydrogen gas at low pressure, a bluish light is emitted. All these lines of H-spectrum have Lyman, Balmer, Paschen, Barckett, Pfund and Humphrey series. These spectral series were named by the name of scientist discovered them. To evalute wavelength of various H-lines Rydberg introduced the following expression,

$$\bar{v} = \frac{1}{\lambda} = \frac{v}{c} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where R is constant known as Rydberg's constant its value is 109, 67800 m^{-1}
Although H-atom consists only one electron yet it's spectra consist of many spectral lines because electrons in different hydrogen atoms absorb different amount of energies and are excited to different energy levels. Since life time of electrons in these excited states is very small, they return to some lower energy level or even to the ground state in one or more jumps.

Maximum number of lines produced when an electron jumps from nth level to ground level = $\frac{n(n-1)}{2}$.

19. Lyman Series

(a) It is a first series of spectral series of H.
(b) It's value of $n_1 = 1$ and $n_2 = 2,3,4$, where 'n_1' is ground state and 'n_2' is called excited state of electron present in *H*-atom.
(c) If the electron goes to $n_1 = 1$ from $n_2 = 2$ — first Lyman series
If the electron goes to $n_1 = 1$ from $n_2 = 3$ — Second Lyman series
If the electron goes to $n_1 = 1$ from $n_2 = 4$ — third Lyman series ----- so on.

(d) $\frac{1}{\lambda} = R_H\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right]$ where $n_2 > 1$ always.

(e) For marginal line of limiting line $n_2 = \infty$. Hence the wavelength of marginal line $= \frac{n_1^2}{R_H}$ for all series. So, for lyman series it is $\frac{1}{R_H}$.

20. Balmer Series :

(i) It is second series of H-spectral series.
(ii) It was found out in 1892 in visible region by **Balmer.**
(iii) Blamer series was found out before all series because it was found in visible region.
(iv) It's value of $n_1 = 2$ and $n_2 = 3,4,5$ ....
(v) If the electron goes to $n_1 = 2$ from $n_2 = 3$ — first Balmer series
If the electron goes to $n_1 = 2$ from $n_2 = 4$ — Second Balmer series
If the electron goes to $n_1 = 2$ from $n_2 = 5$ — third Balmer series ----- so on.

(vi) The wavelength of marginal line of Balmer series $= \frac{n_1^2}{R_H} = \frac{2^2}{R_H} = \frac{4}{R_H}$.

(vii) $\frac{1}{\lambda} = R_H\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$ where $n_2 > 2$ always.

21. Paschen Series :

(i) It is the third series of H – spectrum.
(ii) It was found out in infra red region by ***Paschen.***
(iii) It's value of $n_1 = 3$ and $n_2 = 4,5,6$ ....
(iv) If the electron goes to $n_1 = 3$ from $n_2 = 4$ — first Paschen series
If the electron goes to $n_1 = 3$ from $n_2 = 5$ — Second Paschen series
If the electron goes to $n_1 = 3$ from $n_2 = 6$ — third Paschen series ----- so on.
(v) The wavelength of marginal line of paschen series $= \frac{n_1^2}{R_H} = \frac{3^2}{R_H} = \frac{9}{R_H}$.
(vi) $\frac{1}{\lambda} = R_H\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$ where $n_2 > 3$ always.

22. Brackett Series :

(i) It is fourth series of H – spectrum.
(ii) It was found out in infra red region by **Brackett.**
(iii) It's value of $n_1 = 4$ and $n_2 = 5,6,7$ ....

(iv) If the electron goes to $n_1 = 4$ from $n_2 = 5$ — first Brackett series
If the electron goes to $n_1 = 4$ from $n_2 = 6$ — Second Brackett series
If the electron goes to $n_1 = 4$ from $n_2 = 7$ — third Brackett series ----- so on.

(v) The wavelength of marginal line of Brackett series $= \dfrac{n_1^2}{R_H} = \dfrac{4^2}{R_H} = \dfrac{16}{R_H}$.

(vi) $\dfrac{1}{\lambda} = R_H\left[\dfrac{1}{4^2} - \dfrac{1}{n_2^2}\right]$ where $n_2 > 4$ always.

23. Pfund Series :

(i) It is fifth series of H – spectrum.
(ii) It was found out in infra red region by Pfund.
(iii) It's value of $n_1 = 5$ and $n_2 = 6,7,8$ ....
(iv) If the electron goes to $n_1 = 5$ from $n_2 = 6$ — first Pfund series
If the electron goes to $n_1 = 5$ from $n_2 = 7$ — Second Pfund series
If the electron goes to $n_1 = 5$ from $n_2 = 8$ — third Pfund series ----- so on.

(v) The wavelength of marginal line of Pfund series $= \dfrac{n_1^2}{R_H} = \dfrac{5^2}{R_H} = \dfrac{25}{R_H}$.

(vi) $\dfrac{1}{\lambda} = R_H\left[\dfrac{1}{5^2} - \dfrac{1}{n_2^2}\right]$ where $n_2 > 5$ always.

24. Calculation of Rydberg Constant from bohr's atomic model

Suppose that an electron transit from first energy. level to second energy level. Then, the change of energy. is given by

$\triangle E = E_{n_2} - E_{n_1}$

or, $= \left[\dfrac{-2\pi^2 mZ^2e^4k^2}{n_2^2h^2}\right] - \left[\dfrac{-2\pi^2 mZ^2e^4k^2}{n_1^2h^2}\right]$ or, $\dfrac{hc}{\lambda} = \dfrac{-2\pi^2 mZ^2e^4k^2}{h^2} \times \left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

or, $\dfrac{1}{\lambda} = \dfrac{2\pi^2 mZ^2e^4k^2}{ch^3}\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right] = R_H{\cdot}Z^2\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

or, $R_H = \dfrac{2\pi^2 me^4k^2}{ch^3}$ Rydberg constant

25. Particle and Wave Nature of Electron

In 1924,de Broglie proposed that an electron, like light, behaves both as material particle and as a wave. This proposal gave a new theory, known as wave mechanical theory of matter. According to this theory, the electrons, protons and even atoms, when in motion, posses wave properties.

de Broglie derived an expression for calculating the wavelength of the wave associated with the electron. According to Planck's equation

$$E = h\upsilon = h.\frac{c}{\lambda} \qquad .....(1)$$

The energy of a photon on the basis of Einstein's mass-energy relationship is

$$E = mc^2 \qquad ....(2)$$

Where, c is the velocity of the electron.
From (1) and (2) equation

$$h \times \frac{c}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc} = \frac{h}{p} \qquad .....(3)$$

Momentum of the moving electron is inversely proportional to its wavelength.
Let kinetic energy of the particle of mass 'm' is E.

$$E = \frac{1}{2}mv^2$$

$$2Em = m^2v^2$$

$$\sqrt{2Em} = mv = p\,(\text{momentum})$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2Em}} \qquad .....(4)$$

Let a charged particle, with charge q be accelerated with a potential of V; then the kinetic energy may be given as : E=qV

$$\frac{1}{2}mv^2 = qV$$

$$m^2v^2 = 2qVm$$

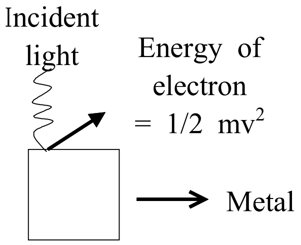
$$mv = \sqrt{2qVm}$$

and,

$$\lambda = \frac{h}{\sqrt{2qVm}} \qquad .....(5)$$

26. Photoelectric Effect

Emission of electrons from a metal surface when exposed to light radiations of appropriate wavelength is called photoelectric effect. The emitted electrons are called photoelectrons.



Work function or threshold energy may be defined as the minimum amount of energy required to eject electrons from a metal surface. According to Einstein,
Maximum kinetic energy of the ejected electron = absorbed energy – work function

$$\frac{1}{2}\,mv_{max}^{2} = hv - hv_0 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$

where, v_0 and λ_0 are threshold frequency and threshold wavelength respectively.

Stopping Potential

The minimum potential at which the photoelectric current becomes zero is called stopping potential.

If V_0 is the stopping potential, then

$$eV_0 = h\left(v - v_0\right) \text{ or } V_0 = \frac{h\left(v - v_0\right)}{e} = \frac{K.E._{max}}{e}$$

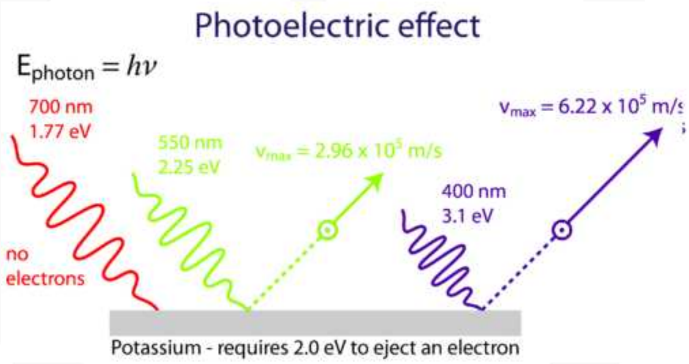
Some facts of Photoelectric Effect

(i) There is no time lag between incidence of light and emission of photoelectrons.

(ii) For emission of photoelectrons, the frequency of incident light must be equal to or greater than the threshold frequency.

(iii) Rate of emission of photoelectrons from a metal surface is directly proportional to the intensity of incident light.

(iv) The maximum kinetic energy of photoelectrons depends on the frequency of incident radiation; but, it is independent of the intensity of light used.



27. Heisenberg's Uncertainty Principle

According to this principle it is impossible to measure simultaneously the exact position and exact momentum of a body as small as an electron. If uncertainty of measurement of position is $\triangle x$ and uncertainty of measurement of momentum is $\triangle p$ or $m\triangle v$, then according to Heisenberg.

$$\triangle x \,.\, \triangle p \;\geq\; \frac{h}{4\pi} \qquad \text{or} \qquad \triangle x \,.\, m\triangle v \;\geq\; \frac{h}{4\pi}$$

where h is planck's constant

Like de Broglie equation, this principle has significance only for microscopic particles.

28. Wave Mechanical Model of Atom

The atomic model which is based on the particle and wave nature of the electron is known as wave mechanical model of the atom. This was developed by Erwin Schrodinger in 1926. This model describes the electron as a three-dimensional wave in the electronic field of positively charged nucleus.

Significance of Ψ :
The wave function may be regarded as the amplitude function expressed in terms of coordinates x, y and z. The wave function may have positive of negative values depending upon the values of coordinates

Significance of Ψ^2 :
Ψ^2 is a probability factor. It describes the probability of finding an electron within a small space. The space in which there is maximum probability of finding an electron is termed as orbital.

29. Quantum Numbers

Quantum numbers are to specify and display to complete information about size, shape ans orientation of the orbital. These are principal, azimuthal and magnetic quantum number, which follows directly from solution of schrodinger wave equation.

30. Principal Quantum Number (n) :

(i) It was proposed by Bohr and denoted by ‘n’.
(ii) It determines the average distance between electron and nucleus, means it is denoted the size of atom.
(iii) It determine the energy of the electron in an orbit where electron is present.
(iv) The maximum number of an electron in an orbit represented by this quantum number as $2n^2$.

31. Azimuthal Quantum Number of Angular Quantum Number (*l*) -

(i) It was proposed by Sommerfield and denoted by ‘*l*’.
(ii) It determines the number of subshells or sublevels to which the electron belongs.
(iii) It tells about the shape of subshells.
(iv) It also expresses the energies of subshells s<p<d<f (Increasing energy).
(v) The value of *l* is integral values upto (n-1), starting from zero where ‘n’ is the number of principle shell.

(vi)

Value of l	0	1	2	3
Name of subshell	s	p	d	f
Shape of orbital	spherical	Dumbbell	Clover leaf (Except d_{z^2} doughtnut)	Complex

(vii) It represent the orbital angular momentum, which is equal to $\frac{h}{2\pi}\sqrt{l(l+1)}$.

(viii) The number of electrons in subshell = $2(2l+1)$.

(ix) For a given value of 'n' the total value of 'l' is always equal to the value of 'n'.

32. Magnetic Quantum Number (m) :

(i) It gives the number of permitted orientation of subshells.

(ii) The value of m varies from $-l$ to $+l$ through zero.

(iii) Degenerate orbitals – Orbitals having the same energy are known as degenerate orbitals. e.g. for p subshell P_x, P_y and P_z are degenerate orbital.

(iv) The number of degenerate orbitals of s subshell = 0.

33. Spin quantum number (s) :

(i) The value of 's' is $+\frac{1}{2}$ or $-\frac{1}{2}$, which is signified as the spin or rotation or direction of electron on it's axis during the movement.

(ii) The spin may be clockwise or anticlockwise.

(iii) It represents the value of spin angular momentum is equal to $\frac{h}{2\pi}\sqrt{s(s+1)}$.

(iv) Maximum spin of an atom = $\frac{1}{2}\times$number of unpaired electron.

34. Shape and size of orbitals

An orbital is the region of space around the nucleus within which the probability of finding an electron of given energy is maximum (90-95%). The shape of this region (electron cloud) gives the shape of the orbital. It is basically determined by the azimuthal quantum number l, while the orientation of orbital depends on the magnetic quantum number (m).

35. s-orbital (l = 0):

These orbitals are spherical and symmetrical about the nucleus. The probability of finding the electron is maximum near the nucleus and keeps on decreasing as the distance from the nucleus increases. There is vacant space between two successive s-orbitals known as radial node. But there is no radial node for 1s orbital since it is starting from the nucleus.

36. p-orbital (l = 1):

The probability of finding the p-electron is maximum in two lobes on the opposite sides of the nucleus. This gives rise to dumb-bell shape for the p-orbital $l = 1$.
Hence, m = –1, 0, +1. Thus, p-orbital have three different orientations. These are designated as p_x, p_y & p_z depending upon whether the density of electron is maximum along the x y and z axis respectively.

37. d-orbital (l = 2):

For d-orbitals, l = 2. Hence m = –2, –1, 0, +1, +2. Thus there are 5d orbitals. They have relatively complex geometry. Out of the five orbitals, the three (d_{xy}, d_{yz}, d_{zx},) project in between the axis and the other two d_{z^2} and $d_{x^2-y^2}$ lie along the axis.

38. Spherical nodes :

The spherical surface where probability of finding the electron is zero, is called spherical nodes.
No. of spherical nodes in any orbital = $n-l-1$

39. Nodal Plane :

This is a plane passing through the nucleus where the probability of finding the electron is zero.
Number of nodal plane in a orbital = l

Orbital	Nodal plane
p_x	yz
p_y	xz
p_z	xy
d_{xy}	yz,zx
d_{yz}	xy,xz
d_{zx}	xy,yz

40. Electronic Configuration

The distribution of electrons in different orbitals of atom is known as electronic configuration of the atoms. Filling up of orbitals in the ground state of atom is governed by the following rules:

41. Aufbau Principle

According to this principle, "In the ground state, the atomic orbitals are filled in order of increasing energies i.e. in the ground state the electrons first occupy the lowest energy orbitals available".
Lower the value of $n+l$, lower is the energy of the orbital and such an orbital will be filled up first.

When two orbitals have same value of $(n+l)$ the orbital having lower value of "n" has lower energy and such an orbital will be filled up first.
Thus, order of filling up of orbitals is as follows:
$1s<2s<2p<3s<3p<4s<3d<4p<5s<4d<5p<6s<4f<5d$

42. Pauli's Exclusion Principle

According to this principle, "No two electrons in an atom can have same set of all the four quantum numbers n, l, m and s.
According to this principle an orbital can accommodate at the most two electrons with spins opposite to each other. It means that an orbital can have 0, 1, or 2 electron. If an orbital has two electrons they must be of opposite spin.

43. Hund's Rule of Maximum Multiplicity

According to this rule "Electron filling will not take place in orbitals of same energy until all the available orbitals of a given subshell contain one electron each with parallel spin".

44. Extra stability If half filled and completely filled orbitals

Symmetry of orbitals
If the shift of an electron from one orbital to another orbital differing slightly in energy results in the symmetrical electronic configuration. It becomes more stable. For example p^3, d^5, f^7 configurations are more stable then their near once.
Exchange energy
In case of half filled and completely filled orbitals, the exchange energy is maximum and is greater than the loss of energy due to the transfer of electron from a higher to a lower sublevel e.g. from 4s to 3d orbitals in case of Cu and Cr.

CBSE Exam Pattern Exercise
Subjective Questions (1)

(Q. No. 1 to 2) One Mark

1. Which of the following are isoelectronic species, i.e., those having the same number of electrons?

   Na^+, K^+, Mg^{2+}, Ca^{2+}, S^{2-}, Ar.

2. A certain particle carries 2.5×10^{-16} C of static electric charge. Calculate the number of electrons present in it.

(Q. No. 3 to 4) Two Marks

3. An element with mass number 81 contains 31.7% more neutrons as compared to protons. Assign the atomic symbol.

4. If the diameter of a carbon atom is 0.15 nm, calculate the number of carbon atoms which can be placed side by side in a straight line across the length of scale of length 20 cm long.

(Q. No. 5 to 6) Three Marks

5. How much energy is required to ionise a hydrogen atom if the electron occupies n = 5 orbit? Compare your answer with the ionisation energy of hydrogen atom (energy required to remove the electron from n = 1 orbit).

6. The quantum numbers of four electrons are given below. Arrange them in order of increasing energies. List if any of these combination (s) has/have the same energy:
   (i) $n = 4,\ l = 2,\ m_l = -2,\ m_s = -1/2$
   (ii) $n = 3,\ l = 2,\ m_l = -1,\ m_s = +1/2$
   (iii) $n = 4,\ l = 1,\ m_l = 0,\ m_s = +1/2$
   (iv) $n = 3,\ l = 2,\ m_l = -2,\ m_s = -1/2$

(Q. No. 7) Four Marks

7. If the position of the electron is measured within an accuracy of ±0.002nm, calculate the uncertainty in the momentum of the electron. Suppose the momentum of the electron is $h/(4\pi \times 0.05)$ nm, is there any problem in defining this value?

(Q. No. 8 to 10) Five Marks

8. What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition, $n = 4$ to $n = 2$ of He^+ spectrum?

9. The work function for caesium atom is 1.9 eV. Calculate (a) the threshold wavelength and (b) the threshold frequency of the radiation. If the caesium element is irradiated with a wavelength

500 nm, calculate the kinetic energy and the velocity of the ejected photoelectron.

10.

(i) Calculate the total number of electrons present in one mole of methane.

(ii) Find (*a*) the total number and (*b*) the total mass of neutrons in 7 mg of ^{14}C (Assume that the mass of neutron = 1.675×10^{-27} kg)

(iii) Find (a) the total number and (b) the total mass of protons in 34 mg of NH_3 at S.T.P. (Assume the mass of proton = 1.6726×10^{-27} kg)

Will the answer change if temperature and pressure are changed?

Answer & Solution

Q1

No. of electron are: $Na^+ = 11 – 1 = 10$, $K^+ = 19 – 1 = 18$, $Mg^{2+} = 12 – 2 = 10$,
$Ca^{2+} = 20 – 2 = 18$, $S^{2–} = 16 + 2 = 18$, $Ar = 18$.

Hence, isoelectronis species are N^{a+} and Mg^{2+}; K^+, Ca^{2+}, $S^{2–}$ and Ar.

Q2

Charges carried by one electron $= 1.6022 \times 10^{-19} C$

$\therefore$ Electrons present in particle carrying $2.5 \times 10^{-16} C$ charges $= \dfrac{2.5 \times 10^{-16}}{1.6022 \times 10^{-19}} = 1560$

Q3

Mass number = 81, *i.e.*, $p + n = 81$

If protons $= x$, then neutrons $= x + \dfrac{31.7}{100} \times x = 1.317x$

$\therefore$ $x + 1.317\ x = 81$ or $2.317\ x = 81$ or $x = \dfrac{81}{2.317} = 35$

Thus , protons= 35, *i.e.*, atomic no. = 35

Hence, the symbol is $^{81}_{35}Br$

Q4

Diameter of carbon atom= 0.15 nm $= 0.15 \times 10^{-9}$ m 1.5×10^{-10} m

Length along which atoms are to be placed = 20 cm 20×10^{-2} m $= 2 \times 10^{-1}$ m

$\therefore$ No. of C-atoms which can be placed along the line $= \dfrac{2 \times 10^{-1}}{1.5 \times 10^{-10}} = 1.33 \times 10^9$

Q5

$$E_n = \frac{21.8 \times 10^{-9}}{n^2} \text{ J atom}^{-1}$$

For ionization from 5th orbit, $n_1 = 5$, $n_2 = \infty$

$$\therefore\ \Delta E = E_2 - E_1 = -21.8 \times 10^{-19} \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) = 21.8 \times 10^{-19} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = 21.8 \times 10^{-19}$$

$\left(\frac{1}{5^2}-\frac{1}{\infty}\right)$=**$8.72\times10^{-20}$J**

For ionization from 1st orbit, n_1 = 1, n_2 = ∞

$\Delta E'$ =21.8×10^{-19} $\left(\frac{1}{1^2}-\frac{1}{\infty}\right)$=**$21.8\times10^{-19}$ J**

$$\frac{\Delta E'}{\Delta E}=\frac{21.8\times10^{-19}}{8.72\times10^{-20}}=25$$

Thus, the energy required to remove electron from 1st orbit is 25 times than that required to removed electron from 5th orbit.

Q6

The orbitals occupied by the electron are

(i) 4 *d*
(ii) 3 *d*
(iii) 4 *p*
(iv) 3 *d*
(v) 3 *p*
(vi) 4 *p*

Their energies will be in the order:

(v) < (ii) = (iv) = < (vi) = (iii) < (i)

Q7

Δx = 0.002 nm = 2×10^{-3} nm = 2×10^{-12} m

$\Delta x\times\Delta p=\frac{h}{4\pi}\ \therefore\ \Delta p=\frac{h}{4\pi\Delta x}=\frac{6.626\times10^{-34}\ \mathrm{kg\ m^2 s^{-1}}}{4\times3.14\times(2\times10^{-12}\mathrm{m})}$=**$2.638\times10^{-23}$ kg m s^{-1}**

Actual momentum $=\frac{h}{4\pi\times0.05\,\mathrm{nm}}=\frac{h}{4\pi\times5\times10^{-11}\mathrm{m}}=\frac{6.626\times10^{-34}\mathrm{kg\,m^2 s^{-1}}}{4\times3.14\times5\times10^{-11}\mathrm{m}}$

= 1.055×10^{-24} kg m s^{-1}

It cannot be defined as the actual magnitude of the momentum is smaller than the uncertainty.

Q8

For H-like particles in general $\overline{V}=\frac{2\pi^2 mZ^2e^4}{ch^3}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)=\mathrm{RZ}^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$\therefore$For He^+ spectrum, for Balmer transition, *n* = 4 to *n* = 2.

$$\overline{V}=\frac{1}{\lambda}=\mathrm{RZ}^2\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\mathrm{R}\times4\times\frac{3}{16}=\frac{3\mathrm{R}}{4}$$

For hydrogen spectrum $\overline{V}=\frac{1}{\lambda}=\mathrm{R}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)=\frac{3}{4}\mathrm{R}$ or $\frac{1}{\mathrm{n}_1^2}-\frac{1}{\mathrm{n}_2^2}=\frac{3}{4}$

which can be so far n^{-1} = 1 and n_2 = 2. *i.e.,* the transition is from ***n* = 2 to n = 1**

Q9

(i) Work function (W_0) = h ν_0

$\therefore\nu_0=\frac{W_0}{h}=\frac{1.9\times1.602\times10^{-19}\mathrm{J}}{6.626\times10^{-34}\mathrm{J\,s}}=4.59\times10^{14}s^{-1}$ (1 eV = 1.602 × 10^{-19}J)

(ii) $\therefore \lambda_0 = \dfrac{c}{v_0} = \dfrac{3.0\times10^{8}\ ms^{-1}}{6.59\times10^{-14}\ s^{-1}} = 6.54\times10^{-7} m$ = 654×10^{-9} m = **654 nm**

(iii) K.E. of ejected electron = $h\ (v\text{-}v_0) = hc\left(\dfrac{1}{\lambda}-\dfrac{1}{\lambda_0}\right)$

$$= (6.626\times10^{-34}\ \text{J s})\ (3.0\times10^{8}\ \text{m s}^{-1})\left(\frac{1}{500\times10^{-9}\text{m}}-\frac{1}{654\times10^{-9}\text{m}}\right)$$

$$= \frac{6.626\times3.0\times10^{-26}}{10^{-9}}\left(\frac{154}{500\times654}\right)\text{J} = \mathbf{9.36\times10^{-20}}\ \text{J}$$

$$\text{K.E.} = \frac{1}{2}m\,v^2 = 9.36\times10^{-20}\,\text{J or kg m}^2\ \text{s}^{-2}$$

$$\therefore \quad \frac{1}{2}\times(9.11\times10^{-31}\ \text{kg})\ v^2 = 9.36\times10^{-20}\,\text{kg m}^2\ \text{s}^{-2}$$

or $v^2 = 2.055\times10^{11}\ \text{m}^2\ \text{s}^{-2} = 20.55\times10^{10}\ \text{m}^2\ \text{s}^{-2}$ or $v = \mathbf{4.53\times10^{5} ms^{-1}}$**.**

Q10

(i) 1 molecule of CH_4 contains electrons = 6+4=10

$\therefore$ 1 mole, *i.e.*, 6.022×10^{23} molecules will contains electrons = $\mathbf{6.022\times10^{23}}$

(ii)

(a) 1 g atom of ^{14}C = 14 g = 6.022×10^{23} atoms = $(6.022\times10^{23})\times8$ neutrons.)

(as each ^{14}C atom has 14−6=8 neutrons)

Thus, 14 g or 14000 mg have $8\times6.022\times10^{23}$ neutrons

$\therefore$ 7 mg will have neutron$=\dfrac{8\times6.022\times10^{23}}{14000}\times7 = 2.4088\times10^{21}$

(b) Mass of 1 neutrons = 1.675×10^{-27} kg

$\therefore$Mass of 2.4088×10^{-21} neutrons = $(2.4088\times10^{21})\ (1.675\times10^{-27}\ \text{kg})$ = $\mathbf{4.0347\times10^{-6}}$ **kg**

(iii)

(a) 1 mol of NH_3 = 17 g NH_3 = 6.022×10^{23} molecules of NH_3

= $(6.022\times10^{23})\times(7+3)$ protons = 6.022×10^{24} protons

$\therefore$ 34 mg, *i.e.*, 0.034 g NH_3 $=\dfrac{6.022\times10^{24}}{17}\times0.034 = 1.2044\times10^{22}$protons.

(b) Mass of one proton = 1.6726×10^{-27} kg

$\therefore$Mass of 1.2044×10^{22} protons = $(1.6726\times10^{27})\times(1.2044\times10^{22})$ kg = $\mathbf{2.0145\times10^{-5}}$ **kg**

There is no effect of temperature and pressure.

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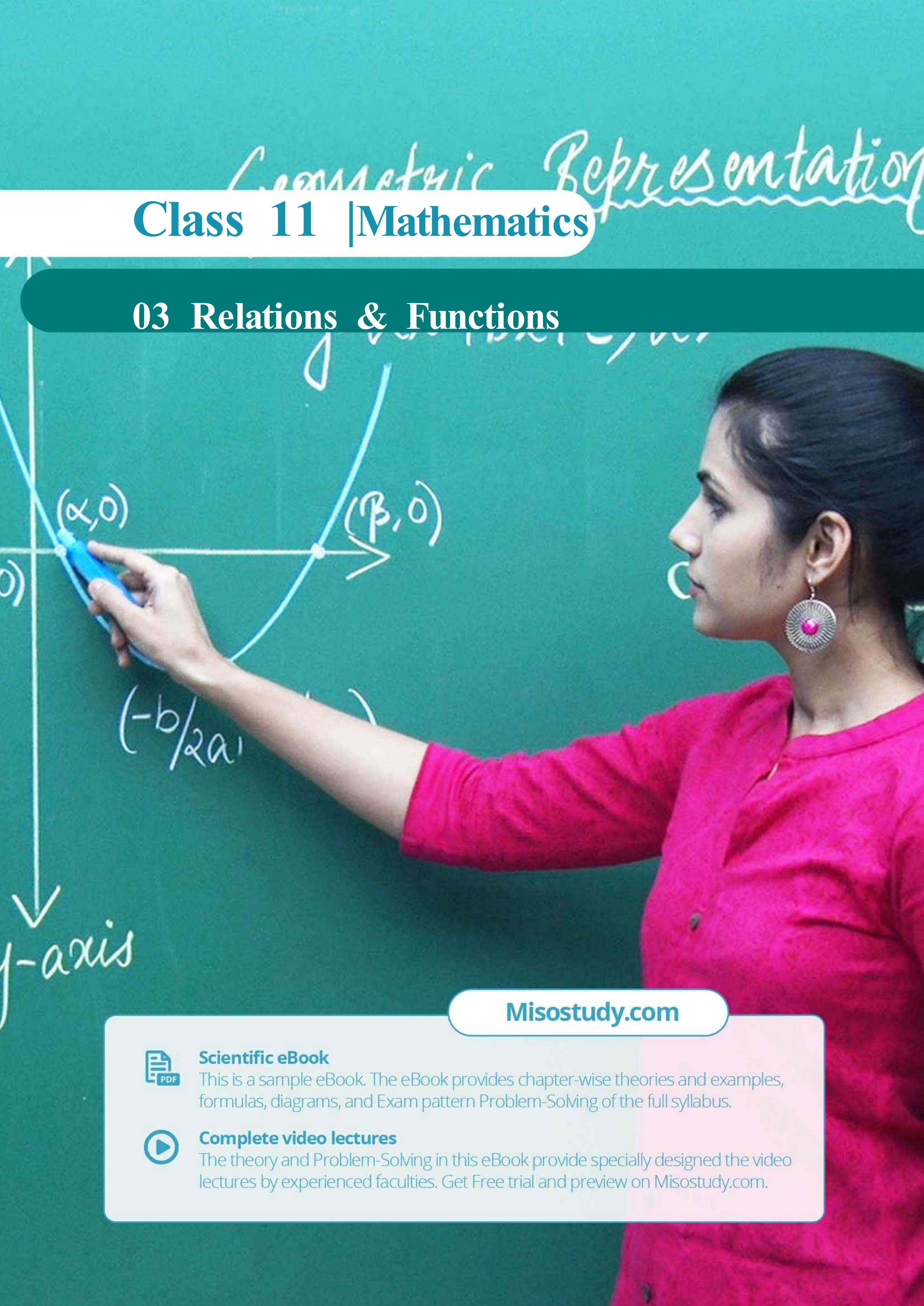


Class 11 |Mathematics

03 Relations & Functions

01. Definition

(i) ORDERED PAIR

An ordered pair consists of two objects or elements in a given fixed order.
For example, if A and B are any two sets, then by an ordered pair of elements we mean pair (a, b) in that order, where $a \in A,\ b \in B$.

NOTE ☞ An ordered pair is not a set consisting of two elements. The ordering of the two elements in an ordered pair is important and the two elements need not be distinct.

(ii) EQUALITY OF ORDERED PAIRS

Two ordered pairs (a_1, b_1) and (a_2, b_2) are equal iff

$$a_1 = a_2 \text{ and } b_2 = b_2$$

i.e., $(a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2$ and $b_2 = b_2$

Example *Find the values of a and b, if* $(3a - 2,\ b + 3) = (2a - 1,\ 3)$.

Solution By the definition of equality of ordered pairs, we have

$$(3a - 2,\ b + 3) = (2a - 1,\ 3)$$
$$\Leftrightarrow \quad 3a - 2 = 2a - 1 \text{ and } b + 3 = 3$$
$$\Leftrightarrow \quad a = 1 \text{ and } b = 0$$

(iii) CARTESIAN PRODUCT OF SETS

Let A and B be any two non-empty sets. The set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$ is called the cartesian product of the sets A and B and is denoted by $A \times B$.

Thus, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

If $A = \phi$ or $B = \phi$, then we define $A \times B = \phi$

Example *If* $A = \{2, 4, 6\}$ *and* $B = \{1, 2\}$, *then*

$$A \times B = \{2, 4, 6\} \times \{1, 2\} = \{(2, 1), (2, 2), (4, 1), (4, 2), (6, 1), (6, 2)\}$$

and, $B \times A = \{1, 2\} \times \{2, 4, 6\} = \{(1, 2), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6)\}$

02. Number of Elements in the Cartesian Product of Two Sets

RESULT

If A and B are two finite sets, then $n(A \times B) = n(A) \times n(B)$.

PROOF

Let $A=\{a_1, a_2, a_3, \ldots, a_m\}$ and $B=\{b_1, b_2, b_3, \ldots, b_n\}$ be two sets having m and n elements respectively. Then,

$$\begin{aligned} A\times B=\{&(a_1, b_1), (a_1, b_2), (a_1, b_3), \ldots, (a_1, b_n) \\ &(a_2, b_1), (a_2, b_2), (a_2, b_3) \ldots, (a_2, b_n) \\ &\vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \\ &(a_m, b_1), (a_m, b_2), (a_m, b_3) \ldots, (a_m, b_n)\} \end{aligned}$$

Clearly, in the tabular representation of $A\times B$ there are m rows of ordered pairs and each row has n distinct ordered pairs.

So, $A\times B$ has mn elements.

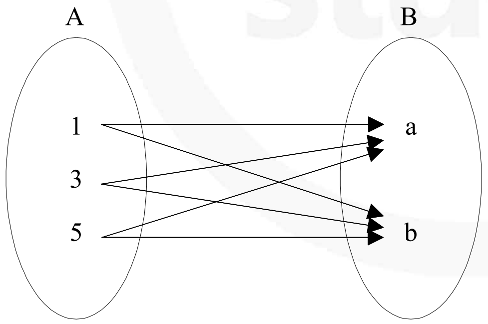
Hence, $n(A\times B)=mn=n(A)\times n(B)$

REMARK (i) If either A or B is an infinite set, then $A\times B$ is an infinite set.
(ii) If A, B, C are finite sets, then $n(A\times B\times C)=n(A)\times n(B)\times n(C)$

03. Diagramatic Representation of Cartesian Product of Two Sets

In order to represent $A\times B$ by an arrow diagram, we first draw Venn diagrams representing sets A and B one opposite to the other as shown in Figure. Now, we draw line segments starting from each element of A and terminating to each element of set B.

If $A=\{1, 2, 3\}$ and $B=\{a, b\}$, then following figure gives the arrow diagram of $A\times B$.



04. Some Useful Results

RESULT 1

For any three sets A, B, C, prove that:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

Proof (i) Let (a, b) be an arbitrary element of $A \times (B \cup C)$. Then,

$(a, b) \in A \times (B \cup C) \Rightarrow a \in A$ and $b \in B \cup C$ [by def.]

$\Rightarrow a \in A$ and $(b \in B$ or $b \in C)$ [by def. of union]

$\Rightarrow (a \in A$ and $b \in B)$ or $(a \in A$ and $b \in C)$

$\Rightarrow (a, b) \in A \times B$ or $(a, b) \in A \times C \Rightarrow (a, b) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ ...(i)

Again, let (x, y) be an arbitrary element of $(A \times B) \cup (A \times C)$. Then,

$(x, y) \in (A \times B) \cup (A \times C) \Rightarrow (x, y) \in A \times B$ or $(x, y) \in A \times C$

$\Rightarrow (x \in A$ and $y \in B)$ or $(x \in A$ and $y \in C)$

$\Rightarrow x \in A$ $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in (B \cup C) \Rightarrow (x, y) \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ ...(ii)

Hence, from (i) and (ii), we have $A \times (B \cap C)$. Then,

Proof (ii) Let (a, b) be an arbitrary element of $A \times (B \cap C)$. Then,

$(a, b) \in A \times (B \cap C) \Rightarrow a \in A$ and $b \in (B \cap C)$ [by def.]

$\Rightarrow a \in A$ and $(b \in B$ and $b \in C)$

$\Rightarrow (a \in A$ and $b \in B)$ and $(a \in A$ and $b \in C)$

$\Rightarrow (a, b) \in A \times B$ and $(a, b) \in A \times C$ [by def.]

$\Rightarrow (a, b) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ...(i)

Again, let (x, y) be an arbitrary element of $(A \times B) \cap (A \times C)$. Then,

$(x, y) \in (A \times B) \cap (A \times C) \Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in A \times C$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in (B \cap C) \Rightarrow (x, y) \in A \times (B \cap C)$

$\therefore (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ...(ii)

Hence, from (i) and (ii), we get

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

RESULT 2

For any three sets A, B, C, prove that:

$A \times (B - C) = (A \times B) - (A \times C)$

Proof Let (a, b) be an arbitrary element of $A \times (B - C)$. Then,

$(a, b) \in A \times (B - C) \Rightarrow a \in A$ and $b \in (B - C) \Rightarrow a \in A$ and $(b \in B$ and $b \notin C)$

$\Rightarrow (a \in A$ and $b \in B)$ and $(a \in A$ and $b \notin C)$

$\Rightarrow (a, b) \in (A \times B)$ and $(a, b) \notin (A \times C) \Rightarrow (a, b) \in (A \times B) - (A \times C)$

$\therefore A \times (B - C) \subseteq (A \times B) - (A \times C)$ ...(i)

Again, let (x, y) be an arbitrary element of $(A \times B) - (A \times C)$. Then,

$(x, y) \in (A \times B) - (A \times C) \Rightarrow (x, y) \in A \times B$ and $(x, y) \notin A \times C$

$\Rightarrow$ $(x \in A$ and $y \in B)$ and $(x \in A$ and $y \notin C) \Rightarrow x \in A$ and $(y \in B$ and $y \notin C)$

$\Rightarrow$ $x \in A$ and $y \in (B - C) \Rightarrow (x, y) \in A \times (B - C)$

$\therefore$ $(A \times B) - (A \times C) \subseteq A \times (B - \mathrm{C})$ ...(ii)

Hence, from (i) and (ii), we get

$$A \times (B - C) = (A \times B) - (A \times C)$$

RESULT 7

For any sets A, B, C, D prove that:

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$$

Proof Let (a, b) be an arbitrary element of $(A \times B) \cap (C \times D)$. Then,

$(a, b) \in (A \times B) \cap (C \times D) \Rightarrow (a, b) \in A \times B$ and $(a, b) \in C \times D$

$\Rightarrow$ $(a \in A$ and $b \in B)$ and $(a \in C$ and $b \in D)$

$\Rightarrow$ $(a \in A$ and $b \in C)$ and $(a \in B$ and $b \in D)$

$\Rightarrow$ $a \in (A \cap C)$ and $b \in B \cap D \Rightarrow (a, b) \in (A \cap C) \times (B \cap D)$

$\therefore$ $(A \times B) \cap (\mathrm{C} \times \mathrm{D}) \subseteq (A \cap C) \times (B \cap D)$

Similarly, $(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (\mathrm{C} \times \mathrm{D})$

Hence, $(A \times B) \cap (\mathrm{C} \times \mathrm{D}) = (A \cap C) \times (B \cap D)$

Corollary For any sets A and B, prove that $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$.

06. Concept of Relations

(i) RELATION

Let A and B be tow sets. Then a relation R from A to B is a subset of $A \times B$. Thus, R is a relation form A to $B \Leftrightarrow R \subseteq A \times B$.

If R is a relation from a non-void set A to a non-void set B and if $(a, b) \in R$, then we write aRb which is read as $'a$ is related to b by the relation R'. If $(a, b) \notin R$, then we write aRb and we say that a is not related to b by the relation R.

(ii) TOTAL NUMBER OF RELATIONS

Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subsets of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines a relation from A to B, so total numbers of relations from A to B is 2^{mn}. Among these 2^{mn} relations the void relation ϕ and the universal relation $A \times B$ are trivial relations from A to B.

07. Representation of a Relation

A relation from a set A to a set B can be represented in any one of the following forms:

(i) ROSTER FORM

In this form a relation is represented by the set of all ordered pairs belonging to R.

Example If R is a relation from set $A = \{-2, -1, 0, 1, 2\}$ to set $B = \{0, 1, 4, 9, 10\}$ by the rule

$$a\ R\ b \Leftrightarrow a^2 = b$$

Then, $0\ R\ 0$, $-2\ R\ 4$, $-1\ R\ 1$, $1\ R\ 1$ and $2\ R\ 4$.

So, R can be described in Roster form as follows:

$$R = \{(0, 0), (-1, 1), (-2, 4), (1, 1), (2, 4)\}$$

(ii) SET-BUILDER FORM

In this form the relation R from set A to set B is represented as

$R = \{(a, b) : a \in A, b \in B$ and a, b satisfy the rule which associates a and $b\}$

Example If $A = \{1, 2, 3, 4, 5\}$, $B = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \ldots\right\}$ and R is a relation from A to B given by

$$R = \left\{(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right), \left(5, \frac{1}{5}\right)\right\}$$

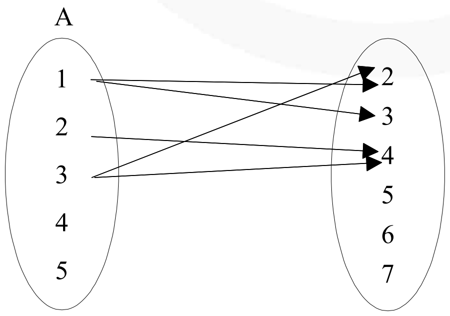
Then, R in set-builder form can be described as follows:

$$R = \left\{(a, b) : a \in A, b \in B \text{ and } b = \frac{1}{a}\right\}$$

(iii) BY ARROW DIAGRAM

In order to represent a relation from set A to a set B by an arrow diagram, we draw arrows from first components to the second components of all ordered pairs belonging to R.

Example Relation $R = \{(1, 2), ((2, 4), (3, 2), (1, 3), (3, 4)\}$ from set $A = \{1, 2, 3, 4, 5\}$ to set $B = \{2, 3, 4, 5, 6, 7\}$ can be represented by the following arrow diagram:



08. Domain and Range of a Relation

Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain or R, while the set of all second components or coordinates of the ordered pairs in R is called the range or R.

Thus, $\boxed{\textbf{Dom } (R) = \{a : (a, b) \in R\} \textbf{ and Range } (R) = \{b : (a, b) \in R\}.}$

★ **RELATION ON A SET** Let A be a non-void set. Then, a relation from A to itself i.e. a subset of $A \times A$, is called a relation on set A.

Example If A = {1, 3, 5, 7}, B = {2, 4, 6, 8, 10} and let R = {(1, 8), (3, 6), (5, 2), (1, 4)} be a relation from A to B. Then,

Dom (R) = {1, 3, 5} and Range (R) = {8, 6, 2, 4}

10. Function as a Special Kind of Relation

DEFINITION

Let A and B be two non-empty sets. A relation f from A to B, i.e., a sub-set of $A \times B$, is called a function (or a mapping or a map) from A to B, if

(i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$

(ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$.

11. Function as a Correspondence

DEFINITION

Let A and B be two non-empty sets. Then a function $'f'$ from set A to st B is a rule or method or correspondence which associates elements of set A to elements of set B such that:

(i) all elements of set A are associated to elements in set B.

(ii) an element of set A is associated to a unique element in set B.

In other words, a function $'f'$ from a set A to a set B associates each element of set A to a unique element of set B.

NOTE ☞ If and element $a \in A$ is associated to an element $b \in B$, then b is called 'the f-image of a' or 'image of a under f' or 'the value of the function f at a'. Also, a is called the pre-image of b under the function f. We write it as : $b = f(a)$

12. Description of a Function

Let $f: A \to B$ be a function such that the set A consists of a finite number of elements. Then, $f(x)$ be described by listing the values which it attains at different points of its domain. For example, if $A = \{-1, 1, 2, 3\}$ and B is the set of real numbers, then a function $f: A \to B$ can be described as $f(-1) = 3, f(1) = 0, f(2) = 3/2$ and $f(3) = 0$. In case, A is an infinite set, then f cannot be described by listing the image at points in its domain. In such cases functions are generally described by a formula. For example, $f: Z \to Z$ given by $f(x) = x^2 + 1$ or $f: R \to R$ given by $f(x) = e^x$ etc.

13. Domain, Co-Domain and Range of a Function

Let $f: A \to B$. Then, the set A is known as the domain of f and the set B is known as the co-domain of f. The set of all f-images of elements of A is known as the range of f or image set of A under f and is denoted by $f(A)$.
Thus, $f(A) = \{f(x) : x \in A\} =$ Range of f
Clearly, $f(A) \subseteq B$.

14. Equal Function

DEFINITION

Two functions f and g are said to be equal iff
(i) domain of $f =$ domain of g,
(ii) co-domain of $f =$ co-domain of g,
and (iii) $f(x) = g(x)$ for every x belonging to their common domain.
If two function f and g are equal, then we write $f = g$.

Example *Let* $A = \{1, 2\}$, $B = \{3, 6\}$ *and* $f : A \to B$ *given by* $f(x) = x^2 + 2$ *and* $g : A \to B$ *given by* $g(x) = 3x$. *Then, we observe that* f *and* g *have the same domain and co-domain. also we have,* $f(1) = 3 = g(1)$ and $f(2) = 6 = g(2)$ Hence, $f = g$.

15. Real Valued Function

A function $f: A \to B$ is called a real valued function, if B is a subset of R (set of all real numbers).
If A and B both are subsets of R, then f is called a real function.

NOTE ☞ In practice, real functions are described by giving the general expression or formula describing it without mentioning its domain and co-domain.

16. Domain of Real Functions

The domain of $f(x)$ is the set of all those real numbers for which $f(x)$ is meaningful.

17. Range of Real Functions

The range of a real function of a real variable is the set of all real values taken by $f(x)$ at points in its domain. In order to find the range of a real function $f(x)$, we may use the following algorithm.

ALGORITHM

STEP I	Put $y = f(x)$
STEP II	Solve the equation $y = f(x)$ for x in terms of y. Let $x = \phi(y)$.
STEP III	Find the values of y for which the values of x, obtained from $x = \phi(y)$, are real and in the domain of f.
STEP IV	The set of values of y obtained in step *III* is the range of f.

Example *Find the domain and range of the function f (x) given by*

$$f(x) = \frac{x-2}{3-x}$$

Solution We have,

$$f(x) = \frac{x-2}{3-x}$$

Domain of f : Clearly, $f(x)$ is defined for all x satisfying $3 - x \neq 0$ i.e. $x \neq 3$.

Hence, Domain $(f) = R - \{3\}$.

Range of f : Let $y = f(x)$, i.e.

$$y = \frac{x-2}{3-x}$$

$$\Rightarrow \quad 3y - xy = x - 2$$

$$\Rightarrow \quad x(y+1) = 3y + 2$$

$$\Rightarrow \quad x = \frac{3y+2}{y+1}$$

Clearly, x assumes real values for all y except $y + 1 = 0$ i.e. $y = -1$.

Hence, Range $(f) = R - \{-1\}$.

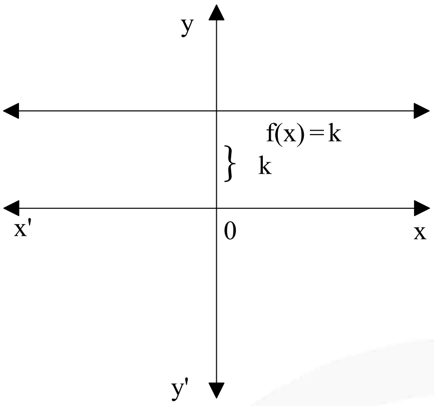
18. Some Standard Real Functions and Their Graphs

CONSTANT FUNCTION

If k is a fixed real number, then a function $f(x)$ given by

$$f(x) = k \text{ for all } x \in R$$

is called a constant function.



$y = k.$

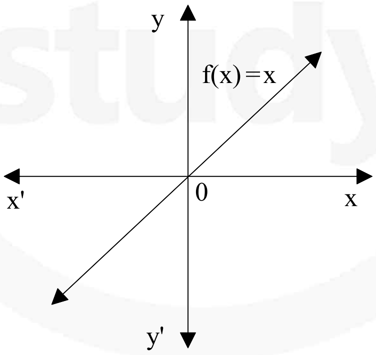
IDENTITY FUNCTION

The function that associates each real number to itself is called the identity function and is usually denoted by I.

Thus, the function $I : R \rightarrow R$ defined by

$$I(x) = x \text{ for all } x \in R$$

is called the identity function.



$y = x.$

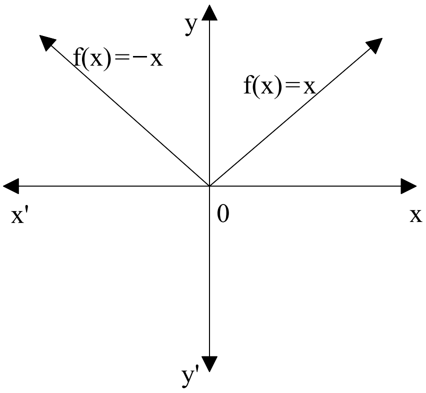
MODULUS FUNCTION

The function $f(x)$ defined by

$$f(x) = |x| = \begin{cases} x, & when\ x \geq 0 \\ -x, & when\ x < 0 \end{cases}$$

is called the modulus function.

It is also called the absolute value function.



$y = |x|$.

GREATEST INTEGER FUNCTION (FLOOR FUNCTION)

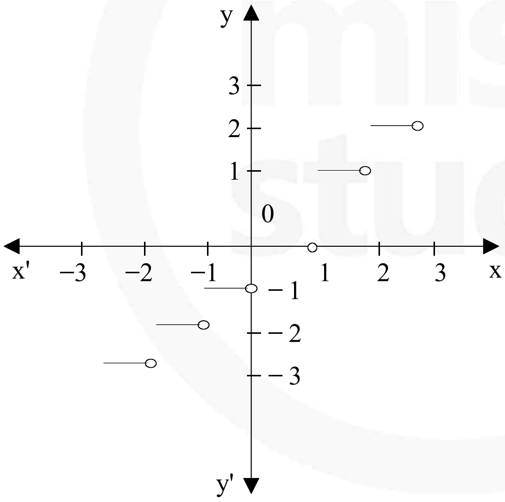
For any real number x, we use the symbol $[x]$ or, $\lfloor x \rfloor$ to denote the greatest integer less than or equal to x.

The function $f : R \to R$ defined by

$$f(x) = [x] \text{ for all } x \in R$$

is called the greatest integer function or the floor function.

It is also called a step function.



$y = [x]$.

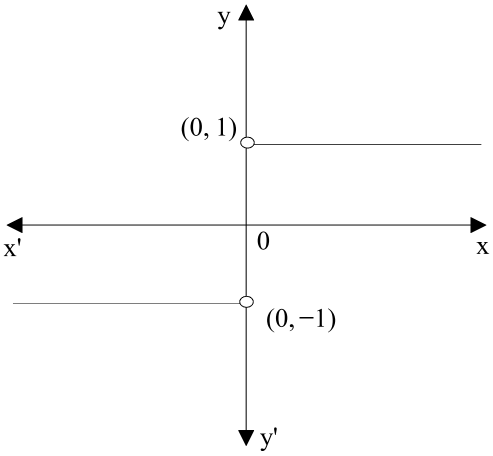
SIGNUM FUNCTION

The function f defined by

$$f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

or, $$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

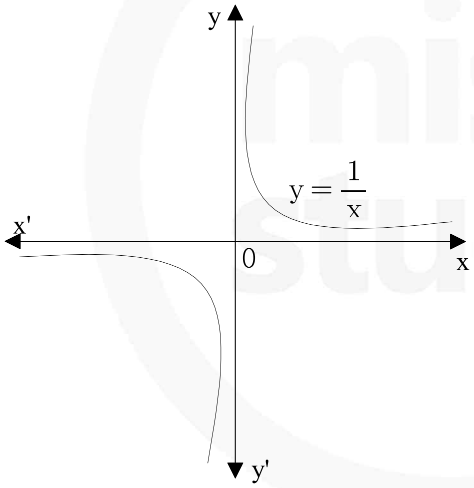
is called the signum function.



$$y=\frac{|x|}{x}.$$

RECIPROCAL FUNCTION

The function $f: R-\{0\} \rightarrow R$ defined by $f(x)=\frac{1}{x}$ is called the reciprocal function.



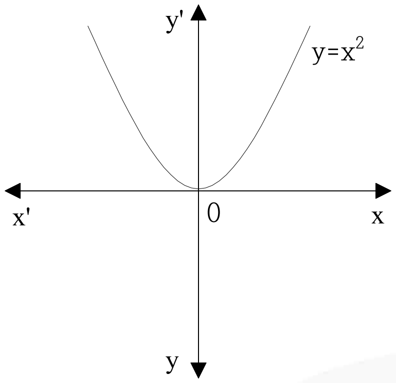
$$y=\frac{1}{x}$$

SQUARE FUNCTION

The function $f: R \rightarrow R$ defined by

$$f(x) = x^2$$

is called the square function.



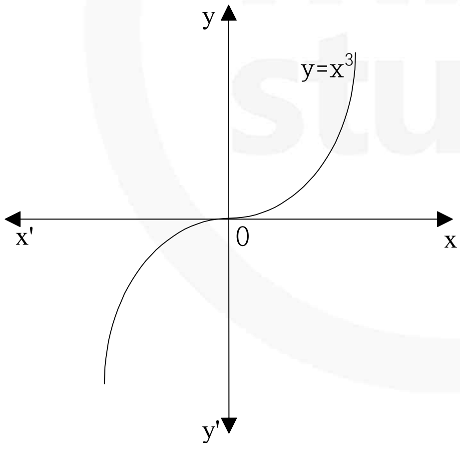
$y = x^2$

CUBE FUNCTION

The function $f: R \rightarrow R$ defined by

$$f(x) = x^3$$

is called the cube function.



$y = x^3$

REMARK (1) A function $f: R \rightarrow R$ is said to be a polynomial function if $f(x)$ is a polynomial in x. For example, $f(x) = x^2 - x + 4$, $g(x) = x^3 + 3x^2 + \sqrt{2}\,x - 1$ etc are polynomial functions.

(2) A function of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$, is called a rational function. The domain of a rational function $f(x) = \frac{p(x)}{q(x)}$ is the set of all real numbers, except points where $q(x) = 0$.

19. REAL FUNCTIONS

OPERATIONS ON REAL FUNCTIONS-

In this section, we shall introduce various operations namely addition, subtraction, multiplication, division etc. on real function.

ADDITION

Let $f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then, their sum $f+g$ is defined as that function from $D_1 \cap D_2$ to R which associates each $x \in D_1 \cap D_2$ to be number $f(x)+g(x)$.

PRODUCT

Let $f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then, their product (or pointwise multiplication) fg is a function from $D_1 \cap D_2$ to R and is defined as

$$(fg)(x) = f(x)g(x) \text{ for all } x \in D_1 \cap D_2$$

DIFFERENCE (SUBTRACTION)

Let $f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then the difference of g from f is denoted by $f-g$ and is defined as

$$(f-g)(x) = f(x) - g(x) \text{ for all } x \in D_1 \cap D_2$$

QUOTIENT

Let $f : D_1 \to R$ and $g : D_2 \to R$ be two real functions. Then the quotient of f by g is denoted by $\frac{f}{g}$ and it is a function from $D_1 \cap D_2 - \{x : g(x) = 0\}$ to R defined by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \text{ for all } x \in D_1 \cap D_2 - \{x : g(x) = 0\}$$

MULTIPLICATION OF A FUNCTION BY A SCALAR

Let $f : D \to R$ be a real function and α be a scalar (real number). Then the product αf is a function from D to R and is defined as

$$(\alpha f)(x) = \alpha f(x) \text{ for all } x \in D.$$

RECIPROCAL OF A FUNCTION

If $f : D \to R$ is a real function, then its reciprocal function $\frac{1}{f}$ is a function from $D - \{x : f(x) = 0\}$ to R and is defined as

Example I $\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)}$

Example I Let $f(x) = x^2$ and $g(x) = 2x + 1$ be two real functions defined over the set of non-negative real numbers. Find $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$.

Solution We have,

$$(f + g)(x) = x^2 + 2x + 1,\ (f - g)(x) = x^2 - 2x - 1,$$

$$(fg)(x) = x^2(2x + 1) = 2x^3 + x^2,\ \left(\frac{f}{g}\right)(x) = \frac{x^2}{2x+1},\ x \neq -\frac{1}{2}$$

REMARK (1) The sum, difference product and quotient are defined for real functions only on their common domain. These operations do not make any sense for general functions even if their domains are same, because the sum, difference, product and quotient may or may not be meaningful for the elements in their common domain.

(2) For any real function $f : D \rightarrow R$ and $n \in N$, we define

$$\underbrace{(fff \dots f)}_{n\text{–times}}(x) = \underbrace{f(x)f(x) \dots f(x)}_{n\text{–times}} = \{f(x)\}^n \text{ for all } x \in D$$

CBSE Exam Pattern Exercise
Subjective Questions (1)

(Q 1 to 2) One Mark

1. Let $f : R \rightarrow R$ be given by $f(x) = x^2 + 3$. Find the pre-images of 39 and 2 under f.

2. If $f(x) = 3x^3 - 5x^2 + 9$, find $f(x - 1)$.

(Q 3 to 4) Two Mark

3. Let $A = \{1, 2, 3\}$ and $B = \{x : x \in N, x$ is prime less than $5\}$. Find $A \times B$ and $B \times A$ & $((A \times B) \cap (B \times A))$

4. *Is* $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ *a function? If this is described by the formula,* $g(x) = \alpha x + \beta$, *then what values should be assigned to* α *and* β?

(Q 5 to 7) Four Marks

5. *Let* $A = \{1, 2, 3, 4, 5, 6\}$. Define a relation R on set A by $R = \{(x, y) : y = x + 1\}$
   (i) Depict this relation using an arrow diagram.
   (ii) Write down the domain, co-domain and range of R.

6. Find the domain and range of each of the following function: $f(x) = \dfrac{3}{2-x^2}$.

7. *Let* A *be a non-empty set such that* $A \times B = A \times C$. *Show that* $B = C$.

(Q 8 to 10) Four Marks

8. *Let f and g be real functions defined by* $f(x) = \sqrt{x+2}$ *and* $g(x) = \sqrt{4-x^2}$. *Then, find each of the following functions*:
   (i) $f + g$ (ii) $f - g$ (iii) fg
   (iv) $\dfrac{f}{g}$ (v) ff (vi) gg

9. The function f is defined by $f(x) = \begin{cases} 1-x, & x<0 \\ 1 & , x=0 \\ x+1, & x>0 \end{cases}$ Draw the graph of $f(x)$.

10. Define the function $f : \boldsymbol{R} \rightarrow \boldsymbol{R}$ by $y = f(x) = x^2$, $x \in \boldsymbol{R}$. Complete the Table given below by using this definition. What is the domain and range of this function? Draw the graph of f.

x	–4	–3	–2	–1	0	1	2	3	4
$y = f(x) = x^2$									

Answer & Solution

Q1.

Let x be the pre-image of 39. Then,

$f(x) = 39 \Rightarrow x^2 + 3 = 39 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

So, pre-images of 39 are – 6 and 6.

Let x be the pre-image of 2. Then,

$f(x) = 2 \Rightarrow x^2 + 3 = 2 \Rightarrow x^2 = -6$

Q2.

$3x^3 - 5x^2 + 9$

$3(x - 1)^3 - 5(x - 1)^2 + 9$

$3(x^3 - 1 - 3x(x - 1)) - 5(x^2 + 1 - 2x) + 9$

$3(x^3 - 1 - 3x^2 + 3x) - 5x^2 - 5 + 10x + 9$

$3x^3 - 3 - 9x^2 + 9x - 5x^2 - 5 + 10x + 9$

$3x^3 - 14x^2 + 19x + 1$

Q3.

We have,

$A = \{1, 2, 3\}$

and, $B = \{x : x \in N, x \text{ is prime less than } 5\} = \{2, 3\}$

$\therefore$ $A \times B = \{1, 2, 3\} \times \{2, 3\} = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\}$

and, $B \times A = \{2, 3\} \times \{1, 2, 3\} = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

$(A \times B) \cap (B \times A)$

$= \{(2, 2), (2, 3,), (3, 2), (3, 3)\}$

Q4.

Since no two ordered pairs in g have the same first component. So, g is a function such that $g(1) = 1$, $g(2) = 3$, $g(3) = 5$ and $g(4) = 7$.

It is given that $g(x) = \alpha(x) = \alpha x + \beta$.

$\therefore$ $g(1) = 1$ and $g(2) = 3 \Rightarrow \alpha + \beta = 1$ and $2\alpha + \beta = 3 \Rightarrow \alpha = 2, \beta = -1$.

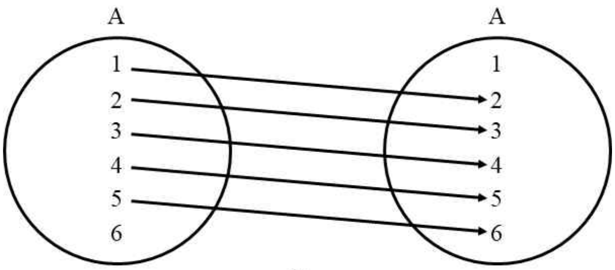
Q5.

(i) Putting $x = 1, 2, 3, 4, 5, 6$ in $y = x + 1$, we get $y = 2, 3, 4, 5, 6, 7$.

For $x = 6$, we get $y = 7$ which does not belong to set A.

$\therefore$ R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

The arrow diagram representing R is as follows:



Figure

(ii) Clearly, Domain (R) = {1, 2, 3, 4, 5}, Range (R) = {2, 3, 4, 5, 6}.

Q6.

We have,

$$f(x) = \frac{3}{2 - x^2}$$

For $f(x)$ to be real, we must have

$2 - x^2 \neq 0 \Rightarrow x \Rightarrow \pm \sqrt{2}$.

$\therefore$ Domain (f) = $R - \{-\sqrt{2}, \sqrt{2}\}$

Let $y(x) = y$. Then,

$y = f(x)$

$\Rightarrow \quad y = \dfrac{3}{2 - x^2}$

$\Rightarrow \quad 2y - x^2y = 3$

$\Rightarrow \quad x^2y = 2y - 3$

$\Rightarrow \quad x = \pm \dfrac{\sqrt{2y - 3}}{y}$

Now, x will take real values other than $-\sqrt{2}$ and $\sqrt{2}$, if

$$\frac{2y - 3}{y} \geq 0$$

Figure

$\Rightarrow \quad y \in (-\infty, 0) \cup \left[\dfrac{3}{2}, \infty\right)$

Hence, range (f) = $(-\infty, 0) \cup [3/2\ \infty)$.

Q7.

Let b be an arbitrary element of B. Then,

$(a, b) \in A \times B$ for all $a \in A$

$\Rightarrow \quad (a, b) \in A \times C$ for all $a \in A$ $\quad [\because A \times B = A \times C]$

$\Rightarrow \quad b \in C$

Thus, $b \in B \Rightarrow b \in C$

$\therefore \quad B \subset C$ ...(i)

Now, let c be an arbitrary element of C. Then,

$(a, c) \in A \times C$ for all $a \in A$ $\quad [\because A \times B = A \times C]$

$\Rightarrow \quad (a, c) \in A \times B$ for all $a \in A$

$\Rightarrow \quad c \in B$

Thus, $\quad c \in C \Rightarrow c \in B$

$\therefore \quad C \subset B \quad$ ...(ii)

From (i) and (ii), we get

$B = C.$

Q8.

We have,

$$f(x) = \sqrt{x+2} \text{ and } g(x) = \sqrt{4-x^2}$$

Clearly, $f(x)$ is defined for

$x + 2 \geq 0 \Rightarrow x \geq -2 \Rightarrow x \in [-2, \infty)$

$\therefore \quad$ Domain $(f) = [-2, \infty]$

$g(x)$ is defined for

$4 - x^2 \geq 0 \Rightarrow x^2 - 4 \leq 0 \Rightarrow (x-2)(x+2) \leq 0 \Rightarrow x \in [-2, 2]$

$\therefore \quad$ Domain $(g) = [-2, 2]$

Now,

Domain $(f) \cap$ Domain $(g) = [-2, \infty) \cap [-2, 2] = [-2, 2]$

(i) $f + g : [-2, 2] \to R$ is given by

$$(f+g)(x) = f(x) + g(x) = \sqrt{x+2} + \sqrt{4-x^2}$$

(ii) $f - g : [-2, 2] \to R$ is given by

$$(f-g)(x) = f(x) - g(x) = \sqrt{x+2} - \sqrt{4-x^2}$$

(iii) $fg : [-2, 2] \to R$ is given by

$$(fg)(x) = f(x)\, g(x)$$

$$\Rightarrow (fg)(x) = \sqrt{x+2} - \sqrt{4-x^2}$$

$$\Rightarrow (fg)(x) = \sqrt{(x+2)^2(2-x)} = (x+2)\sqrt{2-x}$$

(iv) We have,

$$g(x) = \sqrt{4-x^2}$$

$\therefore \quad g(x) = 0 \Rightarrow 4 - x^2 = 0 \Rightarrow x = \pm 2.$

So, $\quad$ Domain $\left(\frac{f}{g}\right) = [-2, 2] - [-2, 2] = (-2, 2)$

$\therefore \quad \frac{f}{g} : (-2, 2) \to R$ is given by

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{(x+2)}}{\sqrt{4-x^2}} = \frac{1}{\sqrt{2-x}}$$

(v) We have,

$$(ff)(x) = f(x)\, f(x) = [f(x)]^2 = \left(\sqrt{x+2}\right)^2 = x + 2 \text{ for all } x \in [-2, \infty]$$

(vi) We have,

$$(gg)(x) = g(x)\, g(x) = [g(x)]^2 = \left(\sqrt{4-x^2}\right)^2 = 4 - x^2 \text{ for all } x \in [-2, 2]$$

Q9.

Here, $f(x) = 1 - x,\ x < 0$, this gives

$f(-4) = 1 - (-4) = 5;$

$f(-3) = 1 - (-3) = 4,$

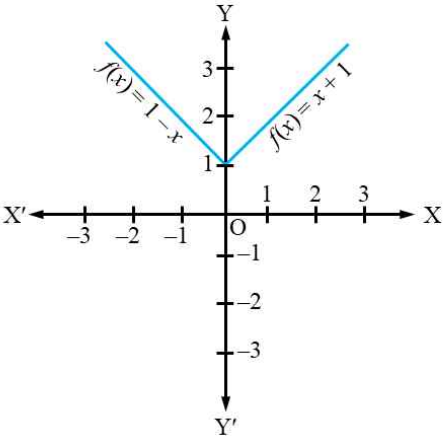
$f(-2) = 1 - (-2) = 3$

$f(-1) = 1 - (-1) = 2;$ etc,

and $f(1) = 2,\ f(2) = 3,\ f(3) = 4$

$f(4) = 5$ and so on for $f(x) = x + 1,\ x > 0$.

Thus, the graph of f is as shown in Figure.



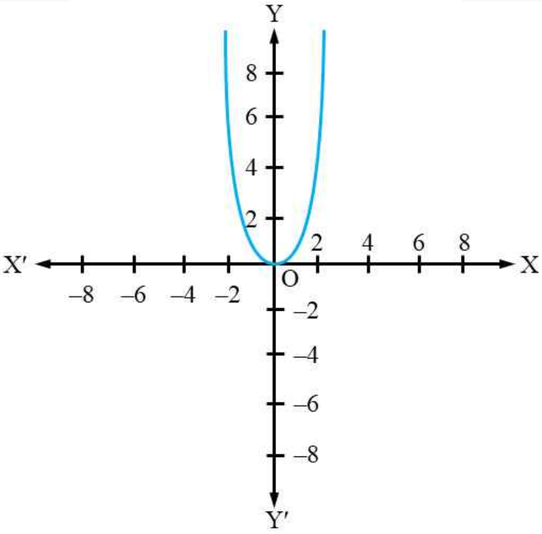
Figure

Q10.

The completed Table is given below:

x	–4	–3	–2	–1	0	1	2	3	4
$y = f(x) = x^2$	16	9	4	1	0	1	4	9	16

Domain of $f = \{x : x \in \boldsymbol{R}\}$. Range of $f = \{x2 : x \in \boldsymbol{R}\}$. The graph of f is given by Figure.



Figure

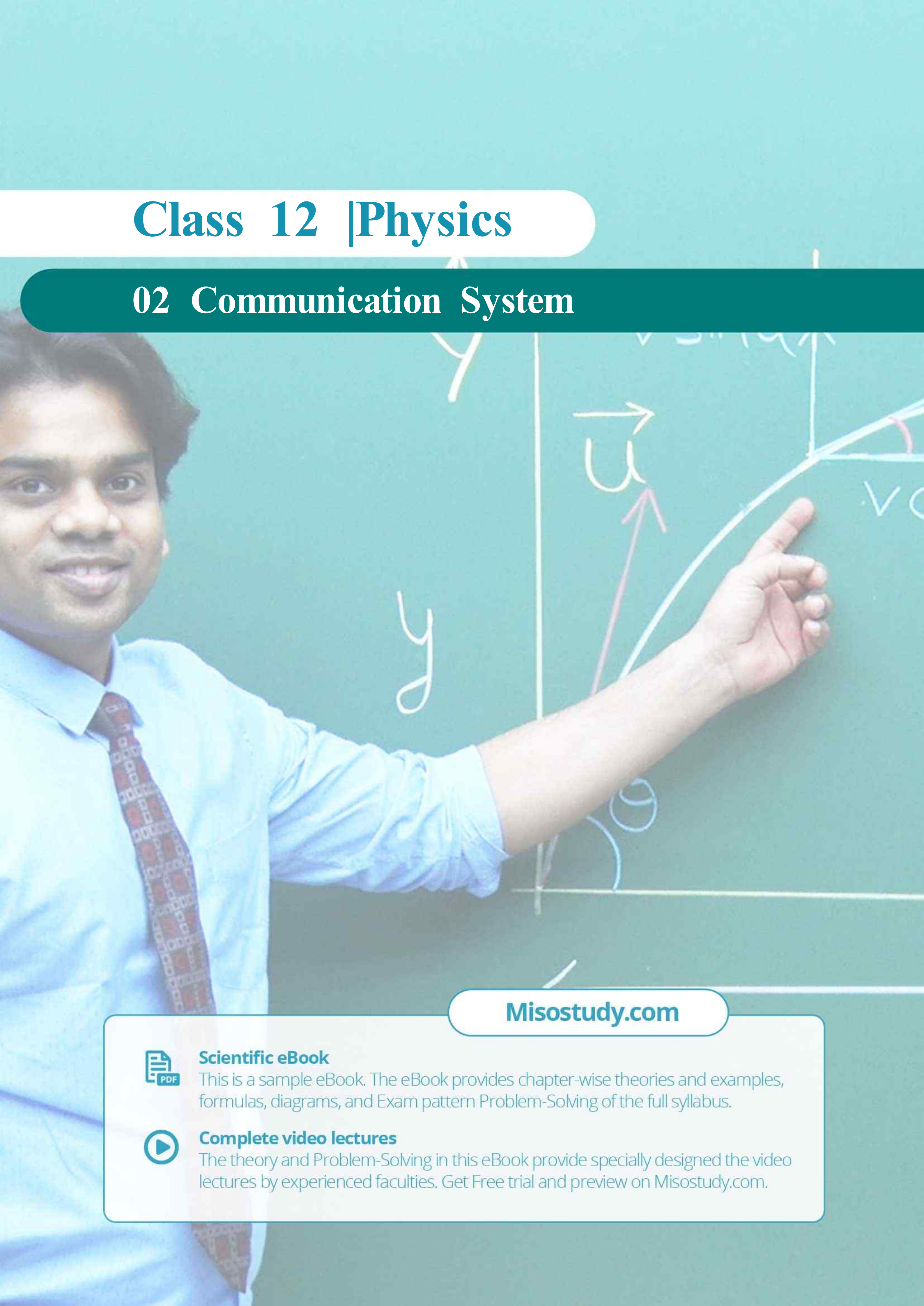
CLASS 12

CBSE

PHYSICS

Class 12 |Physics

02 Communication System

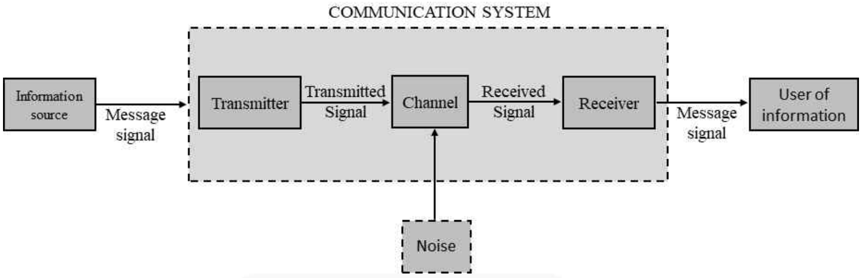


01. Introduction

The act of transmission and reception of information is known as communication.

02. Elements of a Communication System

Every communication system has three essential elements, transmitter, medium and receiver.



There are two basic modes of communication: point-to-point and broadcast.

03 Basic Terminology of Communication Systems

(i) **Transducer :** Device that converts one from of energy into another.
(ii) **Signal:** Information converted in electrical from and suitable for transmission. Signals can be either *analog or digital*.
(iii) **Noise:** The unwanted singles that tend to disturb the transmission and processing of message signals
(iv) **Transmitter:** Processes the incoming message signal so as to make it suitable for transmission through a channel and subsequent reception
(v) **Receiver:** Extracts the desired message signals from the receive signals at the channel output.
(vi) **Attenuation:** It is the loss of strength a signal while propagating through a medium.
(vii) **Amplification:** The process of increasing the amplitude of a signal using an electronic circuit called the amplifier.
(viii) **Range :** Largest distance between a source and a destination up to which the signal is received with sufficient strength
(ix) **Bandwidth:** Frequency range over which an equipment operates or the portion of the spectrum occupied by the signal.

(x) **Modulation:** Original low frequency message/information signal cannot be transmitted to long distances because of obvious reasons. Therefore, at the transmitter, information contained in the low frequency message signal is superimposed on a high frequency wave, which acts as a carrier of the information

(xi) **Demodulation:** The process of extraction of information from the carrier wave at the receiver

(xii) **Repeater:** A combination of receiver and a transmitter. Communication satellite is essentially a repeater station ins space.

NOTE ☞ Undesirable effects in the course of signal transmission are

(i) **Attenuation :** decrease in signal strength due to energy loss.
(ii) **Distortion :** waveform perturbation
(iii) **Interference :** contamination by extraneous signals.
(iv) **Noise :** due to random electrical signal

03. Types of Transmission Media

Broadly, transmission media have been divided into two types

(i) **Guided transmission medium:** That communication medium of channel which is used in point to point communication between a single transmitter and receiver.

(ii) **Unguided transmission medium:** communication medium which is used, where there is no point to point contact between the transmitter and receiver.

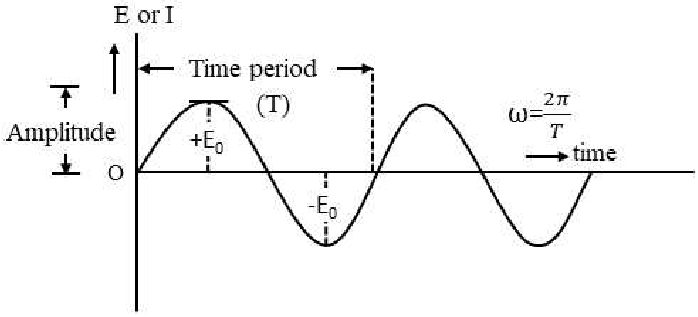
Characteristics and quality of transmission medium depends upon

(i) Nature of transmission medium
(ii) Nature of signal

The electrical signals are of two types:

(i) **Analog signals:** An analog signal is that in which current or voltage value varies continuously with time

$$\boxed{E = E_0 \sin(\omega t + \phi)}$$



Examples of Analog signals are speech, music, sound produced by a vibrating tuning fork.

(ii) **Digital signals:** A digital signal is discontinuous function of time, in contrast to an analog signal, wherein current or voltage value varies continuously with time.

Examples of Digital signals are (i) letters printed in a book (ii) listing of any data (iii) output of a digital computer (iv) electronic transmission of document at a distant place via telephone

04. Propagation of Electromagnetic Waves

An antenna at the transmitter in communication using radio waves, radiates the electromagnetic waves which travel through space and reach the receiving antenna at the other end.

05. Ground Wave

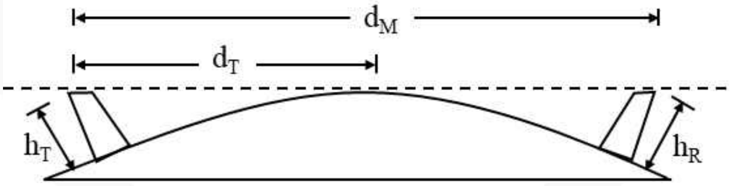
The antennas should have a size comparable to wavelength of signal. At longer wavelengths the antennas have large physical size and they are located on or very near to the ground.

06. Sky Waves

Long distance communication can be achieved by ionospheric reflection of radio waves back towards the earth. The phenomenon of bending of em waves so that they are diverted towards the earth is similar to total internal reflection.

07. Space wave

A space wave travels n a straight line from transmitting antenna to the receiving antenna. Space waves are used for line-of-sight (LOS) communication as well as satellite communication.



Where R is the radius of the earth

$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

where h_R is the height of receiving antenna.

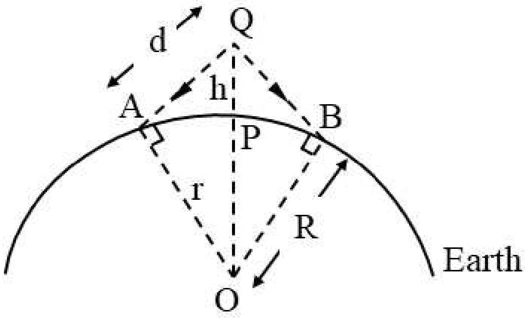
08. Determination of range

The range is determined by the height of transmitting antenna. The range AP or PB can be easily calculated by geometrical consideration. Suppose height of the tower is h and the radius of earth is r (that is OA = OB = OP = r). in the right-angled triangle OQA, we have

$$OQ^2 = QA^2 + OA^2$$
$$\because QA \simeq AP = d$$

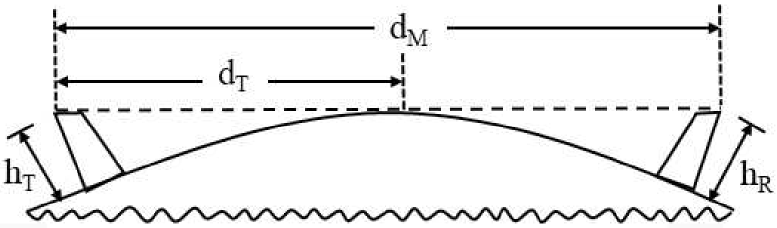
Therefore

$$(R+h)^2 = R^2+d^2$$
$$R^2+h^2+2Rh = R^2+d^2$$
$$\because\ R^2 >> h^2$$
$$\therefore\ d^2 = 2Rh$$
$$d = \sqrt{2Rh}$$



For a height of 500 m, d = 80 km

Hence maximum line-of-sight distance between two antennas will be calculated as follows



$$d_M = \sqrt{2R.h_T} + \sqrt{2R.h_R}$$

Where d_M : Maximum line-of-sight distance between two antennas

d_T : Radio horizon of transmitting antenna

h_T : Height of transmitting antenna

h_R : Height of receiving antennas

09. Modulation And its Necessity

Size of the antenna or aerial : An antenna or aerial is needed, both for transmission and reception. Each antenna should have a size comparable to the wavelength of the signals, (at least $\lambda/4$ in size) so that time variation of the signal is properly sensed by the antenna.

Effective power radiated by antenna : Theoretical studies reveal that power P radiated from a linear.

antenna of length l is proportional $(l/\lambda)^2, i.e., P \propto \left(\frac{1}{\lambda}\right)^2$

Modulation is the phenomenon of superimposing the low audio frequency baseband message or information signals (called the modulation signals) on a high frequency wave (called, the carrier wave). The resultant wave is called the modulated wave, which is transmitted.

10. Amplitude Modulation

The amplitude modulation is produced by varying the amplitude of the carrier waves in accordance with the amplitude of the modulating wave (audio signal). Let the instantaneous values of the voltage of the carrier waves and the modulating signal be represented by

$$e_c = E_c \sin \omega_c t \qquad \text{...(i)}$$

and $$e_m = E_m \sin \omega_m t \qquad \text{...(ii)}$$

respectively. Here, $\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$ are the angular frequencies of the carrier waves and the modulating signal respectively.

The instantaneous voltage of the modulated signal is given by

$$e = (E_c + E_m \sin \omega_m t) \sin \omega_c t$$

or $$e = E_c \left(1 + \frac{E_m}{E_c} \sin \omega_m t\right) \sin \omega_c t \qquad \text{...(iii)}$$

In amplitude modulation, the degree of modulation is defined by a term called ***modulation index***, which is given by

$$m_a = \frac{E_m}{E_c} \qquad \text{...(iv)}$$

The modulation index is also called ***modulation factor*** or ***depth of modulation***.

Therefore, the equation (iii) becomes

$$e = E_c (1 + m_a \sin \omega_m t) \sin \omega_c t$$

or $$e = (E_c + m_a E_c \sin \omega_m t) \sin \omega_c t \qquad \text{...(v)}$$

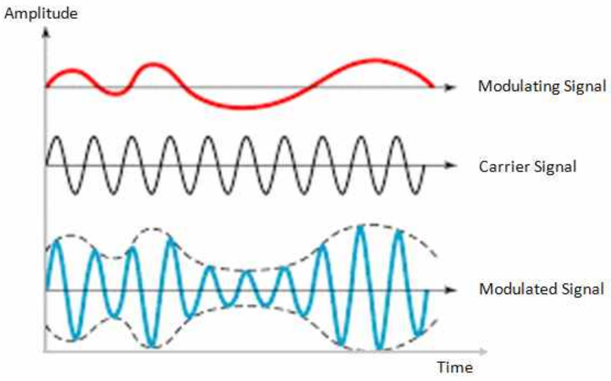
or $$e = E \sin \omega_c t,$$

where $$E = E_c + m_a E_c \sin \omega_m t \qquad \text{...(vi)}$$

represents the amplitude of the modulated signal. It follows that the amplitude of the modulated signal varies with time in accordance with the amplitude of the modulating signal, Fig. gives the sketch of amplitude modulated signal with time. The amplitude of the modulated signal varies between $E_{\min} (= E_c - E_m)$ and $E_{\max} (= E_c + E_m)$. It can be easily proved that

$$m_a = \frac{E_{\max} - E_{\min}}{E_{\max} + E_{\min}} \qquad \text{...(vii)}$$

It follows that the expression for instantaneous value of the voltage of the modulated signal consists of three terms. The first term represents a wave form of carrier frequency ω_c and the second term of frequency $\omega_c - \omega_m$ (slightly less than ω_c), known as ***lower sideband***. The third term represents a wave form of frequency $\omega_c + \omega_m$ (slightly greater than ω_c), called the ***upper sideband***.



Bandwidth :

The modulated signal lies in the frequency range from $\omega_c - \omega_m$ to $\omega_c + \omega_m$ *i.e.* $2\omega_m$. It is called the bandwidth of the modulated signal.

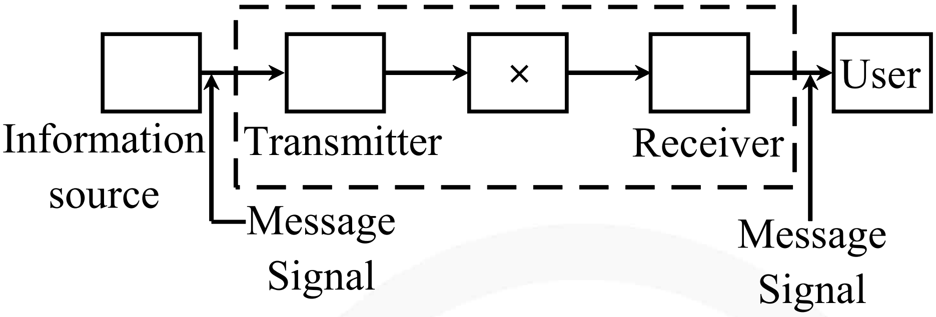
Thus,

$$band\ width = 2 \times frequency\ of\ modulating\ signal$$

CBSE Exam Pattern Exercise
Subjective Questions (1)

(Q 1 to 3) One Mark

1. The figure given below shows the block diagram of a generalised communication system. Identify the element labelled X and write its function.



2. Why are broadcast frequencies (carrier waves) sufficiently spaced in amplitude modulated wave?

3. How does the effective power radiated from a linear antenna depend on the wavelength of the signal to be transmitted?

(Q 4 to 7) Two Marks

4. Mention the function of any two of the following used in communication system.
   (i) Transducer
   (ii) Repeater
   (iii) Transmitter
   (iv) Bandpass filter

5. A message signal of frequency 10 kHz and peak voltage 10 V is used to modulate a carrier of frequency 1 MHz and peak voltage 20 V.
   Determine
   (i) the modulation index
   (ii) the side bands produced.

6.
   (i) What is line of sight communication?
   (ii) Why is it not possible to use sky wave propagation of transmission of TV signals?

7. Define the term modulation. Draw a block diagram of a simple modulator for obtaining AM signal.

(Q 8 to 10) Three Marks

8.
   (i) How is amplitude modulation achieved?
   (ii) The frequencies of two side bands in an AM wave are 640 kHz and 660 kHz, respectively. Find the frequencies of carrier and modulating signal. What is the bandwidth required for amplitude modulation?

9. Optical communication system having an operating wavelength λ (in meters) can use only x% of its source frequency as its channel bandwidth. The system is to be used for transmitting TV signals requiring a bandwidth of F hertz. How many channels can this system transmit simultaneously? Assuming all other factors to remain constant, show graphically the dependence of the number of channels that can be transmitted simultaneously on the operating wavelength of the system.

10. A signal is to be transmitted along a cable system of total length 125 km. The cable has an attenuation of 7 dB km^{-1}. Amplifiers, each having a gain of 43 dB, are placed at 6 km intervals along the cable
   (a) State what is meant by the attenuation of a signal.
   (b) Calculate (i) the total attenuation cased by the transmission of the signal along the cable, (ii) the total signal gain as a result of amplification by all of the amplifiers along the cable.
   (c) The input signal has a power of 450 mW. Use your answers in (b) to calculate the output power of the signal as it leaves the cable system.

Answer & Solution

Q1.

Labelled element X represents the channel. Its function is to transmit information from one place to another.

Q2.

To avoid mixing up of signal from different transmitters the broadcast frequencies are sufficiently spaced in amplitude modulated wave.

This can be done by modulating the signals on high frequency carrier waves, e.g. frequency band for satellite communication is 5.925-6.425 GHz.

Q3

The power radiated by a linear antenna of length L is proportional to $(L/\lambda)^2$, where λ is the signal wavelength. Since for efficient transmission, the signal should be transmitted with high power, the signal should be of small wavelength or high frequency.

Q4

(i) **Transducer :**

Any device which converts one converts pressure, temperature, etc. into varying electrical signals I.e. transducer converts physical signals into electrical signals.

(ii) **Repeater :**

It picks up the signals from the transmitter, amplifies it and transmit it to the receiver. Thus, repeater comprises up of receiver, transmitter and amplifier. Its function is to extend the range of communication.

(iii) **Transmitter :**

It comprises of message signal source, modulator and transmitting antenna. Transmitter make signals compatible for communication channel *via* modulator and antenna.

(iv) **Bandpass filter :**

A device which passes the signals with certain frequency range only.

Q5

(i) Modulation index, $\dfrac{E_m}{E_c} = \dfrac{10}{20} = 0.5$

(ii) Side band frequencies = $f_c \pm f_m$

$f_m = 10\ \text{kHz} \Rightarrow f_c = 1\ \text{MHz} = 1000\ \text{kHz}$

$\therefore$ Side band frequencies = 1000 ± 10

$= 1010\ \text{kHz},\ 990\ \text{kHz}$

Q6

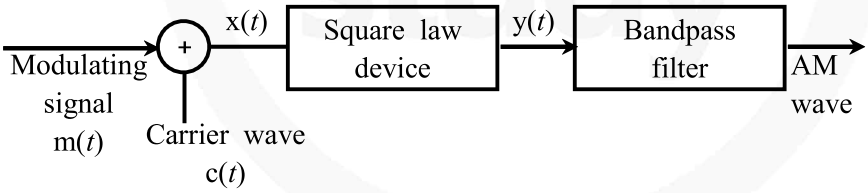
(i) Line of Sight (LOS) is a type of propagation that can transmit & receive data only where transmit & receive stations are in view of each other without any sort of obstacle between them.

Ex – FM radio, Microwave & Satellite Transmission.

(ii) The frequency of waves used for transmission of TV signals are of range 100 MHz-220 MHz. But ionosphere may be able to reflect waves back on earth of frequency up to 30 MHz. Therefore, ionosphere is unable to reflect TV waves (space waves) back on the earth.

Q7

Modulation is the process in which low frequency message signal is superimposed on high frequency carrier wave so that they can be transmitted over long distance. The block diagram for a simple modulator for obtaining AM signal is shown as below :



Q8

(i) For amplitude modulation, message signal is used to modulate amplitude of a high frequency wave in input transistor of CE amplifier.

The output voltage is carrier signal varying in amplitude in accordance with biasing modulating voltage.

(ii) Given, USB frequency = 660 kHz

and LSB frequency = 640 kHz

As USB = $f_c + f_m = 660$ kHz

and LSB = $f_c - f_m = 640$ kHz

$\therefore \quad 2f_c = 660 + 640 = 1300$

So, carrier frequency

$f_c = 650$ kHz

and $2f_m = 20$ kHz

Bandwidth of frequencies required
= USB – LSB = 660 – 640 = 20 kHz

Q9

Here, the wavelength of signal = λ

Therefore, frequency of the signal, $v = \frac{c}{\lambda}$

Since x % of the soure frequency can be used as the bandwidth,

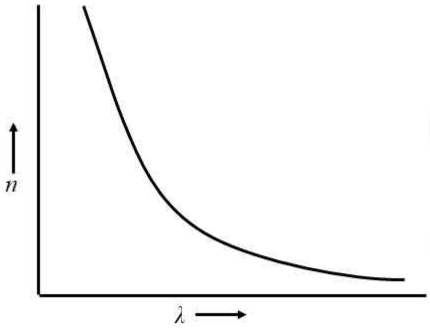
available bandwidth $= \frac{v \times x}{100} = \frac{cx}{\lambda \times 100}$

The bandwidth of the TV signal to be transmitted = F

Therefore, number of channels, the system can transmit,

$$n = \frac{\text{available bandwidth}}{\text{bandwidth of TV signal}}$$
$$= \frac{cx/\lambda \times 100}{F} = \frac{cx}{100\lambda F}$$

As n ∝ $1/\lambda$, the dependence of number of channels on the operating wavelength of the system will be as shown in Fig. below.



Q10

(a) The loss of power in a signal, as it travels, is called attenuation.

(b) (i) Here, attenuation = 7 dB km^{-1}
length of the cable = 125 km
Hence, total attenuation in the cable
= 7 × 125 = **875 dB**

(ii) Since amplifiers are placed at 6 km intervals along the cable of length 125 km, it follows that in total 20 amplifiers will have to be placed.
Gain of each amplifier = 43 dB
Therefore, total signal gain = 20 × 43 = **860 dB**

(c) Here, power of the input signal,
$P_1 = 450 \text{ mW} = 450 \times 10^{-3}$ W
The overall gain = 860–875 = 15 dB
If P_2 is the output power, then

gain (or loss) in signal power = $10\log\frac{P_2}{P_1}$

or $-15 = 10\log\frac{P_2}{450\times10^{-3}}$ or $\log\frac{P_1}{450\times10^{-3}} = -1.5$

or $\frac{P_2}{450\times10^{-3}} = 0.0316.$

or $P_2 = 0.0316 \times 450 \times 10^{-3} = 14 \times 10^{-3} =$ **14 mW**

CBSE Exam Pattern Exercise
Objective Questions (2)

1. Which process is used in optical fibres
   (a) T.I.R
   (b) Scattering
   (c) Reflection
   (d) Dispersion

2. An antenna behaves as a resonant circuit only when its length
   (a) equal $\frac{\lambda}{4}$
   (b) equal $\frac{\lambda}{2}$
   (c) equal $\frac{\lambda}{2}$ or its integral multiple
   (d) equal to 3λ

3. In modulation process, radio frequency wave is termed as
   (a) modulating wave
   (b) modulated wave
   (c) carrier wave
   (d) modified wave

4. A payload that is invariably found on all communication satellites is
   (a) optical telescope
   (b) camera
   (c) transponder
   (d) spectrometer

5. Attenuation in optical fibre is mainly due to
   (a) Scattering
   (b) Dispersion
   (c) Absorption and scattering
   (d) Reflection

Answer & Solution

1. (a)
   T.I.R

2. (c)
   equal $\frac{\lambda}{2}$ or its integral multiple

3. (c)
   carrier wave

4. (c)
   transponder

5. (c)
   Absorption and scattering

CLASS 12

CBSE

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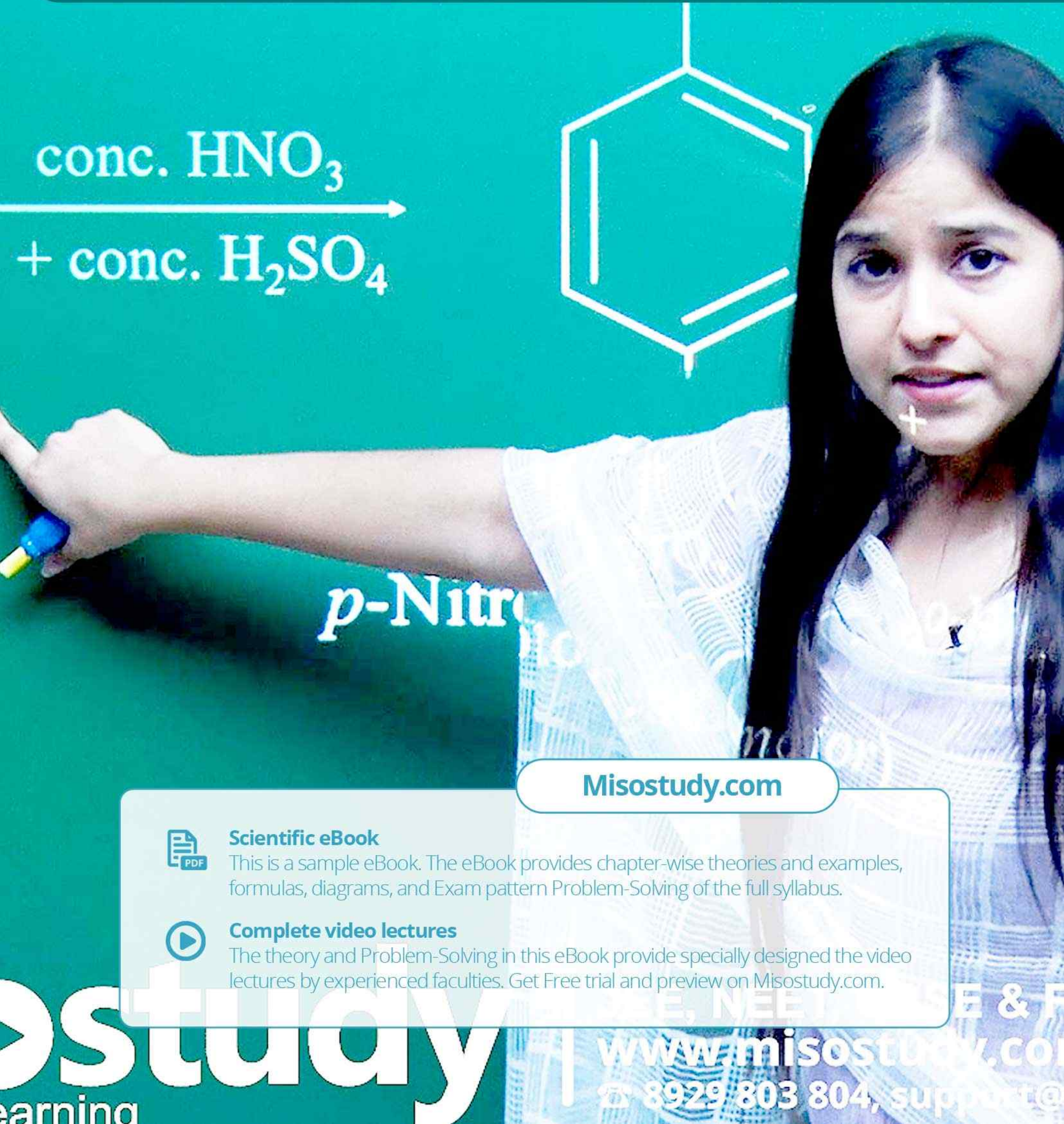
Class 12 | Chemistry

02 Solution



01. Introduction

When two or more chemically non-reacting substances are mixed and form homogeneous mixture it is called solution.

02. Types of Solution

	Solvent	Solute	Examples
1.	Gas	Gas	Mixture of gases, eg. air
2.	Gas	Liquid	Water vapour in air, mist. $CHCl_3(l)$ + $N_2(g)$
3.	Gas	Solid	Smoke, camphor (s) + $N_2(g)$
4.	Liquid	Gas	CO_2 gas dissolve in water (aerated drink), soda water.
5.	Liquid	Liquid	Mixture of miscible liquids e.g. alcohol in water.
6.	Liquid	Solid	Salt in water, sugar in water.
7.	Solid	Gas	hydrogen over palladium.
8.	Solid	Liquid	Mercury in zinc, mercury in gold i.e. all amalgams.
9.	Solid	Solid	Alloys e.g. copper in gold. zinc in copper.

03. Mass Percentage

It may be defined as the number of parts of mass of solute per hundred parts by mass of solution.

$$\% \text{ by mass } \left(\frac{w}{W}\right): \; = \frac{\text{Wt. of solute}}{\text{Wt. of solution}} \times 100$$

[X % by mass means 100 gm solution contains X gm solute; (100−X) gm solvent]

04. Mass-Volume Percentage (W/V %) :

It may be defined as the mass of solute present in 100 cm^3 of solution. For example, If 100 cm^3 of solution contains 5 g of sodium hydroxide, than the mass-volume percentage will be 5% NaOH solution.

$$\%\left(\frac{w}{V}\right) = \frac{\text{wt. of solute (in gm)}}{\text{volume of solution (in mL)}} \times 100$$

$$[X\%\left(\frac{w}{V}\right) \text{means 100 ml solution contains X gm solute}]$$

05. Volume Percent

It can be represented as % v/v or % volume and used to prepare such solutions in which both components are in liquids state. It is the number of parts of by volume of solute per hundred parts by volume of solution

Therefore,

$$\%\left(\frac{v}{V}\right) = \frac{\text{volume of solute}}{\text{volume of solution}} \times 100$$

06. Parts Per Million (ppm)

This method is used for expressing the concentration of very dilute solutions such as hardness of water, air pollution etc.

$$\text{ppm of substance} = \frac{\text{Mass of solute} \times 10^6}{\text{Mass of solution}} = \frac{\text{Volume of solute} \times 10^6}{\text{Volume of solution}}$$

07. Mole Fraction:

The ratio of the number of moles of one component to the total number of all the components present in the solution, is called the mole fraction of that component.

Mole fraction of solute X_A is given by $X_A = \frac{n_A}{n_A + n_B} = \frac{n_A}{\sum n}$

Mole fraction of solvent X_B is given by $X_B = \frac{n_B}{n_A + n_B} = \frac{n_B}{\sum n}$

where n_A is moles of solute A and n_B is moles of solvent B.

For binary solution of A & B $X_A + X_B = 1$

08. Molarity (Molar Concentration) :

It is defined as the number of moles of the solute dissolved in per litre of the solution, i.e.,

$$\text{Molarity (M)} = \frac{\text{Number of moles of solute}}{\text{Volume of solution (in } l)} = \frac{w_A}{m_A \times V} = \frac{c(gm/l)}{m_A} = \frac{\%\frac{W}{W} \times d \times 10}{m_A}$$

where let w_A g of the solute of molecular mass m_A be dissolved in V litre of solution, d = density of solution in g/mL.

09. Molarity of Dilute Solution :

Before dilution		After dilution
M_1V_1	=	M_2V_2

Molarity of mixing :

Let there be three samples of solution (containing same solvent and solute) with their molarity M_1, M_2, M_3 and volumes V_1, V_2, V_3 respectively. These solutions are mixed; molarity of mixed solution may be given as:

$M_1V_1 + M_2V_2 + M_3V_3 = M_R (V_1 + V_2 + V_3)$
where, M_R = Resultant molarity
$V_1 + V_2 + V_3$ = Resultant volume after mixing

10. Some Important Point :

[Note : Molarity is dependent on volume, therefore, it depends on temperature.]

1 M	Molar solution, i.e., molarity is 1
0.5 M or M/2	Semimolar
0.1 M or M/10	Decimolar
0.01 M or M/100	Centimolar
0.001 M or M/1000	Millimolar

11. Molality (m) :

The number of moles or gram-mole of solute dissolve in 1000 gram of the solvent is called molality of the solution.

$$\text{Molality of a solution} = \frac{\text{Number of moles of solute}}{\text{Amount of solvent in kg.}} = \frac{\text{Number of moles of solute} \times 1000}{\text{Amount of solvent in grams.}}$$

12. Normality (N) :

The number of equivalents or gram equivalents of solute dissolved in one litre of the solution is known as normality (N) of the solution.

$$\text{Normality (N)} = \frac{\text{Number of gram equivalents of solute}}{\text{volume of solution in litre}} = \frac{\text{weight of solute in gram}}{\text{equivalent weight} \times \text{volume of sol}}$$

$$= \frac{\text{strength of solution in gram/litre}}{\text{Equivalent weight of solute}}$$

Equivalent weight of a substance is that weight which reacts with or displaces one gram of hydrogen, 8 grams of oxygen or 35.5 grams of chlorine.

13. Solubility of Gases

Gases dissolve in liquids in solids. For example, soda-water contains carbon dioxide dissolved in water under high pressure. Oxygen is sufficiently soluble in water to allow survival of aquatic life in lakes, rivers and oceans.
The solubility of a gas in a liquid is determined by several factors. In addition to the nature of the gas and the liquid, solubility of the gas depends on the temperature and pressure of the system. The solubility of a gas in a liquid is governed by Henry's Law which states that the solubility of a gas in a liquid is directly proportional to the pressure of the gas.

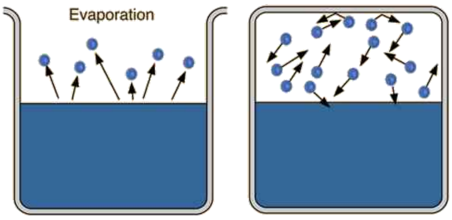
Mole fraction of the gas in a solution is proportional to the partial pressure of the gas. Or, partial pressure of the gas in solution = K_H × mole fraction of the gas in solution. Here K_H is Henry's law constant.

14. Vapour Pressure :

The Evaporation of a Liquid in a Closed Container

When a liquid is taken in a closed vessel at constant temperature, then there are two process which takes place.

(i) **evaporation**
(ii) **condensation**



In the constant evaporation from the surface particles continue to break away from the surface of the liquid.

As the gaseous particles bounce around, some of them will hit the surface of the liquid again, and will be trapped there. This is called condensation. The rate of condensation increases with time, but rate of evaporation remain constant. There will rapidly be an equilibrium set up in which the number of particles leaving the surface is exactly balanced by the number rejoining it.

This pressure at equilibrium is called the saturated vapour pressure (also known as saturation vapour pressure).

15. Effect of Temperature on Vapour Pressure

When the space above the liquid is saturated with vapour particles, you have this equilibrium occuring on the surface of the liquid :

$$\text{liquid} \rightleftharpoons \text{vapour}, \quad \Delta H > 0$$

The forward change (liquid to vapour) is endothermic. It needs heat to convert the liquid into the vapour. According to Le Chatelier, increasing the temperature of a system in a dynamic equilibrium favours the endothermic change. That means increasing the temperature increase the amount of vapour present, and so increases the saturated vapour pressure.

16. Raoult's Law for Volatile Solute :

For a two component solution A (volatile) and B (volatile) the vapour pressure of solution is given by.

Vapour pressure of solution = Vapour pressure of solvent in solution + Vapour pressure of solute

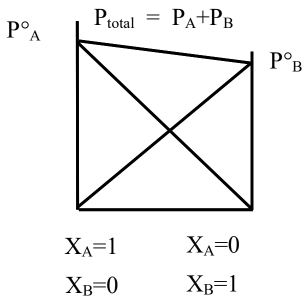
According to Raoult's Law partial. pressure of a component is equal to its mole fraction in solution multiplied with partial pressure in pure state

Thus $P_A = P^\circ_A X_A$

P°_A = vapour pressure in pure state of A

X_A = Mole fraction of A in solution

P_A = Partial pressure of A in solution



Total vapour pressure of solution $P = P^\circ_A X_A + P^\circ_B X_B$

17. Types of Solutions :

(a) **Ideal Solution :** An ideal solution may be defined as the one which obeys Raoult's law over all concentration ranges at a given temperature. The total vapour pressure of an ideal solution containing liquids A and B is given by the following equation.

$$P = P_A + P_B = P°_A X_A + P°_B X_B$$

Example		
	benzene + toluene	chlorobenzene + bromobenzene
	ethyl bromide + ethyl iodide	n-butyl chloride + n-butyl bromide
	ethyl alcohol + methyl alcohol	

(b) **Non-ideal solutions :** Solutions which do not over Raoult's law over all concentration ranges at constant temperature are called non-ideal solutions.

18. Distinction Between Ideal & Non Ideal Solutions

S.No.	Ideal Solution	Non Ideal Solution
1.	They obey Raoult's Law	They do not obey Raoult's Law
2.	$\Delta H_{mixing} = 0 \Rightarrow$ no heat is absorbed or released during dissolution	$\Delta H_{mixing} \neq 0 \Rightarrow$ no heat is absorbed or released during dissolution
3.	$\Delta V_{mixing} = 0 \Rightarrow$ the total volume of the solution is equal to the sum of the volume of the pure liquids mixed to form the solution	$\Delta V_{mixing} \neq 0 \Rightarrow$ the total volume of the solution is not equal to the sum of the volume of the pure liquids mixed to form the solution
4.	In ideal solution $P_A = P_A^0 X_A$	In non ideal solution $P_A \neq P_A^0 X_A$
5.	Components of ideal solution can be separated in pure form by fractional distillation	Components of non ideal solution can not be separated in pure form by fractional distillation

19. Types of Non – Ideal Solutions

(I) Non ideal solutions showing positive deviation.
(II) Non ideal solutions showing negative deviation.

20. Non Ideal Solutions Showing Positive Deviation

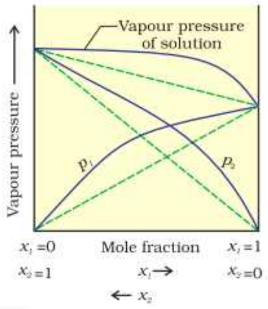
Condition for forming non-ideal solution showing positive deviation from Raoult's law. Two liquids A and B on mixing form this type of solution when

(i) A—B attractive force should be weaker than A—A and B—B attractive forces.
(ii) 'A' and 'B' have different shape, size and character.
(iii) 'A' and 'B' escape easily showing higher vapour pressure than the expected value.

Example

acetone + ethanol
water + ethanol
acetone + benzene
acetone + CS_2
CCl_4 + toluene
CCl_4 + CH_3OH
water + methanol
CCl_4 + $CHCl_3$
cyclohexane + ethanol

21. Graphical Representation of Vapour Pressure of Non-Ideal Solution Showing Positive Deviation



22. Non Ideal Solutions Showing Negative Deviation.

Condition for forming non-ideal solution showing negative deviation from Raoult's law.

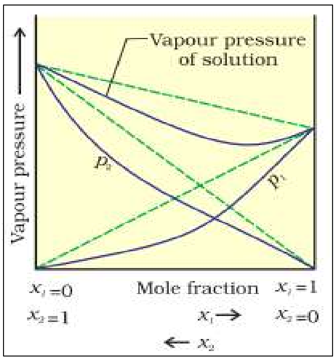
Two liquids A and B on mixing from this type of solution when

(i) A—B attractive force should be greater than A—A and B—B attractive forces.
(ii) 'A' and 'B' have different shape, size and character.
(iii) Escaping tendency of both components 'A' and 'B' is lowered showing lower vapour pressure then expected ideally.

Example

acetone + aniline;
chloroform + diethyl ether,
chloroform + benzene
water + HCl
CH_3OH + CH_3COOH;
acetic acid + pyridine;
H_2O +HNO_3;

23. Graphical Representation of Vapour of non-ideal Solution Showing Negative Deviation



24. Distinction Between Non Ideal Solutions Showing Positive Deviation & Negative Deviation

S.NO.	Showing positive deviation	Showing negative deviation
1.	$\Delta H_{mix} > 0$. (endothermic dissolution i.e. heat is absorbed)	$\Delta H_{mix} < 0$. (Exothermic dissolution i.e. heat is evolved)
2.	$\Delta V_{mix} > 0$. (Volume is increased after dissolution)	$\Delta V_{mix} > 0$. (Volume is decreased after dissolution)
3.	$p_A > p_A^o X_A;\ p_B > p_B^o X_B$, $\therefore p_A + p_B > p_A^o X_A + p_B^o X_B$	$p_A < p_A^o X_A;\ p_B < p_B^o X_B$, $\therefore p_A + p_B < p_A^o X_A + p_B^o X_B$

25. Colligative Properties

The properties of dilute solutions containing nonvolatile solute, which depends upon relative number of solute and solvent particles but do not depend upon their nature are called colligative properties.

Some of the colligative properties are
(i) Relative lowering of vapour pressure
(ii) Elevation in boiling point
(iii) Depression in freezing point and
(iv) Osmotic pressure.

26. Factor that Affect the Colligative Property

The number of solute particles in solution. To be more accurate, the colligative property depends upon the fraction of solute and solvent particles in solution.
(1) Nature of the solvent
(2) Independent of the nature of the solute
(3) Extent of association and dissociation of solute particles in solution.

27. Relative Lowering of Vapour Pressure

As shown earlier the mathematical expression for relative lowering of vapour pressure is as follows

$$\frac{\Delta P}{P^o{}_A} = X_B = \text{mole fraction of solute}$$

$\Delta P = P^o{}_A - P_A$ = lowering of vapour pressure
P_A = vapour pressure of pure solvent

Molecular mass of non-volatile substance can be determined from relative lowering of vapour pressure

$$\frac{P^o{}_A - P}{P^o{}_A} = \frac{W_B/M_B}{W_A/M_A + W_B/M_B}$$

For dilute solution $W_B/M_B < W_A/M_A$ and hence W_B/M_B may be neglected in the denominator. So

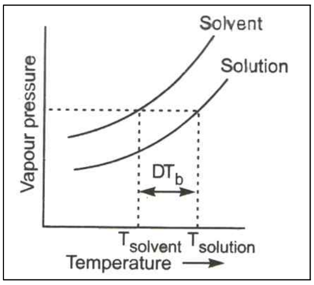
$$\frac{P^o{}_A - P_A}{P^o{}_A} = \frac{W_B/M_B}{W_A/M_A} = \frac{W_B}{W_A} \times \frac{M_A}{M_B} \text{ or } M_B = \frac{W_B/M_A}{W_A}\left(\frac{P^o_A}{P^o_A - P_A}\right)$$

28. Elevation in Boiling Point

The boiling points elevates when a non-volatile solute is added to a volatile solvent. Which occurs due to lowering of vapour pressure. The boiling point of a liquid may be defined as the temperature at which its vapour pressure becomes equal to atmospheric pressure.

So when a non-volatile solute is added to a volatile solvent results lowering of vapour pressure and consequent elevation of boiling point.
where

$\Delta T_b = T_b - T^o{}_b$
ΔT_b =elevation in B.P.
ΔP =lowering of V.P.
X_B =mole fraction of solute
K = elevation constant
$T^o{}_b$ = boiling point of solvent
T_b =boiling point of solution



It is found that elevation of boiling point is directly proportional to the number of moles of the solute in given amount of the solvent(m).

$$\Delta T_b \propto m$$

$$\Delta T_b \propto K_b m$$

Where 'm' is the molality of solution.
Where K_b is ebullioscopic or boiling point elevation constant. When molality of the solution is equal to one. Then

$$\Delta T_b = K_b$$

Determination of K_b of solvent:

$$K_b = \frac{RT^2_b}{1000L_v}$$

where R is molar gas constant, T_b is the boiling point of the solvent on Kelvin scale and L_v the latent heat of vaporization of solvent in calories per gram.

For water $K_b = \frac{2\times(373)^2}{1000\times540} = 0.515$ K-kg/mol

29. Depression in Freezing Point

The freezing point of a pure liquid is fixed. If a non-volatile solute is dissolved in a liquid the freezing point of the solution is lowered. The freezing point is that temperature at which the solvent has the same vapour pressure in two phases liquid solution and solid solvent. Since the solvent vapour pressure in solution is depressed. Its vapour pressure will become equal to that of the solid solvent at a lower temperature.

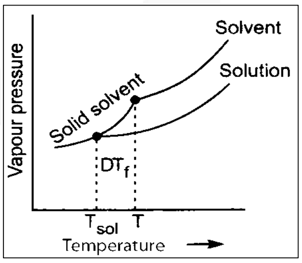
$$\Delta Tf = T^o_f - T_f$$

It is found that depression in freezing point is directly proportional to the number of moles of the solute in given amount of the solvent(m).

Hence $\Delta T_f \propto m$

$$\Delta T_f \propto K_f\ m$$

Where m = molarity of the solution.



K_f = molal depression constant
When molarity (m) of the solution is one. then

$$\Delta T_f = K_f$$

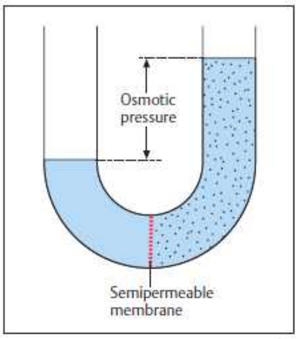
Hence molal depression constant or molal cryoscopic constant may be defined as "the depression in freezing point when one mole of non-volatile solute is dissolved per kilogram (1000 gm) of solvent" and molal depression constant is defined as "the depression in freezing point when one mole of non-volatile solute is dissolved per litre (1000 ml) of solvent."
The molecular mass of the non-volatile solute may be calculated by the use of following mathematical equation

$$M_B = \frac{K_f \times W_B \times 1000}{W_A \times \Delta T_f}$$

Where W_A = mass of solvent, W_B = mass of solute,
M_A = Molar mass of solvent, M_B = Molar mass of solute.

30. Osmotic Pressure

(i) Osmosis : Spontaneous flow of solvent molecules through a semipermeable membrane from a pure solvent to the solution (or from a dilute solution to a concentrated solution) is termed as osmosis.



(ii) Osmotic Pressure :
When a solution is separated from the pure solvent with the help of a semipermeable membrane. There occurs the flow of solvent molecules from the pure solvent to the solution side. The flow of solvent molecules from a region of higher concentration of solvent to the region of lower concentration of solvent is termed as the phenomenon of osmosis. This also happens when two solution of different concentrations are separated by s semipermeable membrane.
The excess pressure that must be applied to the solution side to prevent the passage of solvent into it through a semipermeable membrane.

(iii) Theory of Dilute Solutions :
The osmotic pressure of dilute solution was the same as the solute would have exerted if it were a gas at the same temperature as of the solution and occupied a volume equal to that of the solution. This generalization is known as Van't Hoff theory of dilute solutions. The osmotic pressure is a colligative property. So the osmotic pressure is related to the number of moles of the solute by the following relation
$\pi V = nRT$

$$\pi = \frac{n}{V} RT \qquad (\therefore \frac{n}{V} = C)$$

$\pi = CRT$
Here C = concentration of solution in miles per litre;
R = solution constant;
T = temperature in Kelvin degree;
n = number of moles of solute; and
V = volume of solution.

This equation is called Van‘t Hoff‘s equation.

31. Type of Solution :

(i) **Isotonic solution :**
The two solutions having equal osmotic pressure are termed as isotonic solution.

(ii) **Hypertonic solution :**
A solution having higher osmotic pressure than some other solution is said to be called hypertonic solution.

(iii) **Hypotonic solution :**
A solution having a lower osmotic pressure relative to some other solution is called hypotonic solution.

Semipermeable membrane :

A membrane which allows the passage of solvent molecules but not that of solute. When a solution is separated from the solvent by it is known as semipermeable membrane.
Some example of it are as follows
(a) Copper ferrocyanide $Cu_2[Fe(CN)_6]$
(b) Calcium phosphate membrane and
(c) Phenol saturated with water.

Reverse Osmosis :

If a pressure greater than the osmotic pressure is applied on the concentrated solution, the solvent starts to flow concentrated solution to dilute solution (or pure solvent). This is reverse osmosis. One of its chief uses is desalination of sea water to get pure drinking water.

32. Abnormal Molecular Masses

Vapour pressure lowering, boiling point elevation, freezing point depression and osmotic pressure are colligative properties which depend upon the fraction of solute and solvent particles in solution and not upon the chemical nature of the solute. If solute molecules dissociates in solution, there are more particles in solution and therefore, lowering of vapour pressure shows an increased effect.

$$NaCl_{(s)} \rightleftharpoons Na^{+}_{(aq)} + Cl^{-}_{(aq)}$$

If the solute molecules associates in solution, there are less particles in solution, and therefore lowering of vapour pressure shows a decreased effect.

$$nAB \rightleftharpoons (AB)_n$$

$$2C_6H_5COOH \rightleftharpoons (C_6H_5COOH)_2$$

33. Vant Hoff Factor

In 1886, Van't Hoff, Jacobs Henricus (Dutch chemist, 1859,-1911) introduced a factor 'i' known as Van't Hoff factor to express the extent to association or dissociation of a solute in solution. It can be calculated as:

$$i = \frac{\text{number of solute particles actually present in solution}}{\text{number of solute particles dissolved}}$$

$$= \frac{\text{Observed colligative property}}{\text{normal colligative property}} = \frac{\text{observed molality}}{\text{normal molality}} = \frac{\text{normal molecular weight of solute}}{\text{observed molecular weight of solut}}$$

The Van't Hoff factor for a solute can be calculated by the following modified equations:

(i) $\frac{P_A^o - P_A}{P_A^o} = iX_B$

(ii) $\Delta T_f = iK_f m$

(iii) $\Delta T_b = iK_b m$

(iv) $\pi = iCRT$

where C is molarity of the solution.

NOTE ☞ For non-electrolytes; $i = 1$
For electrolytes; $i > 1$ (If solute particles undergo Dissociation in the solution)
$i < 1$ (If solute particles undergo Association in the solution)

34. Application of Van't Hoff Factor :

(a) **Calculation of Degree of Dissociation of solute particles:**

	A_n	$\rightleftharpoons$	nA
No. of moles dissolved	1 mol		0
No. of moles after dissociation	$1 - \alpha$		$n\alpha$

Total number of moles present in solution = $(1 - \alpha) + n\alpha$

(b) Calculation of Degree of Dissociation of solute particles:

	A_n $\rightleftharpoons$	nA
No. of moles dissolved	1 mol	0
No. of moles after dissociation	$1-\alpha$	$n\alpha$

Total number of moles present in solution = $(1-\alpha)+n\alpha$

$$\text{Van't Hoff factor, } \mathbf{i} = \frac{\text{Moles of solute actually present in solution}}{\text{Moles of solute dissolved}}$$

$$= \frac{(1-\alpha)+n\alpha}{1} = 1+(n-1)\alpha \quad \text{or} \quad \alpha = \frac{i-1}{n-1}$$

(c) Calculation of Degree of Association of solute particles:

Let n moles of the solute, A, associate to form $(A)_n$. If α is the degree of association.

	nA $\rightleftharpoons$	A_n
No. of moles dissolved	1 mol	0
No. of moles after dissociation	$1-\alpha$	α/n

Total number of moles present in solution = $(1-\alpha)+\alpha/n$

$$i = \left[1-\alpha\left(1-\frac{1}{n}\right)\right] \qquad \text{Hence } \alpha = \frac{i-1}{\frac{1}{n}-1} = (i-1)\times\frac{n}{1-n}.$$

CBSE Exam Pattern Exercise
Subjective Questions (1)

(Q 1 to 2) One Mark

1. What role does the molecular interaction play in solution of alcohol and water?

2. Why do gases nearly always tend to be less soluble in liquid as the temperature is raised?

(Q 3 to 4) Two Marks

3. If the solubility product of CuS is 6 × 10^{-16}, calculate the maximum molarity of CuS in aqueous solution.

4. State Henry's law and mention some of its important applications.

(Q 5 to 6) Three Marks

5.
   (a) Determine the amount of $CaCl_2$ (i = 2.47) dissolved in 2.5 litre of water such that its osmotic pressure is 0.75 atm at 27°C.
   (b) Amongst the following compounds, identify which are insoluble, partially soluble and highly soluble in water (i) phenol, (ii) toluene, (iii) formic acid, (iv) ethylene glycol, (v) chloroform, (vi) pentanol

6. Calculate the mass of a non-volatile solute (molar mass 40 g mol^{-1}) which should be dissolved in 114g octane to reduce its vapour pressure to 80%.

(Q 7 to 8) Four Marks

7. 100 g of liquid A (molar mass 140 g mol^{-1}) was dissolved in 1000 g of liquid B (molar mass 180 g mol^{-1}). The vapour pressure of pure liquid B was found to be 500 torr. Calculate the vapour pressure of pure liquid A and its vapour pressure in the solution if the total vapour pressure of the solution is 475 torr.

8. The air is a mixture of a number of gases. The major components are oxygen and nitrogen with approximate proportion of 20% is to 79% by volume at 298 K. The water is in equilibrium with air at a pressure of 10 atm. At 298 K, if the Henry's law constants for oxygen and nitrogen are 3.30 × 10^7 mm and 6.51 × 10^7 mm respectively, calculate the composition of these gases in water.

(Q 9 to 10) Five Marks

9. Calculate the depression in the freezing point of water when 10 g of $CH_3CH_2CHClCOOH$ is added to 250 g of water. K_a = 1.4 × 10^{-3}, K_f = 1.86 K kg mol^{-1}.

10. 19.5g of CH_2FCOOH is dissolved in 500g of water. The depression in the freezing point observed is 1.0°C. Calculate the van't hoff factor and dissociation constant of fluoroacetic acid. K_f for water is 1.86 K kg mol^{-1}.

Answer & Solution

Q1

There is strong is hydrogen bonding in alcohol molecules as well as water molecules. On mixing, the molecular interactions are weakened. Hence, their solution will show positive deviations from ideal behaviour. As a result, the solution will have higher vapour pressure and lower boiling point that of water and alcohol.

Q2

Dissolution of gas in liquid is an exothermic process (Gas + Solvent $\rightleftharpoons$ Solvent + Heat). As the temperature is increased, equilibrium shifts backward.

Q3

Maximum molarity of CuS in aqueous solution = Solubility of CuS in mol L–^{1}

If S is the solubility of Cush in mol L^{-1}, then

$$CuS \rightleftharpoons \underset{S}{Cu^{2+}} + \underset{S}{S^{2-}} \;,\; K_{sp}=[Cu^{2+}][S^{2-}]=S\times S=S^2$$

$$\therefore \quad S^2 = 6\times10^{-6} \text{ or } S=\sqrt{6\times10^{-16}}=\mathbf{2.45\times10^{-8}\ mol\ L^{-1}.}$$

Q4

The mass of a gas dissolved in a given volume of the liquid at constant temperature is directly proportional to the pressure of the gas present in equilibrium with the liquid.

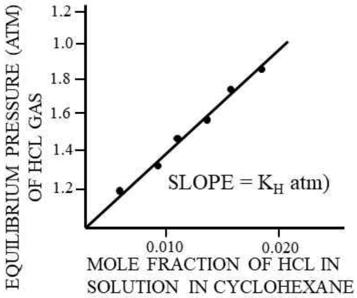
Mathematically, $m \propto P$ **or** $m = Kp$

Or

The solubility of a gas in a liquid at a particular temperature is directly proportional to the pressure of the gas in equilibrium with the liquid at that temperature.

From eqn (ii), $p_A = \frac{1}{K'}x_A$ or $P_A = K_H x_A$

The partial pressure of a gas in vapour phase (p) is directly proportional to the mole fraction (x) of the gas in the solution.



Plot of mole fraction in solution versus equilibrium pressure

Application of Henry's law:

(i) **In the production of carbonated beverages.**

(ii) **In the deep sea diving.**
bend or decompression sickness

(iii) **In the function of lungs.**

(iv) **For climbers or people living at high altitudes.**

Q5

$\pi = i\ \ CRT = i\frac{n}{V}RT$ or $n = n = \frac{\pi\times V}{i\times R\times T} = \frac{0.75\,\text{atm}\times 2.5\,\text{L}}{2.47\times 0.0821\ \text{L atm K}^{-1}\text{mol}^{-1}\times 300\,\text{K}}$ **=0.0308 mole**

Molar mass of $CaCl_2$ = 40 + 2×35.5 = 111 g mol^{-1} ∴ Amount dissolved = 0.0308 × 111 g = **3.42g**

Q6

Reduction of vapour pressure to 80% means that if $p° = 100$ mm, then $p_s = 80$ mm. Applying complete formula

$$\frac{p°-p_s}{p°} = \frac{n_2}{n_1+n_2} = \frac{w_2M_2}{w_1/M_1 + w_2/M_2}$$

$$\frac{100-80}{100} = \frac{w_2/40}{114/114 + w_2/40}$$ (Mol. mass of octane C_8H_{18} =114 g mol^{-1})

or $\frac{20}{100} = \frac{w_2/40}{1+w_2/40}$ or $\frac{1}{5}\left(1+\frac{w_2}{40}\right) = \frac{w_2}{40}$ or $w_2 =$ **10g**

Note that complete formula is required because concentration of solution I greater than 5%

Complete formula can also be applied in the form

$\frac{p°-p_s}{p°_s} = \frac{w_2/M_2}{w_1/M_1}$ or $\frac{100-80}{80} = \frac{w_2/40}{114/114}$ or $\frac{1}{4} = \frac{w_2}{40}$ or $w_2 =$ **10g**

Alternatively, suppose mass of solute dissolved = w g

Moles of solute $= \frac{w}{40}g$

Moles of solvent (octane) $= \frac{114}{114} = 1\,\text{mole}$ (Mol. mass of C_8H_{18}=114g mol^{-1})

∴Mole fraction of solvent $= \frac{1}{1+w/40}$

For a non-volatile solute,

Vapour pressure of solution =Mole fraction of solvent in the solution ×Vapour pressure of pure

solvent
$P_s = x_1 \times p°$

$$\therefore\ 80 = \frac{1}{1+w/40} \times 100 \text{ or } 1+\frac{w}{40} = \frac{100}{80} \text{ or } \frac{w}{40} = \frac{10}{8} - 1 = \frac{2}{8} = \frac{1}{4} \text{ or } w = \mathbf{10g}$$

Q7

No of moles of liquid A (solute) $= \dfrac{100\,\text{g}}{140\,\text{g mol}^{-1}} = \dfrac{5}{7}\,\text{mole}$

No. of moles of liquid B (solvent) $= \dfrac{1000\,\text{g}}{180\text{ g mol}^{-1}} = \dfrac{50}{9}\,\text{mole}$

$\therefore$ Mole fraction of A in the solution (x_A) $= \dfrac{5/7}{5/7+50/9} = \dfrac{5/7}{395/63} = \dfrac{5}{7} \times \dfrac{63}{395} = \dfrac{45}{395} = 0.114$

$\therefore$ Mole fraction of B in the solution $(x_B) = 1 - 0.114 = 0.886$

Also, given $P_B° = 500$ torr
Applying Raoult's law, $P_A = x_A\ P_A° = 0.114 \times P_A°$
$P_B = x_B\ P_B° = 0.886 \times 500 = 443$ torr
$P_{Total} = p_A + p_B$

$475 = 0.114\ P_B° + 443$ or $p_A° = \dfrac{475-443}{0.114} =$ **280.7 torr**

Substituting this value in eqn. (*i*), we get $P_A = 0.114 \times 280.7$ torr = **32 torr.**

Q8
Total pressure of air in equilibrium with water =10 atm
As sir contains 20% oxygen and 79% nitrogen by volume,

$\therefore$ Partial pressure of oxygen $(p_{O_2}) = \dfrac{20}{100} \times 10$ atm = 2 atm = 2×760 mm =1520 mm

Partial pressure of oxygen $(p_{N2}) = \dfrac{79}{100} \times 10\,\text{atm}$ = 7.9 atm 7.9×760 mm = 6004 mm

$K_H(O_2) = 3.30 \times 10^7$ mm, $K_H(N_2) = 6.51 \times 10^7$mm

Applying Henry's law, $Po_2 = K_H \times xo_2$ or $x_{O_2} = \dfrac{p_{O_2}}{K_H} = \dfrac{1520\text{ mm}}{3.30 \times 10^7\text{ mm}} = 4.61 \times 10^{-5}$

$p_{N_2} = K_H \times x_{N_2}$ or $x_{N_2} = \dfrac{P_{N_2}}{K_H} = \dfrac{6004\text{ mm}}{6.51 \times 10^7\text{mm}} =$ $\mathbf{9.22 \times 10^{-5}}$.

Q9
Molar mass of $CH_3CH_2CHClCOOH = 15+14+13+35.5+45 = 122.5$ g mol^{-1}

10 g of $CH_3CH_2CHClCOOH = \dfrac{10}{122.5}\text{mole} = 8.16 \times 10^{-2}$ mole

$\therefore$ Molality of the solution (*m*) $\dfrac{8.16 \times 10^{-2}\text{mole}}{250\,\text{g}} \times 1000\,\text{g kg}^{-1} = 0.3264$

Of *a* is the degree of dissociation of $CH_3CH_2CHClCOOH$, then

$$CH_3CH_2CHClCOOH \rightleftharpoons CH_3CH_2CHClCOO^- + H^+$$

Initial conc. C mol L^{-1} 0 0

At eqm. C (1–α) C α C α

$$\therefore \quad Ka = \frac{C\alpha . C\alpha}{C(1-\alpha)} \simeq C\,\alpha^2 \quad \text{or} \quad \alpha = \sqrt{\frac{K_a}{C}} = \sqrt{\frac{1.4\times 10^{-3}}{0.3264}} = 0.065$$

To calculate van't Hoff factor:

$$CH_3CH_2CHClCOOH \rightleftharpoons CH_3CH_2CHClCOO^- + H^+$$

Initial moles 1

Moles at eqm. 1 – α α α Total = 1 + α

$$i = \frac{1+\alpha}{1} = 1+\alpha = 1+0.065 = 1.065 \;;\; \Delta T_f = i\ K_f m = (1.065)(1.86)(0.3264) = \mathbf{0.65°}.$$

Q10

Hence, W_2=19.5g, w_1=500 g, Kf =1.86 K kg mol^{-1}, (ΔT_f $)_{obs}$ = 1.0°

$$\therefore M2\ (\text{observed}) = \frac{100K_f w^2}{w_1 \triangle T_f} = \frac{(1000\,g\ kg^{-1})(1.86\,K\ kg\ mol^{-1})(19.5g)}{(500\,g)\,(1.0\,K)} = 72.54\ g$$

M_2 (calculated) for CH_2FCOOH =14+19+45=78 g mol^{-1}

$$\text{van't Hoff factor } (i) = \frac{(M_2)_{cal}}{(M_2)_{obs}} = \frac{78}{72.54} = \mathbf{1.0753.}$$

Calculation of dissociation constant. Suppose degree of dissociation at the given concentration is α.

Then $CH_2FCOOH \rightleftharpoons CH_2FCOO^- + H^+$

Initial C mol L^{-1} 0 0

At eqm. C(1–α) Cα Cα, Total = C (1+α)

$$i \quad i = \frac{C(1+\alpha)}{C} = 1+\alpha \quad \text{or} \quad \alpha = i-1 = 1.0753-1 = 0.0753$$

$$K_\alpha = \frac{[CH_2FCOO^-][H^+]}{[CH_2FCOOH]} = \frac{C\,\alpha . C\,\alpha}{C(1-\alpha)} = \frac{C\alpha^2}{1-\alpha}$$

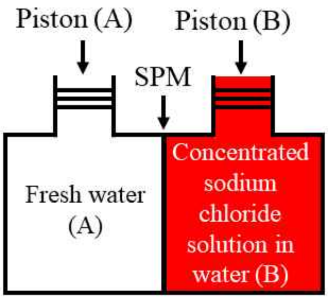
Taking volume of the solution as 500 mL,

$$C = \frac{19.5}{78}\times\frac{1}{500}\times 1000 = 0.5M \qquad \therefore \qquad K_a = \frac{C\,\alpha^2}{1-\alpha} = \frac{(0.5)(0.0753)^2}{1-0.0753} = 3.07\times 10^{-3}.$$

CBSE Exam Pattern Exercise
Objective Questions (2)

1. Considering the formation, braking and strength of hydrogen bond, predict which of the following mixtures will show a positive deviation from Raoult's law ?
   (a) Methanol and acetone
   (b) Chloroform and acetone
   (c) Nitric acid and water
   (d) Phenol and aniline

2. Consider the figure and mark the correct option.
   (a) water will move form side (A) to side (B) if a pressure lower than osmotic pressure is applied on piston (B)



   (b) water will move from side (B) to side (A) if a pressure greater than osmotic pressure is applied on piston (B)
   (c) water will move from side (B) to side (A) if a pressure equal to osmotic pressure is applied on piston (B)
   (d) water will move from side (A) to side (B) if pressure equal to osmotic pressure is applied on piston (A)

3. If two liquids A and B form minimum boiling azeotrope at some specific composition, then ....
   (a) A–B interactions are stronger than those between A–A or B–B
   (b) vapour pressure of solution increases because more number of molecules of liquids A abd B can escape from the solution
   (c) vapour pressure of solution decreases because less number of molecules of only one of the liquids escape from the solution
   (d) A–B interactions are weaker than those between A–A or B–B

4. 4L of 0.02 M aqueous solution of NaCl was diluted by adding one litre of water. The molality of the resultant solution is ....
   (a) 000.4
   (b) 0.008
   (c) 0.012
   (d) 0.016

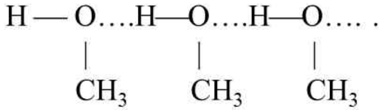
5. K_H value for Ar(g), CO_2(g), HCHO (g) and CH_4(g) are 40.39, 1.67, 1.83×10^{-5} and 0.413 respectively.
Arrange these gases in the order of their increasing solubility.
(a) HCHO < CH_4 < CO_2 < Ar
(b) HCHO < CO_2 < CH_4 < Ar
(c) Ar < CO_2 < CH_4 < HCHO
(d) Ar < CH_4 < CO_2 < HCHO

The questions given below consists of an Assertion and the Reason. Use the following key to choose the appropriate answer.
(a) If both assertion and reason are CORRECT and reason is the CORRECT explanation of the assertion.
(b) If both assertion and reason are CORRECT, but reason is NOT THE CORRECT explanation of the Assertion.
(c) If assertion is CORRECT, but the reason is INCORRECT.
(d) If both assertion and reason are INCORRECT.

6. **Assertion :** When NaCl is added to water, a depression in freezing point is observed.
Reason : The lowering of vapour pressure of a solution causes depression in the freezing point.

Answer & Solution

Q1. (a)



$$H-\underset{CH_3}{\underset{|}{O}}\ldots H-\underset{CH_3}{\underset{|}{O}}\ldots H-\underset{CH_3}{\underset{|}{O}}\ldots.$$

On adding acetone, its molecules get in between the molecules of methanol braking hydrogen bonds and reducing methanol-methanol attractions.

Q2. (b)

The process represented in the image is the reverse osmosis (R.O) which takes place when pressure more than osmotic pressure is applied to the solution.

Due to this, the solvent will flow from the solution into the pure solvent through semi permeable membrane.

Q3. (d)

Minimum boiling azeotrope is formed when actual vapour pressure in higher than expected, i.e. solution shows +ve deviation from Raoult's law which is so when A–B interactions are weaker than A–A or B–B interactions.

Q4. (d)

$M_1V_1 = M_2V_2$, $0.02 \times 4 = M_2 \times 5$ or $M_2 = 0.016$

Q5. (c)

acceleration to the henry's law

$$P_A = K_H\ x_A$$

If x_A has a lower value, then K_H will increase, $\therefore$ solubility decrease.

Q6. (a)

Reason is the correct explanation of the assertion.

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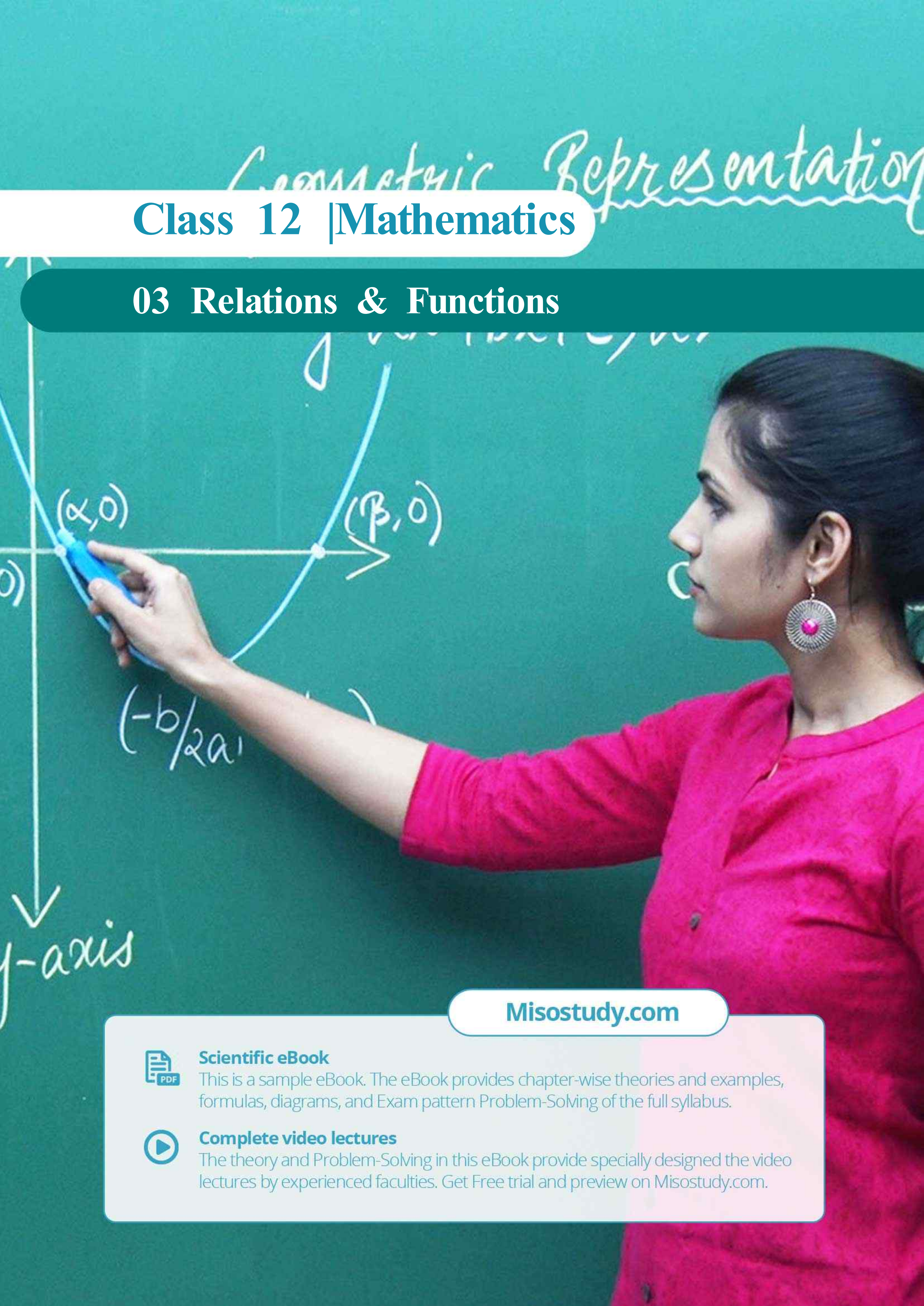


Class 12 |Mathematics

03 Relations & Functions

01. Types of Relations

(A) VOID, UNIVERSAL AND IDENTITY RELATIONS

Void Relation-

Let A be a set. Then, $\phi \subseteq A \times A$ and so it is a relation on A. This relation is called the void or empty relation on set A.

Universal Relation-

Let A be a set. Then, $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.

NOTE ☞ It is to note here that the void relation and the universal relation on a set A are respectively the smallest and the largest relations on set A.
Both the empty (or void) relation and the universal relation are sometimes. They are called trivial relations.

Identity Relation-

Let A be a set. Then, the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.
In other words, a relation I_A on A is called the identity relation i.e., if every element of A is related to itself only.

(B) REFLEXIVE, SYMMETRIC, TRANSITIVE RELATIONS

Reflexive Relation-

A relation R on a set A is said to be reflexive if every element of A is related to itself.
Thus, R is reflexive $\Leftrightarrow (a, a) \in R$ for all $a \in A$.
A relation R on a set A is not reflexive if there exists an element $a \in A$ such that $(a, a) \notin R$.

Illustration I Let A = {1, 2, 3} be a set. Then R = {(1, 1), (2, 2), (3, 3), (1, 3), (2, 1) is a reflexive relation on A. But R_1 = {(1, 1), (3, 3), (2, 1), (3, 2)} is not a reflexive relation on A, because 2 ∈ A but (2, 2) ∉ R_1.

Symmetric Relation-

A relation R on a set A is said to be a symmetric relation iff
$(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$
i.e. $aRb \Rightarrow bRa$ for all $a, b \in A$.

Illustration I Let A = {1, 2, 3, 4} and let R_1 and R_2 be relations on A given by R_1 = {(1, 3), (1, 4), (3, 1), (2, 2) (4, 1)} and R_2 = {(1, 1), (2, 2), (3, 3), (1, 3)}. Clearly, R_1 is a symmetric relation on A. However, R_2 is not ∈ R_2 but (3, 1) ∉ R_2.

NOTE ☞ A reflexive relation on a set A is not necessarily symmetric. For example, the relation R = {(1, 1), (2, 2), (3, 3), (1, 3) is a reflexive relation on set A = {1, 2, 3} but it is not symmetric.

Transitive Relation-

Let A be any set. A relation R on A is said to be a transitive relation iff

$(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow$ $(a, c) \in R$ for all $a, b, c \in A$.

i.e. aRb and bRc

$\Rightarrow$ aRc for all $a, b, c \in A$.

Illustration I *On the set N of natural numbers, the relation R defined by* $xRy \Rightarrow x$ *is less than* y *is transitive, because for any* $x, y, z \in N$
$x < y$ *and* $y < z \Rightarrow x < z$ *i.e.,* xRy *and* $yRz \Rightarrow xRz$

(C) **Equivalence Relation**

A relation R on a set A is said to be an equivalence relation on A iff

(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$

(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$

(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Let R be a relation on the set of all line in a plane defined by $(l_1, l_2) \in R \Leftrightarrow$ line l_1 is parallel to line l_2.

Then, R is an equivalence relation.

Solution : Let L be the given set of all lines in a plane. Then we observe the following properties.

Reflexive : For each line $l \in L$, we have
$l \parallel l \Rightarrow (l, l) \in R$ for all $l \in L \Rightarrow R$ is reflexive

Symmetric : Let $l_1, l_2 \in L$ such that $(l_1, l_2) \in R$. Then,
$(l_1, l_2) \in R \Rightarrow l_1 \parallel l_2 \Rightarrow l_2 \parallel l_1 \Rightarrow (l_2, l_1) \in R$. So, R is symmetric on R.

Transitive : Let $l_1, l_2, l_3 \in L$ such that $(l_1, l_2) \in R$ and $(l_2, l_3) \in$ R. Then,
$(l_1, l_2) \in R$ and $(l_2, l_3) \in R \Rightarrow l_1 \parallel l_2$ and $l_2 \parallel l_3 \Rightarrow l_1 \parallel l_3 \Rightarrow (l_1, l_3) \in R$

So, R is transitive on L, Hence R being reflexive symmetric and transitive is an equivalence relation on L.

02. Kinds of Functions

ONE-ONE FUNCTION (INJECTION)

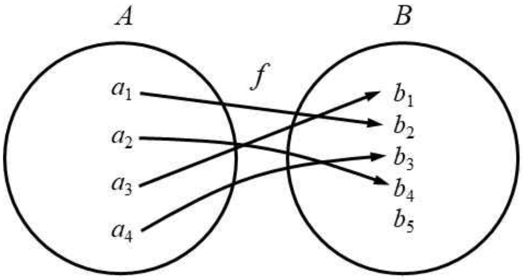
A function $f : A \to B$ is said to be a one-one function or an injection if different elements of A have different images in B.

Thus, $f : A \to B$ is one-one

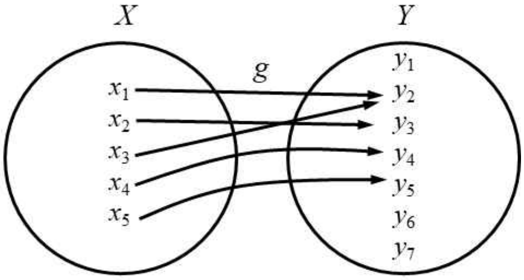
$\Leftrightarrow$ $a \neq b \Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow$ $f(a) = f(b) \Rightarrow a = b$ for all $a, b \in A$

Example : Let $f : A \to B$ and $g : X \to Y$ be two functions represented by the following diagrams :



Figure



Figure

Clearly, $f : A \to B$ is a one-one function. But, $g : X \to Y$ is not one-one because two distinct elements x_1 and x_3 have the same image under function g.

Let $f : A \to B$ be a function such that A is an infinite set and we wish to check the injectivity of f. In such a case it is not possible to list the images of all elements of set A to see whether different elements of A have different images or not. The following algorithm provides a systematic procedure to check the injectivity of a function.

Algorithm

(i) Take two arbitrary elements x, y (say) in the domain *of* f.

(ii) Put $f(x) = f(y)$

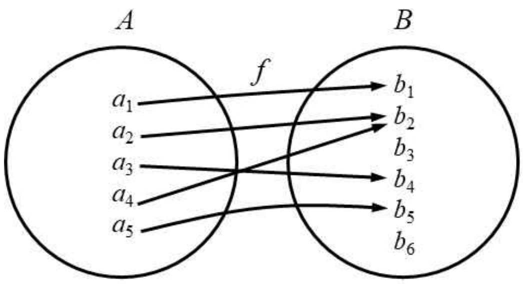
(iii) Solve $f(x) = f(y)$. If $f(x) = f(y$ gives x = y only, them $f : A \to B$ is a one-one function (or an injection). Otherwise not.

MANY-ONE FUNCTION

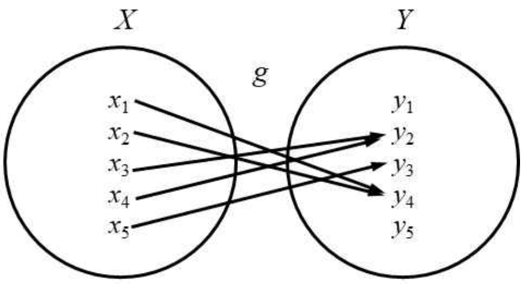
A function $f : A \to B$ is said to be a many-one function if two or more elements of set A have the same image in B.

Thus, $f : A \to B$ is a many-one function if there exist $x, y \in A$ such that $x \neq y$ but $f(x) = f(y)$.

Example : Let $f : A \to B$ and $g : X \to Y$ be two functions represented by the following diagrams :



Figure



Figure

Clearly, $a_2 \neq a_4$ but $f(a_2) = f(a_4)$ and $x_1 \neq x_2$ but $g(x_1) = g(x_2)$.
So, f and g are many-one functions.

NOTE ☞ In other words, $f : A \rightarrow B$ is many-one function if it is not a one-one function.

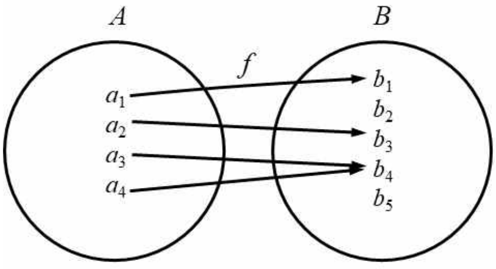
ONTO FUNCTION (SURJECTION)

A function $f : A \rightarrow B$ is said to be an onto function or a surjection if every element of B is the f-image of some element of A i.e., if f(A) = B of range of f is the co-domain of f. Thus, $f : A \rightarrow B$ is a surjection iff for each $b \in B$, there exists $a \in A$ such that f(a) = b.

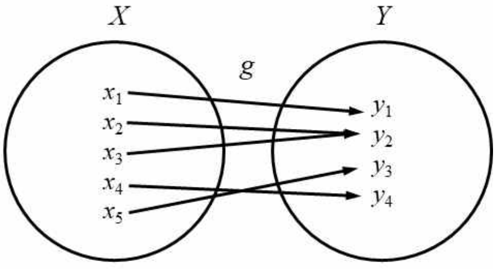
INTO FUNCTION. A function $f : A \rightarrow B$ is an into function if there exists an element in B having no pre-image in A.

In other words, $f : A \rightarrow B$ is an into function if it is not an onto function.

Example : Let $f : A \rightarrow B$ and $g : X \rightarrow Y$ be two functions represented by the following diagrams :



Figure



Figure

Clearly, b_2 and b_5 are two elements in B which do not have their pre-images in A.
So, $f : A \rightarrow B$ is an into function.
Under function g every elements in Y has its pre-image X. So, $g : X \rightarrow Y$ is an onto function.

The following algorithm can be used to check the surjectivity of a real function.

Algorithm

Let $f : A \rightarrow B$ be the given function.

(i) Choose an arbitrary element y in B.
(ii) Put f(x) = y
(iii) Solve the equation f(x) = y for x and obtain x in terms of y. Let x = g(y).
(iv) If for all values of $y \in B$, the values of x obtained from x = g(y) are in A, then f is onto. If there are some $y \in B$ for which x, given by x = g(y), is not in A. Then, f is not onto.

BIJECTION (ONE-ONE ONTO FUNCTION)

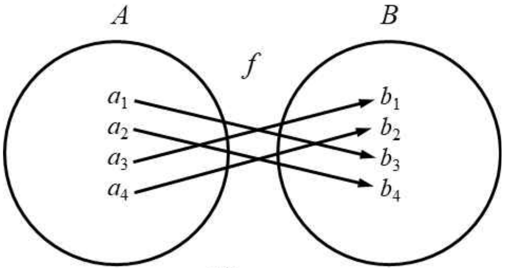
A function $f : A \to B$ is a bijection if it is one-one as well as onto. In other words, a function $f : A \to B$ is a bijection, if

(i) it is one-one i.e. $f(x) = f(y) \Rightarrow x = y$ for all x, $y \in A$.

(ii) it is onto i.e. for all $y \in B$, there exists $x \in A$ such that f(x) = y.

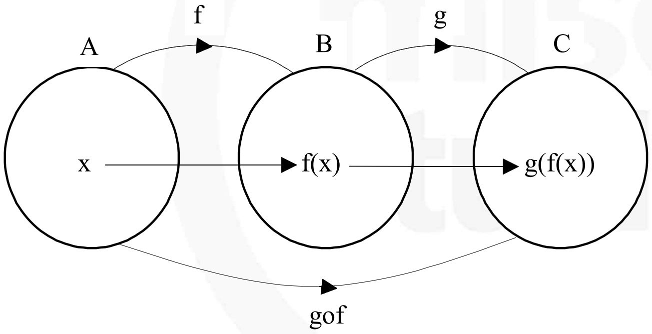
Example : *Let f : A → B be a function represented by the following diagram :*

Clearly, *f* is a bijection since it is both injective as well as surjective.



Figure

03. Composition of Functions



Definition:

Let $f : A \to B$ and $g : B \to C$ be two functions. Then a function go$f : A \to C$ defined by

(*gof*)(*x*) = *g*(*f*(*x*)), for all $x \in A$

is called the composition of *f* and *g*.

NOTE ☞ (i) It is evident from the definition that gof is defined only if for each $x \in A$, f(x) is an element of g so that we can take its g-image. Hence, for the composition gof to exist, the range of f must be subset of the domain of g.

(ii) It should be noted that gof exists iff the range of f is a subset of domain of g. Similarly, fog exists if range of g is a subset of domain of f.

PROPERTIES OF COMPOSITION OF FUNCTIONS

RESULT 1 The composition of functions is not commutative i.e. $fog \neq gof$.

RESULT 2 The composition of functions is associative i.e. if f, g, h are three functions such that $(fog)oh$ and $fo(goh)$ exist, then

$$(fog)oh = fo(goh)$$

Proof Let A, B, C, D be four non-void sets. Let $h : A \to B$, $g : B \to C$ and $f : C \to D$ be three functions. Then,

$h : A \to B,\ g : B \to C,\ f : C \to D$

$\Rightarrow$ $fog : B \to D$ and $h : A \to B$

Again, $h,\ A \to B,\ g : B \to C,\ f : C \to D$

$\Rightarrow$ $fog : C \to D$ and $goh : A \to C$

$\Rightarrow$ $fo\ (goh) : A \to D$

Thus, $(fog)oh$ and $fo(goh)$ are functions from set A to set D.

Now, we shall show that $\{(fog)oh\}\ (x) = \{fo\ (goh)\}\ (x)$ for all $x \in A$.

Let x be an arbitrary element of A and let $y \in B$, $z \in C$ such that $h\ (x) = y$ and $g(y) = z$. Then,

$h : A \to B,\ g : B \to C,\ f : C \to D$

$\Rightarrow$ $fog : B \to D$ and $h : A \to B$

Again, $h,\ A \to B,\ g : B \to C,\ f : C \to D$

$\Rightarrow$ $fog : C \to D$ and $goh : A \to C$

$\Rightarrow$ $fo\ (goh) : A \to D$

Thus, $(fog)oh$ and $fo(goh)$ are functions from set A to set D.

Now, we shall show that $\{(fog)oh\}\ (x) = \{fo\ (goh)\}\ (x)$ for all $x \in A$.

Let x be an arbitrary element of A and let $y \in B$, $z \in C$ such that $h\ (x) = y$ and $g(y) = z$. Then,

$\{(fog)\ oh\}\ (x) = (fog)\ \{h\ (x)\}$

$\Rightarrow$ $\{(fog)\ oh\}\ (x) = (fog)\ (y)$ $\quad [\because\ h(x) = y]$

$\Rightarrow$ $\{(fog)\ oh\}\ (x) = f(g\ (y))$

$\Rightarrow$ $\{(fog)\ oh\}\ (x) = f(z)$ ...(i) $\quad [\because\ g(y) = z]$

And, $\{fo\ (goh)\}\ (x) = f\{(goh)\ (x)\}$

$\Rightarrow$ $\{fo(goh)\}\ (x) = f\{g\ (h\ (x))\}$

$\Rightarrow$ $\{fo(goh)\}\ (x) = f\{g\ (y)\}$ $\quad [\because\ h(x) = y]$

$\Rightarrow$ $\{(fog)\ oh\}\ (x) = f(z)$ ...(ii) $\quad [\because\ g(y) = z]$

From (i) and (ii), we have

$\{(fog)\ oh\}\ (x) = \{fo\ (goh)\}\ (x)$ for all $x \in A$.

Hence, $(fog)\ oh = fo\ (goh)$

RESULT 3 The composition of two bijections is a bijection i.e. if f and g are two bijections, then gof is also a bijection.

Proof Let $f : A \to B$ and $g : B \to C$ be two bijections. Then, gof exists such that $gof : A \to C$.

We have to prove that gof is injective as well as surjective map.

Injectivity : Let x, y be two arbitrary elements of A. Then,

$\Rightarrow \quad (gof)\,(x) = (gof)\,(y)$

$\Rightarrow \quad g(f(x)) = g(f\,(y))$

$\Rightarrow \quad f(x) = f(y) \qquad [\because\ g$ is an injective map]

$\Rightarrow \quad x = y \qquad [\because\ f$ is an injective map]

Thus, $(gof)\,(x) = (gof)\,(y)$ for all, $x, y \in A$

So, gof is an injective map.

Surjectivity : In order to prove the surjectivity of gof, we have to show that every element in C has its pre-image in A *i.e.* for all $z \in \mathrm{C}$, there exists $x \in A$ such that $(gof)\,(x) = z$.

Let z be an arbitrary element of C. Then,

$z \in C \Rightarrow$ there exists $y \in B$ *s.t.g.* $(y) = z \qquad [\because\ g$ is a surjective map]

and, $\quad y \in B \Rightarrow$ there exists $x \in A$ *s.t.f.* $(x) = y \qquad [\because\ f$ is a surjective map]

Thus, we find that for every $z \in C$, there exists $x \in A$ such that

$(gof)\,(x) = g(f(x)) = g(y) = z.$

i.e. every element of C is the gof-image of some element of A.

So, gof is a surjective map.

RESULT 4 Let $f : A \to B$. Then, $foI_A = I_B of = f$ i.e. the composition of any function with the identity function is the function itself.

Proof Since $I_A : A \to A$ and $\mathrm{f} : A \to B$, therefore, of $I_A : A \to B$. Now let x be an arbitrary element of A. Then,

$(fo\ I_{\mathrm{A}})\,(x) = f\,(I_{\mathrm{A}}(x)) = f(x) \qquad [\because\ I_A\,(x) = x$ for all $x \in A]$

$\therefore \quad fo\ I_{\mathrm{A}} = f$

Again, $\quad f : A \to B$ and $I_B : B \Rightarrow I_B$ of $: A \to B$.

Now, let x be an arbitrary element of B. Let $f(x) = y$. Then, $y \in B$

$\therefore \quad (I_B\ \mathrm{of})\,(x) = I_B\,(f(x))$

$\Rightarrow \quad (I_B\ \mathrm{of})\,(x) = I_B\,(y) \qquad [\because\ f(x) = y]$

$\Rightarrow \quad (I_B\ \mathrm{of})\,(x) = y$

$\Rightarrow \quad (I_B\ \mathrm{of})\,(x) = f\,(x) \qquad [\because\ I_B(y) = y$ *for all* $y \in B]$

$\therefore \quad I_{\mathrm{B}}\ of = f$

Hence, $\quad fo\ I_A = I_B\ of = f$

RESULT 5 Let $f : A \to B$, $g : B \to A$ be two functions such that $gof = I_A$. Then, f is an injection and g is a surjection.

Proof *f is an injection* : Let $x, y \in A$ such that $f(x) = f(y)$. Then,

$$f(x) = f(y)$$
$$\Rightarrow \quad g(f(x)) = g(f(y))$$
$$\Rightarrow \quad gof(x) = gof(y)$$
$$\Rightarrow \quad I_A(x) = I_A(y) \qquad [\because gof = I_A \text{ (Given)}]$$
$$\Rightarrow \quad x = y \qquad [\text{By def. of } I_A]$$

Thus, $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$

So, f is an injective map.

g is a surjection : We have, $g : B \to A$. In order to prove that g is a surjection. It is sufficient to prove that every element in A has its pre-image in B.

Let x be an arbitrary element of A. Then, as $f : A \to B$ is a function therefore $f(x) \in B$.

Let $f(x) = y$. Then,

$$g(y) = g(f(x))$$
$$\Rightarrow \quad g(y) = gof(x)$$
$$\Rightarrow \quad g(y) = I_A(x) \qquad [\because gof = I_A]$$
$$\Rightarrow \quad g(y) = x$$

Thus, for every $x \in A$ there exists $y = f(x) \in B$ such that $g(y) = x$.
So, g is a surjection.

RESULT 6 Let $f : A \to B$ and $g : B \to A$ be two function such that $fog = I_B$. Then, f is a surjection and g is an injection.

Proof *f is a surjection* : In order to prove that $f : A \to B$ is a surjection, it is sufficient to prove that every element in B has its pre-image in A.

Let b be an arbitrary element of B.

Since g : B → A. Therefore, $g(b) \in A$.

Let $\quad g(b) = a$.

$$\text{Now,} \quad f(a) = f(g(b)) \qquad [\because a = g(b)]$$
$$\Rightarrow \quad f(a) = fog(b)$$
$$\Rightarrow \quad f(a) = I_B(b) \qquad [\because fog = I_B]$$
$$\Rightarrow \quad f(a) = b$$

Thus, for every $b \in B$ there exists $a \in A$ such that $f(a) = b$.

So, f is a surjection.

g is an injection : Let x, y be any two elements of B such that $g(x) = g(y)$. Then,

$$g(x) = g(y)$$
$$\Rightarrow \quad f(g(x)) = f(g(y))$$
$$\Rightarrow \quad fog(x) = fog(y)$$
$$\Rightarrow \quad I_B(x) = I_B(y)$$
$$\Rightarrow \quad x = y$$

Thus, $g(x) = g(y) \Rightarrow x = y$ for all $x, y, \in B$.

So, g is an injection.

RESULT 7 Let $f : A \rightarrow B$ and $g : B \rightarrow A$ be two functions. Then,

(i) $gof \ : A \rightarrow C$ is into $\Rightarrow g : B \rightarrow C$ is onto

Proof In order to prove that $g : B \rightarrow C$ is onto whenever gof : $A \rightarrow$ is onto, it is sufficient to prove that for all $z \in C$ there exists $y \in B$ such that $g(y) = z$.

Let z be an arbitrary element of C. Since gof : $A \rightarrow C$ is onto. Therefore, there exists $x \in A$ such that

$$gof(x) = z$$
$$\Rightarrow \quad g(f(x)) = z$$
$$\Rightarrow \quad g(y) = z, \text{ where } y = f(x) \in B.$$

Thus, for all $z \in C$, there exists $y = f(x) \in B$ such that $g(y) = z$.

Hence, $g : B \rightarrow C$ is onto.

(ii) $gof \ : A \rightarrow C$ is one-one $\Rightarrow f : A \rightarrow B$ is one-one.

Proof In order to prove that $f : A \rightarrow B$ is one-one, it is sufficient to prove that $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$.

Let $x, y \in A$ such that $f(x) = f(y)$. Then,

$$f(x) = f(y)$$
$$\Rightarrow \quad g = (f(x)) = g(f(y)) \qquad [\because g : B \rightarrow \text{C is a function}]$$
$$\Rightarrow \quad gof(x) = gof(y)$$
$$\Rightarrow \quad x = y \qquad [\because gof : A \rightarrow \text{C is a function}]$$

Hence, $f : A \rightarrow B$ is one-one.

(iii) $gof : A \to C$ is onto and $\Rightarrow g : B \to C$ is one-one $\Rightarrow f : A \to B$ is onto.

Proof In order to prove that $f : A \to B$ is onto, it is sufficient to prove that for all $y \in B$ there exists $x \in A$ such that $f(x) = y$.

Let y be an arbitrary element of B. Then,

$$g(y) \in C \qquad [\because g : B \to C]$$

Since $gof : A \to C$ is an onto function. Therefore, for any $g(y) \in C$ there exists $x \in A$ such that $go\, f(x) = g(y)$

$\Rightarrow \quad g = (f(x)) = g(y)$

$\Rightarrow \quad f(x) = y \qquad [\because g$ is one-one]

Thus, for all $y \in b$ there exists $x \in A$ such that $f(x) = y$.

Hence, $f : A \to B$ is onto.

(iv) $gof : A \to C$ is one-one and $\Rightarrow f : A \to B$ is onto $\Rightarrow g : B \to C$ is one-one.

Proof Let $y_1, y_2, \in B$ such that $g(y_1) = g(y_2)$. In order to prove that g is one-one, it is sufficient to prove that $y_1 = y_2$.

Now, $f : A \to B$ is onto and $y_1, y_2, \in B$. So, there exist $x_1, x_2 \in A$ such that

$$f(x_1) = y_1 \text{ and } f(x_2) = y_2$$

Now, $\quad g(y_1) = g(y_2)$

$\Rightarrow \quad g(f(x_1)) = g(f(x_2))$

$\Rightarrow \quad gof(x_1) = gof(x_2)$

$\Rightarrow \quad x_1 = x_2 \qquad [\because gof : A \to C$ is one-one]

$\Rightarrow \quad f(x_1) = (x_2) \qquad [\because f : A \to B$ is a function]

$\Rightarrow \quad y_1 = y_2$

Hence, $g : B \to C$ is one-one.

04. Inverse of An Element

Let A and B be two sets and let $f : A \to B$ be a mapping. If $a \in A$ is associated to $b \in B$ under the function f, then $'b'$ is called the f image of $'a'$ and we write it as $b = f(a)$. We also say that $'a'$ is the pre-image or inverse element of $'b'$ under f and we write $a = f^{-1}(b)$.

NOTE ☞ The inverse of an element under a function may consist of a single element, two or more elements or no element depending on whether function is injective or many-one; onto or into.

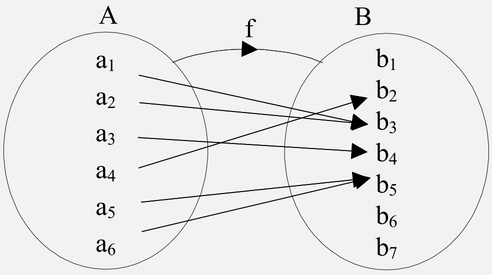
If f is represented by Figure, then we find that

$$f^{-1}(b_1) = \phi,\ f^{-1}(b_2) = a_{4},$$

$$f^{-1}(b_3) = \{a_1, a_2\},\ f^{-1}(b_4) = a_{3},$$

$$f^{-1}(b_5) = \{a_5, a_6\},\ f^{-1}(b_6) = \phi$$

and, $$f^{-1}(b_7) = \phi$$



05. Inverse of A Function

Definition:

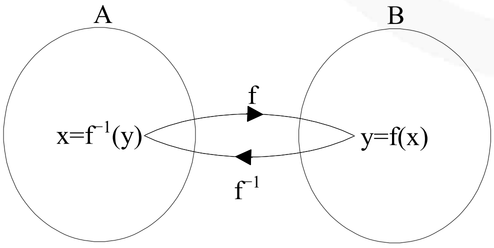
Let $f: A \to B$ be a bijection. Then a function $g: B \to A$ which associates each element $y \in B$ to a unique element $x \in A$ such that $f(x) = y$ is called the inverse of f.

i.e., $f(x) = y \Leftrightarrow g(y) = x$

The inverse of f is generally denoted by f^{-1}

Thus, if $f: A \to B$ is a bijection, then $f^{-1}: B \to A$ is such that

$$f(x) = y \Leftrightarrow f^{-1}(y) = x$$



In order to find the inverse of a bijection, we may follow the following algorithm.

Algorithm

Let $f: A \to B$ be a bijection. To find the inverse of f we follow the following steps:

STEP I Put $f(x) = y$, where $y \in B$ and $x \in A$.

STEP II Solve $f(x) = y$ to obtain x in terms of y.

STEP III In the relation obtained in step II replace x by $f^{-1}(y)$ to obtain the required inverse of f.

06. Properties of Inverse of a Function

RESULT 1 If $f: A \to B$ is a bijection and $g: B \to A$ is the inverse of f, then $fog = I_B$ and $gof = I_A$, where I_A and I_B are the identity function on the sets A and B respectively.

Proof In order to prove that gof = I_A and $fog = I_B$, we have to prove that $(gof)(x) = x$ for all $x \in A$ and $(fog)(y) = y$ for all $y \in B$.

Let x be an element of A such that $f(x) = y$. Then,

$$g(y) = x \qquad [\because g \text{ is inverse of } f\,]$$

Now, $(gof)(x) = g(f(x)) = g(y) = x$

$(gof)(x) = x$ for all $x \in A$

$\Rightarrow$ $gof = I_A$.

We have,

$(fog)(y) = f(g(y)) = f(x) = y$

$\therefore$ $fog\ (y) = y$ for all $y \in B$

$\Rightarrow$ $fog = I_B$.

Hence, $gof = I_A$ and fog = I_B.

RESULT 2 If $f: A \to B$ and $g: B \to C$ are two bijections, then $gof: A \to C$ is a bijection and $(gof)^{-1} = f^{-1}og^{-1}$

Proof We have,

$$\left.\begin{matrix} f: A \to B \text{ is a bijection} \\ g: B \to C \text{ is a bijection} \end{matrix}\right\} \Rightarrow gof: A \to C \text{ is a bijection} \Rightarrow (gof)^{-1}: C \to A \text{ exists.}$$

Again,

$$\left.\begin{matrix} f: A \to B \text{ is a bijection} \Rightarrow f^{-1};\ B \to A \text{ is a bijection} \\ g: B \to C \text{ is a bijection} \Rightarrow f^{-1};\ B \to A \text{ is a bijection} \end{matrix}\right\} \Rightarrow f^{-1}og^{-1}: C \to A$$

Let $x \in A$, $y \in B$ and $z \in C$ such that $f(x) = y$ and $g(y) = z$. Then,

$(gof)(x) = g(f(x)) = g(y) = z$

$\Rightarrow$ $(gof)^{-1}(z) = x$ ...(i)

Now,

$f(x) = y$ and $g(y) = z$

$\Rightarrow \quad f^{-1}(y) = x$ and $g^{-1}(z) = y$

$\therefore \quad (f^{-1} o g^{-1})(z) = (f^{-1}(g^{-1}(z)) = f^{-1}(y) = x$ ...(ii)

From (i) and (ii), we have

$(gof)^{-1}(z) = (f^{-1} o g^{-1})(z)$ for all $z \in C$.

Hence, $(gof)^{-1} = f^{-1} o g^{-1}$.

RESULT 3 If $f : A \rightarrow B$ and $g : B \rightarrow A$ be two functions such that $gof = I_A$ and $fog = I_B$.

Then, f and g are bijections and $g = f^{-1}$.

Proof *f is one-one* : Let $x, y \in A$ such that $f(x) = f(y)$. Then,

$f(x) = f(y)$

$\Rightarrow \quad g(f(x)) = g(f(y))$

$\Rightarrow \quad (gof)(x) = (gof)(y)$

$\Rightarrow \quad I_A(x) = I_A(y)$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is a one-one map.

f is onto : Let $y \in B$ and let $g(y) = x$. Then,

$g(y) = x$

$\Rightarrow \quad f(g(y)) = f(x)$

$\Rightarrow \quad (fog)(y) = f(x)$

$\Rightarrow \quad I_B(y) = f(x)$ $[\because fog = I_B]$

$\Rightarrow \quad y = f(x)$ $[\because I_B(y) = y]$

Thus, for each $y \in B$, there exists $x \in A$ such that $f(x) = y$.

So, f is onto.

Hence, f is a bijection.

Similarly, it can be proved that g is a bijection.

Now we shall show that $g = f^{-1}$.

Since $f : A \rightarrow B$ is a bijection. Therefore, f^{-1} exists.

Now, $fog = I_B$

$\Rightarrow \quad f^{-1} 0 (fog) = f^{-1} 0 I_B$

$\Rightarrow \quad (f^{-1} of) og = f^{-1} 0 I_B$ [By associativity]

$\Rightarrow \quad I_A 0 g = f^{-1} 0 I_B$ $[\because f^{-1} of I_A]$

$\Rightarrow \quad g = f^{-1}$ $[\because I_A og = g$ and $f^{-1} 0 I_B = f^{-1}]$

Hence, $g = f^{-1}$

07. Binary Operation

DEFINITION

A binary operation * on a set A is a function * : A × A → A. We denote * (a, b) by a * b.

Eg. : Addition, subtraction and multiplication are binary operations on **R**, but division is not a binary operation on **R.** Further division is a binary operation on the set **R**. of non-zero real numbers.

Solution + : **R** × **R** → **R** is given by

$(a, b) \to a + b$

– : **R** × **R** → **R** is given by

$(a, b) \to a - b$

× : **R** × **R** → **R** is given by

$(a, b) \to ab$

Since '+', '–' and '×' are functions, they are binary operation on **R.**

But ÷ : **R** × **R** → **R,** given by $(a, b) \to \frac{a}{b}$ is a function and hence a binary operation on **R**.

Types of Binary Operation

Type I **Commutativity** *A binary operation '*' on a set S is said to be a communicative binary operation, if*

$$a * b = b * a \text{ for all } a, b \in S$$

Eg.: The binary operations addition (+) and multiplication (×) are commutative binary operation on Z. However, the binary operation subtraction (–) is not a commutative binary operation on Z as $3 - 2 \neq 2 - 3$.

Type II **Associativity** *A binary operation '*' on a set S is said to be an associative binary operation, if*

$$(a * b) * c = a * (b * c) \text{ for all } a, b \in S.$$

Eg.: The binary operations of addition (+) and multiplication (×) are associative binary operation on Z. However, the binary operation subtraction (–) is not a associative binary operation on Z as $(2 - 3) - 5 \neq 2 - (3 - 5)$.

Identity Element

Let "' be a binary operation on a set S. If there exists an elements e ∈ S such that*

$$a*e = a = e * a \text{ for all } a \in S.$$

Then, e is called an identity element for the binary operation '' on set S.*

Eg. : Consider the binary operation of addition (+) on Z. we know that $0 \in Z$ such that

$$a + 0 = a = 0 + a \text{ for all } a \in Z$$

So, '0' is the identity element for addition on Z.

If we consider multiplication on Z, then '1' is the identity element for multiplication on Z, because

$$1 \times a = a = a \times 1 \text{ for all } a \in Z.$$

Eg. : We know that addition (+) and multiplication (·) are binary operations on N such that

$$n \times 1 = n = 1 \times n \text{ for all } n \in N$$

But, there do not exists any natural number e such that

$$n + e = n = e + n \text{ for all } n \in N.$$

So, 1 is the identity element for multiplication on N, but N does not have identity element for addition on N.

Inverse of an Element

Invertible Element *Let '*' be a binary operation on a set S, and let e be the identity element in S for this binary operation * on S. Then, an element* $a \in S$ *is called an invertible element if there exists an element* $b \in S$ *such that*

$$a * b = e = b * a$$

The element b is called an inverse of element a.

Thus, an element $b \in S$ *is called an inverse of an element* $a \in S$*, if*

$$a * b = e = b * a.$$

Eg.: Consider the binary operation addition (+) on Z. Clearly, 0 is the identity element for addition on Z and for any integer a, we have

$$a + (-a) = 0 = (-a) + a$$

So, $-a$ is the inverse of $a \in Z$.

Multiplication is also a binary operation on Z and 1 is the identity element for multiplication on Z. But, no element, other than $1 \in Z$, is invertible.

Composition Table

A binary operation on finite set can be completely describe by means of a table known as a composition table. Let $S = \{a_1, a_2, ..., a_n\}$ be a finite set and * be a binary operation on S. Then the composition table for * is constructed in the manner indicated below.

We write the elements $a_1, a_2, ..., a_n$ of the set S in the top horizontal row and the left vertical column in the same order. Then we put down the element $a_i * a_j$ at the intersection of the row headed by a_i $(1 \le j \le n)$ to get the following table :

*	a_1	a_2	…	a_i	…	a_j	…	a_n
a_1	$a_1 * a_1$	$a_1 * a_2$	…	$a_i * a_i$	…	$a_1 * a_j$	…	$a_1 * a_n$
a_2	$a_2 * a_1$	$a_2 * a_2$	…	$a_2 * a_1$	…	$a_2 * a_j$	…	$a_2 * a_n$
⋮								
a_i	$a_i * a_1$	$a_i * a_2$	…	$a_i * a_i$	…	$a_i * a_j$	…	$a_i * a_n$
⋮								
a_j	$a_j * a_1$	$a_j * a_2$	…	$a_j * a_i$	…	$a_j * a_j$	…	$a_j * a_n$
⋮								
a_n	$a_n * a_1$	$a_n * a_2$	…	$a_n * a_i$	…	$a_n * a_j$	…	$a_n * a_n$

(i) From the composition table we infer the following results.

If all the entires of the table are elements of set S and each element of S appears once and only once in each row and in each column, then the operation is a binary operation.

Sometimes we also say that the binary operation is well defined which means that the operation*. But for us, this is a consequence of the definition of binary operation.

(ii) If the entries in the table are symmetric with respect to the diagonal which starts at the upper left corner of the table and terminates at the lower right corner, we say that the binary operation is commutative on S, otherwise it is said to be non-commutative on S.

(iii) If the row headed by an element say a_j coincides with the row at the top and the column headed by a_j coincides with the column on extreme left, then a_j is the identity element in S for the binary operation *on S.

(iv) If each row except the top most row or each column except the left most column contains the identity element every element of S is invertible with respect to *. To find the inverse of an element say a_i, we consider row (or column) headed by a_i. Then we determine the position of identity element e in this row (or column). If e appears in the column (or row) headed by a_j, then a_i and a_j are inverse of each other.

CBSE Exam Pattern Exercise
Subjective Questions (1)

(Q 1 to 4) One Mark

1. Let R is the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class [0].

2. If $f : R \to R$ is defined by $f(x) = (3 - x^3)^{1/3}$, then find $fof(x)$.

3. Let $* : R \times R \to$ given by $(a, b) \to a + 4b^2$ be a binary operation. Compute $(-5) * (2 * 0)$.

4. If $*$ is a binary operation on set Q of rational number defined as $a * b = \frac{ab}{5}$. Write the identity for $*$, if any.

(Q 5 to 8) Four Marks

5. If the function $f : R \to R$ is given by $f(x) = x^2 + 2$ and $g : R \to R$ is given by $g(x) = \frac{x}{x-1}$; $x \neq 1$, then find fog and gof, and hence find fog (2) and gof (–3).

6. Show that $f : N \to N$, given by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{if} x \text{ is even} \end{cases}$ is bijective (both one-one and onto).

7. Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.

8. Consider the binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a * b = \min \{a, b\}$. Write operation table of operation $*$.

(Q 9 to 10) Six Marks

9. Let $f : N \to R$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f : N \to S$, where S is the range of f, is invertible. Also, find the inverse of f.

10. Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for (a, b) $(c, b) \in A$. Determine, whether $*$ is commutative and associative. Then, with respect to

* on A

(i) Find the identity element in A.

(ii) Find the invertible elements of A.

Answer & Solution

Q1.

$[0] = \{b \in A;\ bR0\}$

$= \{b \in A;\ (b, 0) \in R\}$

$= \{b \in A;\ 2 \text{ divides } b - 0\}$

$= \{0, 2, 4\}$

Q2.

$\text{fof}(x) = f((3 - x)^{1/3})$

$= (3 - y^3)^{1/3}$ where $y = (3 - x^3)^{1/3}$

$= (3 - ((3 - x^3)^{1/3})^3)^{1/3}$

$= (3 - (3 - x^3))^{1/3} = (x^3)^{1/3} = x = I(x)$

$\therefore \quad fof = I_{\mathbf{R}}$

Q3.

$a * b = a + 4b^2$

$\Rightarrow -5 * (2 * 0)$

$\Rightarrow -5 * (2 + 4(0^2))$

$= -5 * 2$

$= -5 + 4(2)^2$

$= -5 + 16 = 11$

Q4.

Let e be the identity element in Q under $*$

$\Rightarrow a * e = a \ \forall\ a \in Q$

$= e * a$

$\Rightarrow \dfrac{ae}{5} = a \ \forall\ a \in Q$

$\Rightarrow e = 5 \in Q$

$\therefore$ Identity element $= 5$

Q5.

$f(x) = x^2 + 2 \ \forall\ x \in \text{IR}$

$D(f) = \text{IR}$

$R(f) = [2, \infty]$

$(\because\ x^2 \geq 0\ \forall\ x \in \mathbf{R} \Rightarrow x^2 + 2 \geq 2\ \forall\ x \in \mathbf{R}\ \therefore\ x^2 + 2 \in [2, \infty))$

$g(x) = \dfrac{x}{x-1}$ & $g(x) \notin \mathbf{R}$ for $x = 1$

$\therefore\ \mathrm{D}(g) = \mathbf{R}/\{1\}$

& $\mathrm{R}(g) = \mathbf{R}/\{1\}$.

$$\left(\frac{x}{x-1} = y \Rightarrow x = yx - y \right.$$
$$\left. \Rightarrow x = \frac{y}{y-1} \in R \,\forall\, y \in R/\{1\}\right)$$

As $\mathrm{R}(g) \subseteq \mathrm{D}(f)$

$\therefore$ *fog* exists

for $x \in D(g) = \mathbf{R} - \{1\}$,

$$\begin{aligned}(fog)\ (x) &= f(g(x)) \\ &= f\left(\frac{x}{x-1}\right) \\ &= \left(\frac{x}{x-1}\right)^2 + 2 \\ &= \frac{x^2}{(x+1)^2} + 2 \\ &= \frac{3x^2 - 4x + 2}{(x+1)^2}\end{aligned}$$

$$\therefore\ fog(2) = \frac{3(2)^2 - 4(2) + 2}{(2-1)^2}$$
$$= 6$$

And, as $R(g) \subseteq D(f)$

$\therefore$ *gof* is defined

$\therefore$ for $x \in \mathbf{R} = D(f)$

$$\begin{aligned}gof(x) &= g(f(x)) \\ &= g(x^2 + 2) = \frac{x^2+2}{x^2+2-1} \\ &= \frac{x^2+2}{x^2+1}\end{aligned}$$

$$\therefore gof\ (-3) = \frac{(-3)^2 + 2}{(-3)^2 + 1} = \frac{11}{10}$$

Q6.

Injective :

case (i) if x_1 & x_2 both are odd.

Then, $f(x_1) = f(x_2)$

$\Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

case (ii) if x_1 & x_2 both are even

Then, $f(x_1) = f(x_2)$

$\Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$

case (iii) If x_1 is even & x_2 is odd

$\Rightarrow x_1 \neq x_2$

to show $f(x_1) \neq f(x_2)$

As $x_1 \neq x_2 \Rightarrow x_1 - 1 \neq x_2 + 1$

$\Rightarrow f(x_1) \neq f(x_2)$

$\therefore$ From all three cases f is one-one

Surjective :

let $y \in \mathbf{N} = C(f)$

If y is odd then $y + 1$ is an even natural no ;

$$f(y + 1) = (y + 1) - 1 = y$$

$\therefore \exists x = y + 1 \in \mathbf{N}$ such that

$$f(x) = y \in \mathbf{N}$$

And, if y is even then $y - 1$ is an odd natural no;

$f(y - 1) = (y - 1) + 1 = y$

$\therefore \exists x = y - 1 \in \mathbf{N}$; $f(x) = y \in \mathbf{N}$

$\therefore$ f is onto.

Q7.

Reflexive :

Claim : $1/2 \not R\ 1/2$

$$\because \frac{1}{2} \not\leq \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$\therefore$ $1/2 \not R\ 1/2$

$\therefore$ $(1/2, 1/2) \notin R$

$\therefore$ R is not reflexive.

Symmetric :

Claim : $-2R3$ but $3 \not R -2$

As $-2 \leq 3^3 \Rightarrow (-2, 3) \in R$

But $3 \not\leq (-2)^3 = 8$ $\therefore$

$(3, -2) \notin R$

$\therefore$ R is not symmetric.

Transitive :

Claim : $2R3/2$ & $3/2\ R4/3$ but $2 \not R\ 4/3$

As $2 \leq (3/2)^3$ & $3/2 \leq (4/3)^3$ but $2 \not\leq (4/3)^3$

$\Rightarrow (2, 3/2) \in R, (3/2, 4/3) \in R$ but $(2, 4/3) \notin R$

$\therefore$ R is not transitive.

Q8.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

Q9.

ONE ONE :

Let $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9x_1^2 - 9x_2^2 + 6x_1 - 6x_2 = 0$

$9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) = 0$

$(x_1 - x_2)(3x_1 + 3x_2 + 2) = 0$

But $3x_1 + 3x_2 + 2 \neq 0$

$\because x_1, x_2 \in \mathbf{N}$

$\Rightarrow x_1 = x_2$

$\therefore f$ is one one

ONTO :

As S = R(f)

& $f : \mathbf{N} \to S$

$\therefore$ C(f) = R(f)

$\therefore f$ is onto

$\therefore f$ is invertible $\Rightarrow \exists f^{-1} : S \to \mathbf{N}$

Let $y = 9x^2 + 6x - 5$

$y = (3x)^2 + 2.3x.1 + (1)^2 - (1)^2 - 5$

$y = (3x + 1)^2 - 6$

$\Rightarrow y + 6 = (3x + 1)^2$

$\Rightarrow 3x + 1 = +\sqrt{y+6}$

$(\because x \in \mathbf{N})$

$\therefore 3x = \sqrt{y+6} - 1$

$$x = \frac{\sqrt{y+6} - 1}{3}$$

$$\therefore f^{-1}(x) = \frac{\sqrt{x+6} - 1}{3}$$

Q10.

Commutative :

let (a, b), (c, d) $\in$ Q × Q = A

$\Rightarrow$ (a, b) * (c, d) = (ac, b + ad)

& (c, d) * (a, b) = (ca, d + bc)

As (a, b) * (c, d) $\neq$ (c, d) * (a, b)

$\therefore$ * is not commutative

Associative :

Let (a, b), (c, d), (e, f) $\in$ Q × Q

Consider

((a, b) * (c, d)) * (e, f) = (ac, b + ad) * (e, f) = (ace, b + ad + acf) = (a, b) * ((c, d) * (e, f))

$\therefore$ * is associative.

Identity Element

Let (e, f) be the identity element in A w.r.t. *

$\Rightarrow$ (a, b) * (e, f) = (a, b) $\forall$ (a, b) $\in$ A = Q × Q

$\Rightarrow$ (ae, b + af) = (a, b)

$\Rightarrow$ ae = a & b + af = b

$\Rightarrow$ e = 1 & f = 0
$\therefore$ (1, 0) $\in$ Q $\times$ Q = A
is the identity element of A w.r.t. *

Inverse Element

let (c, d) $\in$ Q $\times$ Q = A be the inverse element of (a, b) $\in$ Q $\times$ Q
$\Rightarrow$ (a, b) * (c, d) = (1, 0)
$\Rightarrow$ (ac, b + ad) = (1, 0)
$\Rightarrow$ ac = 1 & b + ad = 0
$\Rightarrow$ c = 1/a & d = –b/a ; a $\neq$ 0
$\therefore$ for (a, b) $\in$ Q $\times$ Q; a $\neq$ 0, inverse of (a, b) exists & is given by (1/a, –b/a) $\in$ Q $\times$ Q.

CBSE Exam Pattern Exercise
Objective Questions (2)

1. Which of the following functions are equal
   (a) $\sin^{-1}(\sin x)$ and $\sin(\sin^{-1}x)$
   (b) $\frac{x^2-4}{x-2}, x+2$
   (c) $\frac{x^2}{x}, x$
   (d) A={1,2}, B={3,6}
   f:A→B given by $f(x) = x^2+2$ and
   g:A→B given by $g(x) = 3x$

2. Let $f : \left[\frac{1}{2}, \infty\right] \rightarrow \left[\frac{3}{4}, \infty\right]$, where $f(x) = x^2 - x + 1$ is
   (a) one–one onto
   (b) many one-into
   (c) many one-onto
   (d) one-one into

3. IF A = {1,2,3}
   B = {4,5,6,7} and
   f = {(1,4) (2,5) (3,6)} is a function from A to B then f is
   (a) one-one
   (b) onto
   (c) many one
   (d) both (a) and (b)

4. The range of the function $f(x) = \frac{|x-2|}{x-2}, x \neq 2$ is
   (a) {1,0,–1}
   (b) {1}
   (c) {1,–1}
   (d) None of these

5. f:R→R and g:R→R are given by $f(x) = |x|$ and $g(x) = |5x-2|$, then fog is
   (a) $|5x-2|$
   (b) $5x-2$
   (c) $2-5x$
   (d) None of these

6. Range of the function $f(x) = \frac{|x^2+1|}{x^2+1}$ is
   (a) {1}
   (b) {1,–1}
   (c) {1,0,–1}
   (d) R

7. If set A has 5 elements and set B has three elements then total no. of one-one functions from A to B are
   (a) 0
   (b) $^5P_3$
   (c) 5
   (d) 5!

8. If f(x) = [x] and g(x) = |x| then fog $\left(\frac{-5}{2}\right)$ is
   (where [.] represents greatest integer function of x)
   (a) 2
   (b) 3
   (c) –3
   (d) –2

Answer & Solution

1. (d)

(a) $\sin^{-1}(\sin x) \neq \sin(\sin^{-1}x)$

$\because$ Those functions are equal

whose range f domain are equal

But $\sin^{-1}(\sin x) = x \Rightarrow x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

But $\sin(\sin^{-1}x) = x \Rightarrow x \in [-1,1]$

(b) $\frac{(x^2-4)}{(x-2)} = x+2\ ,\ x \neq 2$

Domain of $\frac{x^2-4}{x-2} \Rightarrow R-2$

& Domain of $x+2$ is R

Hence they are not equal

(c) same explanation as (b)

(d) $A = \{1,2\}$, $B = \{3,6\}$

$f(1) = 3$

$f(2) = 6$

$g(x) = 3x$

$g(1) = 3$

$g(2) = 6$

Since range & domain in both functions is equal.

Hence functions are equal function

2. (a)

$f(x) = x^2 - x + 1$

$f: \left(\frac{1}{2}, \infty\right) \rightarrow \left(\frac{3}{4}, \infty\right)$

For one-one

$f(x_1) = f(x_2)$

$x^2_1 - x_1 + 1 = x^2_\alpha - x_1 + 1$

$x^2_1 - x^2_\alpha - x_1 + x_2 = 0$

$(x_1 - x_2)(x_1 + x_2) - 1\ (x_1 - x_2) = 0$

$(x_1 - x_2)(x_1 + x_\alpha - 1) = 0$

either $x_1 = x_\alpha$ or $x_1 + x_\alpha = 1$

But $x_1 + x_\alpha = 1$
only when $x_1 = x_\alpha$
& for no other value
$\therefore\ x_1 = x_2$
Hence one-one
onto
$f(x) = x^2 - x + 1$

$$= x^2 - x + \frac{1}{4} - \frac{1}{4} + 1$$

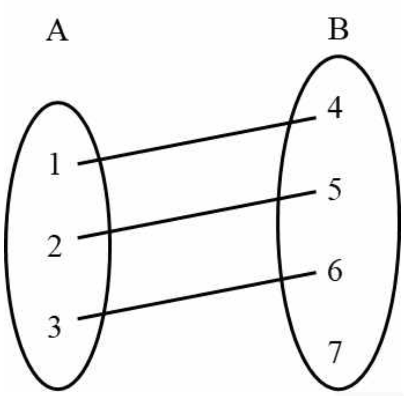
$$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$$ [Using completing the square method]

For $x \geq \frac{1}{2}$

$y \geq \frac{3}{4}$

Hence range = Codomain
Hence function is onto

3. (a)



Clearly f is one but not onto

4. (c)

$f(x) = \dfrac{|x-2|}{x-\alpha}, x \neq 2$ is

$$f(x) = \begin{cases} 1 & x-2>0 \\ -1 & x-2<0 \end{cases}$$

$\therefore$ Range is $\{1, -1\}$

5. (a)
$f(x) = |x|$
$g(x) = |5x - 2|$
$f(g(x)) = ||5x - 2||$
$= |5x - 2|$

6. (a)

$f(x) = \frac{|x^2+1|}{x^2+1}$

x^2+1 is always +ve

$\therefore$ f(x) = 1

{1}

7. (a)
Set A has 5 elements
Set B has 3 elements
$\therefore$ one-one function = zero

8. (a)
f(x) = [x]
g(x) = |x|
f(g(x) = [|x|]

$$f\left(g\left(\frac{-5}{2}\right)\right) = \left[\left|\frac{-5}{2}\right|\right] = \left[\frac{5}{2}\right] = 2$$