CRASH COURSE

JEE MAIN 2021-22

PHYSICS

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SET-1

1. Force F is given in terms of time t and distance x by

$$F = A \sin Ct + B \cos Dx$$

Then the dimensions of $\frac{A}{B}$ and $\frac{C}{D}$ are given by

(a) $[MLT^{-2}, M^0L^0T^{-1}]$
(b) $[MLT^{-2}, M^0L^{-1}T^0]$
(c) $[M^0L^0T^0, M^0LT^{-1}]$
(d) $[M^0LT^{-1}, M^0L^0T^0]$

2. What are the dimensions of electrical resistance?

(a) $ML^2T^{-2}I^2$
(b) $[ML^2T^{-3}I^{-2}]$
(c) $[ML^2T^{-3}I^2]$
(d) $[ML^2T^{-2}I^{-2}]$

3. From some instruments, current measured is i = 10.0 Amp, potential difference measured is V = 100.0 V, length of wire is 31.4 cm, and diameter of wire is 2.00 mm (all in correct significant figures) will be (Use π = 3.14)
   (a) 1.00×10^{-4} Ω-m
   (b) 1.0×10^{-4} Ω-m
   (c) 1×10^{-4} Ω-m
   (d) 1.000×10^{-4} Ω-m

4. The external and internal diameters of a hollow cylinder are measured to be (4.23 ± 0.01) cm and (3.89 ± 0.01) cm. The thickness of the wall of the cylinder is
   (a) (0.34 ± 0.02) cm
   (b) (0.17 ± 0.02) cm
   (c) (0.17 ± 0.01) cm
   (d) (0.34 ± 0.01) cm

5. 1 cm on the main scale of a vernier callipers is divided into 10 equal. If 10 divisions of vernier coincide with B small divisions of main scale, then the least count of the vernier calliper is
   (a) 0.01 cm
   (b) 0.02 cm
   (c) 0.05 cm
   (d) 0.005 cm

6. A small metal sphere of radius r and density ρ falls from rest in a viscous liquid of density σ and coefficient of viscosity η. Due to friction heat is produced. The expression for the rate of production of heat when the sphere has acquired the terminal velocity is

   (a) $\left[\frac{8\pi g}{27\eta}(\rho-\sigma)^2\right]r^5$
   (b) $\left[\frac{8\pi g^2}{27\eta}(\rho-\sigma)^2\right]r^5$
   (c) $\left[\frac{8\pi g^2}{27\eta}(\rho-\sigma)\right]r^5$
   (d) $\left[\frac{8\pi g^2}{27\eta^2}(\rho-\sigma)\right]r^5$

SET-2

1. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then, $x_1 ; x_2 ; x_3$ is

   (a) 1 : 2 : 4
   (b) 1 : 2 : 5
   (c) 1 : 3 : 5
   (d) 1 : 3 : 9

2. A ball is dropped from the top of a building. They ball takes 0.5 s to fall past the 3 m length of a window some distance from the top of the building. If the speed of the ball at the top and at the bottom of the window are v_T and v_B respectively, then
   (a) $v_T + v_B = 12\ \text{ms}^{-1}$
   (b) $v_T - v_B = 4.9\ \text{ms}^{-1}$
   (c) $v_B + v_T = 1\ \text{ms}^{-1}$
   (d) $\dfrac{v_B}{v_T} = 2$

3. A particle starts from rest with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in the last two seconds is
   (a) $\dfrac{2v(n-1)}{n}$
   (b) $\dfrac{v(n-1)}{a}$
   (c) $\dfrac{v(n+1)}{n}$
   (d) $\dfrac{2v(2n+1)}{a}$

4. A person walks up a stationary escalator in time t_1. If he remains stationary on the escalator, then it can take him up in time t_2. How much time would it take him to walk up the moving escalator?
   (a) $\dfrac{t_1 + t_2}{2}$
   (b) $\sqrt{t_1 t_2}$
   (c) $\dfrac{t_1 t_2}{t_1 + t_2}$
   (d) $t_1 + t_2$

Answer & Solutions

SET-1

1. (c)

$$\frac{A}{B} = M^0L^0T^0 \qquad \frac{C}{D} = \frac{X}{t} = LT^{-1}$$

2. (b)

3. (a)

$$s = \frac{\pi D^2}{4L}.\frac{V}{I}$$

$$\Rightarrow \frac{3.14 \times (2 \times 10^{-3})^2}{4(.314)}\left(\frac{100.0}{10.0}\right)$$

$s = 1.00 \times 10^{-4}$ Ω-m

4. (c)

$(R_1 \pm \Delta R_1) = (2.11 \pm 0.005)$ cm

$(R_2 \pm \Delta R_2) = (1.945 \pm 0.005)$ cm

Thickness $t = R_1 - R_2 = 0.17$ cm

$\Delta t = \Delta R_1 + \Delta R_2 = 0.01$ cm

$(t \pm \Delta t) = (0.17 \pm 0.01)$ cm

5. (b)

10 VSD = 8 MSD

1 VSD = 0.8 MSD

LC = 1 MSD – 1 VSD

= 1 MSD – 0.8 MSD

$= 0.2$ MSD $= 0.2 \times \frac{1}{10}$ cm

= 0.02 cm

6. (b)

Rate of heat production = – (Power of viscous force)

$\frac{dH}{dt}$ = (viscous force) (terminal velocity)

$= (6\pi\eta rv)(v)$ ...(i)

$$v = \frac{2}{9}\frac{(s - \sigma)r^2g}{\eta}$$

$$\frac{dx}{dt} = \left[\frac{8\pi g^2}{2\eta}(s-\sigma)\right]r^5$$

SET-2

1. (c)

$$x_1 = \frac{1}{2}a(10)^2 = 50a$$

$$x_2 = \frac{1}{2}a(20)^2 - \frac{1}{2}(a)(10)^2 = 150a$$

$$x_3 = \frac{1}{2}a(30)^2 - \frac{1}{2}a(20)^2 = 250a$$

$x_1 : x_2 : x_3 = 1 : 3 : 5$

2. (a)

$v_B = v_T + gt$

$= v_T + 9.8 \times 0.5$

$= v_B - v_T = 4.9$

$v_B^2 - v_T^2 = 58.8$

Solving $v_T + v_B = 12 \text{ ms}^{-1}$

3. (a)

$v = an \quad \Rightarrow a = v/n$

$$s = \frac{1}{2}a(n)2 - \frac{1}{2}a(n-2)^2$$

$= v/2n\ [4n - 4]$

$$= \frac{2v(n-1)}{n}$$

4. (c)

Solution : Speed of man w.r.t. escalator $v_{mc} = L/t_1$

speed of escalator $v_c = L/t_2$

speed of man w.r.t. grand

$$v_m = v_{mc} + v_c = L\left[\frac{1}{t_1} + \frac{1}{t_2}\right]$$

$$\text{time } t = \frac{L}{v_m} = \frac{t_1 t_2}{t_1 + t_2}$$

CHEMISTRY

JEE Main 2021-22 CRASH COURSE

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1. 5.85g NaCl is dissolved in 1 L water. The number of ions of Na^+ and Cl^- is 1mL of this solution will be :
   (a) 6.02×10^{19}
   (b) 1.2×10^{22}
   (c) 1.2×10^{20}
   (d) 6.02×10^{20}

2. If mass of neutron is assumed to half of its original value, whereas that of proton is assumed to be twice of its original value then the atomic mass of ${}^{14}_{6}C$ will be:
   (a) same
   (b) 14.28% more
   (c) 14.28% less
   (d) 28.56% less

3. The density of a liquid is 1.2 g/mL. There are 35 drops in 2mL. The number of molecules in 1 drop is (molecular weight of liquid = 70).
   (a) $\frac{1.2}{35}N_A$
   (b) $\left(\frac{1}{35}\right)^2 N_A$
   (c) $\frac{1.2}{(35)^2}N_A$
   (d) 1.2 N_A

4. 26.8 gm of $Na_2SO_4.nH_2O$ contains 12.6 gm of water. The value of 'n' is :
   (a) 1
   (b) 10
   (c) 6
   (d) 7

5. How many moles of Na^+ ions are in 20mL of 0.4M Na_3PO_4 ?
   (a) 0.008
   (b) 0.024
   (c) 0.05
   (d) 0.20

6. In the reaction ;

$$CO + \frac{1}{2}O_2 \rightarrow CO_2; \quad N_2 + O_2 \rightarrow 2NO$$

   10mL of mixture containing carbon monoxide and nitrogen required 7mL oxygen to form CO_2 and NO, on combustion. The volume of N_2 in the mixture will be :
   (a) 7/2mL
   (b) 17/2mL
   (c) 4mL
   (d) 7mL

7. A mixture of ethane and ethene occupies 40 litre at 1.00 atm and at 400K. The mixture reacts completely with 130g of O_2 to produce CO_2 and H_2O. Assuming ideal gas behaviour, calculate the mole fractions of C_2H_6 and C_2H_4 in the mixture.
   (a) 11.12, 12.13
   (b) 18.34, 34.36
   (c) 66.66, 33.34
   (d) 35.34, 31.34

8. A mixture of HCOOH and $H_2C_2O_4$ is heated with concentrated H_2SO_4. The gas produced is collected and on treating with KOH solution, the volume of the gas decreases by 1/6th. Calculate the molar ratio of the two acids in the original mixture.
   (a) 4:1
   (b) 3:4
   (c) 4:4
   (d) 2:1

9. 3.6g mixture of sodium chloride and potassium chloride is dissolved in water. The solution is treated with excess of silver nitrate solution. 7.74g of silver chloride is obtained. Find the percentage of sodium chloride and potassium chloride in the mixture.
   (a) 52.2, 48.2
   (b) 42.7, 57.3
   (c) 58.2, 65.4
   (d) 38.5, 55.1

10. A mixture in which the mole ration of H_2 and O_2 is 2:1, is used to prepare water by the reaction :

$$2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$$

The total pressure in the container is 0.8 atm at 20°C before the reaction. Determine the final pressure at 120°C after the reaction, assuming 80% yield of water.

(a) 0.742
(b) 2.423
(c) 1.256
(d) 0.787

Answer & Solutions

1. (c)

No. of moles of NaCl $= \frac{5.85}{58.5} = 0.1$

1 NaCl $\rightarrow$ $(Na^+ + Cl^-) = 2$ ion

$\therefore$ 1 mole NaCl $\rightarrow$ $2\times6.022\times10^{23}$ ions

$= \frac{1.2\times10^{23}}{1000}$ ion

$= 1.2\times10^{20}$ ions

2. (b)

${}^{14}_{6}C$ $\rightarrow$ no of neutrons = 8

no of Protons = 6

new atomic mass $= \frac{1}{2}\times8+2\times6$

$= 4 + 12 = 16$

original atomic mass = 14

% increase $= \frac{16-14}{14}\times100 = \frac{2}{14}\times100$

$= \frac{100}{7} = 14.28\%$

3. (c)

$\delta = 1.2$ g/ml

volume of one drop $= \left(\frac{2}{35}\right)$

density $= \frac{\text{mass}}{\text{volume}}$

$1.2 = \frac{W}{\left(\frac{2}{35}\right)}$

$1.2 \times \left(\frac{2}{35}\right) = W$

$\Rightarrow$ $1.2 \times \left(\frac{2}{35}\right) = nM \rightarrow 70$

$n = \frac{1.2}{(35)^2}$

no of molecules $= \frac{1.2}{(35)^2}\times6.022\times10^{23}$

no of molecules $= \frac{1.2}{(35)^2}\times NA$

4. (d)

molecular mass of $Na_2So_4.nH_2O$ = (142 + 18n)

(142 + 18n) $Na_2So_4.nH_2O \rightarrow 18n\ H_2O$

26.8 gm $Na_2So_4.nH_2O \rightarrow \frac{18n}{(142+18n)} \times 26.8$

Now, $\frac{18n}{(142+18n)} \times 26.8 = 12.6$

$\Rightarrow \frac{12.6}{26.8} = \frac{18n}{(142+18n)}$

$\Rightarrow n = \frac{142}{18} \approx 7.88 \approx 7$

5. (b)

1 mole $Na_3PO_4 \rightarrow$ 3 mole Na^+

$\left(\frac{1}{125}\right)$ mole $Na_3PO_4 \rightarrow \left(\frac{3}{125}\right)$ mole of Na^+

= 0.024

6. (c)

$CO + \frac{1}{2}O_2 \rightarrow CO$

x $(x/2)$

$N_2 + O_2 \rightarrow 2NO$

y y

$x + y$; $\frac{x}{2}+y = 7$

$x + 2y = 14$

$x + y = 10$

$x + 2y = 14$

$y = 4 \Rightarrow y = 4$

Volume of N_2 in the mixture = 4

7. (c)

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{1 \times V_1}{2+3} = \frac{1 \times 40}{400}$

$V_1 = \frac{2+3}{10} = 27.3$ litre

Mixture $\rightarrow C_2H_6$, C_2H_4

Let

the volume of C_2H_6 = x litre

the volume of C2H4 = (27.3 – x) litre

$C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$ + Energy

$$1 \text{ vol} \quad \frac{7}{2}\text{vol}$$

$$x \text{ vol} \quad \frac{7}{2}x \text{ vol}$$

$$C_2H_4 + 3O_2 \rightarrow 2CO_2 + 2H_2O + \text{Energy}$$

$$1 \text{ vol} \quad 3 \text{ vol}$$

$$(27.3 - x) \quad 3(27.3 - x)$$

Total volume of oxygen required

$$\frac{7}{2}x + 3(27.3 - x)$$

Now,

$$\text{mass of oxygen} = 130$$

$$\therefore \quad \left\{\frac{7}{2}x + 3(27.3 - x)\right\} \times \frac{32}{22.4} = 130$$

$$x = 18.2$$

$$\text{mole fraction } (C_2H_6) = \frac{18.2}{27.3} \times 100$$

$$\approx 66.66\%$$

$$\text{mole fraction } (C_2H_4) = 100 - 66.66\%$$

$$= 33.34\%$$

8. (a)

$$\underset{(a)}{HCOOH} \xrightarrow{\text{Conc.}H_2SO_4} \underset{(a)}{CO} + H_2O$$

$$\underset{(b)}{\begin{array}{c} COOH \\ | \\ COOH \end{array}} \xrightarrow{\text{Conc.}H_2SO_4} \underset{(b)}{CO} + \underset{(b)}{Co_2} + H_2O$$

Total no. of m oles of gas = a + b + b

= a + 2b

on treating KOH solution, Co_2 gas is absorbed

$$b = \frac{a + 2b}{6}$$

$$\Rightarrow \quad 6b = a + 2b$$

$$\Rightarrow \quad \frac{a}{b} = \frac{4}{1}$$

9. (b)

let the mass of NaCl = x gm

mass of KCl = $(3.6 - x)$gm

$$\left.\begin{array}{l} NaCl \\ KCl \end{array}\right] AgNo_3 \longrightarrow \begin{array}{c} AgCl \\ \downarrow \\ 7.74 \text{ gm} \end{array}$$

$$NaCl + AgNo_3 \longrightarrow AgCl + NaNo_3$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

58.5 gm $\qquad$ 143.5 gm

58.5 gm NaCl $\longrightarrow$ 143.5 gm AgCl

x gm NaCl $\longrightarrow$ $\frac{143.5}{58.5} \times x$ gm AgCl

$$KCl + AgNo_3 \longrightarrow AgCl + KNo_3$$

74.5 gm $\qquad$ 143.5 gm

74.5 gm KCl $\longrightarrow$ 143.5 gm AgCl

$(3.6 - x)$ gm KCl $\longrightarrow$ $\frac{143.5}{74.5} \times (31.6 - x)$ gm AgCl

$$\text{Total AgCl} = \frac{143.5}{58.5} \times x + \frac{143.5}{74.5}(3.6 - x)$$

$$7.74 = \frac{143.5}{58.5} \times x + \frac{143.5}{74.5}(3.6 - x)$$

$x = 1.54$

$$\% \text{ NaCl} = \frac{1.54}{3.6} \times 100 = 42.7\%$$

% KCl = 100 – 42.7
= 57.3%

10. (d)

$$2H_2 + O_2 \longrightarrow 2H_2O$$

2n $\quad$ n $\qquad$ O

$(2n - 2x)$ $\quad$ $(n - x)$ $\qquad$ $2x$

Now, $\quad \frac{2x}{2n} \times 100 = 80$

$$\frac{x}{n} = \frac{80}{100} = 0.8$$

$x = 0.8$ n

moles of H_2 = 2n – 2x
= 2n – 2 × 0.8 n
= 2n – 1.6 n
= 0.4 n

moles of O_2 = n – x
= n – 0.8 n
= 0.2 n

moles of H_2O = 2×0.8 n
= 1.6 n

Total no of moles at final = 0.4n + 0.2n + 1.6n

Total no of moles before the react = 2n + n + 0 = 3n

Now,

PV = nRT

(0.8) × V = 3n × R × 293 $\qquad$ ...(i)

$P \times V = 2.2n \times R \times 393$ ...(ii)

eq (i) divided eq (ii)

$$\frac{0.8 \times V}{P \times V} = \frac{3n \times R \times 293}{2.2n \times R \times 393}$$

$$\Rightarrow \quad \frac{0.8}{P} = \frac{3 \times 293}{2.2 \times 393} = P = \frac{2.2 \times 393 \times 0.8}{3 \times 293}$$

$P = 0.787$

MATHEMATICS

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1. The distance of the point (1,0,2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ & the plane $x - y + z = 16$ is
   (a) 8
   (b) $3\sqrt{21}$
   (c) 13
   (d) $2\sqrt{14}$

2. The distance of the point (1,–5,9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is
   (a) $3\sqrt{10}$
   (b) $10\sqrt{3}$
   (c) $\frac{10}{\sqrt{3}}$
   (d) $\frac{20}{3}$

3. If the line $\frac{x-3}{2}=\frac{y+2}{-1}=\frac{z+4}{3}$ lies in the plane $lx + my - z = 9$ them $l^2 + m^2$ is equal to
   (a) 26
   (b) 18
   (c) 5
   (d) 2

4. If the image of the point $(1, -2, 3)$ in the plane $2x + 3y - 4z + 22 = 0$ measured parallel to the line $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q them PQ is equal to

(a) $\sqrt{42}$
(b) $6\sqrt{5}$
(c) $3\sqrt{5}$
(d) $2\sqrt{42}$

5.

(i) The length of the projection of the line segment joining the points $(5, -1, 4)$ and $(4, -1, 3)$ on the plane $x + y + z = 7$ is

(a) $\frac{2}{\sqrt{3}}$
(c) $\frac{1}{3}$
(b) $\frac{2}{3}$
(d) $\frac{\sqrt{2}}{\sqrt{3}}$

(ii) Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(a) $x + 2y - 2z = 0$
(b) $3x + 2y - 2z = 0$
(c) $x - 2y + z = 0$
(d) $5x = 2y - 4z = 0$

6.

(i) Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{c}$ is a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ & $\vec{a}.\vec{c} = 4$ them $|\vec{c}|^2 = ?$

(ii) Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\,\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\,\hat{k}$ & $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\,\hat{k}$ be three vectors such that the projection of $\vec{b}$ on $\vec{a}$ is $\vec{a}$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$ then $|\vec{b}| = ?$

7.

(i) Let A be a point on the line $\vec{r} = (1 - 3\mu)\hat{i} + (\mu - 1)\hat{j} + (2 + 5\mu)\hat{k}$ & B(3, 2, 6) be a point in space then value of μ for which $\overrightarrow{AB}$ is parallel to the plane $x - 4y + 3z = 1$ is

(ii) Let $\vec{\alpha} = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ & $\vec{b}$ are non – collinear. The value of λ for which $\vec{\alpha}$ & $\vec{b}$ are collinear is

8.

(i) Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$ & $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ & $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors then, the non zero vector $\vec{a} \times \vec{c}$ is

(ii) Let $\sqrt{3}\,\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\,\hat{j}$ & $\beta\hat{i} + (1-\beta)\hat{j}$ respectively be the position vectors of points A, B, C with respect to origin O. If the distance of C from the bisector of the acute angle between OA & OB is $\frac{3}{\sqrt{2}}$ then sum of all possible values of β is

(iii) Let $\vec{a}, \vec{b}, \vec{c}$ be 3 unit vectors out of which vectors $\vec{b}$ & $\vec{c}$ are non parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $|\alpha - \beta|$ is equal to :

9. Let $[x]$ denote the greatest integer less than or equal to x. Then,

$$\lim_{x \to 0} \frac{\tan(\pi \sin^2 x) + (|x| - \sin(x[x]))^2}{x^2} :$$

(a) Does not exist
(b) equals π
(c) equals 1 + π
(d) equals 0

10.

(i) $\lim_{y \to 0} \dfrac{\sqrt{1 + \sqrt{1 + y^4}} - \sqrt{2}}{y^4}$ is :

(a) exists and equals $\frac{1}{4\sqrt{2}}$

(b) exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

(c) exists and equals $\frac{1}{2\sqrt{2}}$

(d) Does not exist

Answer & Solutions

1. (c)

Eqn. of line : $\dfrac{x-2}{3} = \dfrac{y+1}{4} = \dfrac{z-2}{12} = \lambda$

$\therefore$ Any point on this line is

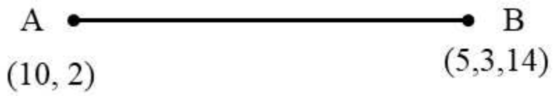
$(x,y,z) = (3\lambda + 2,\ 4\lambda - 1,\ 12\lambda + 2)$

Let (x,y,z) lies on the plane x – y + z = 16

$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (12\lambda + 2) = 16$

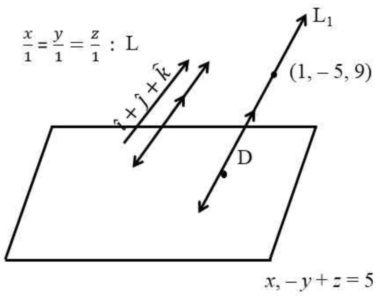
$\Rightarrow \lambda = 1$

$\therefore$ Point of intersection of line & plane is (5, 3, 14)



$AB = \sqrt{16+9+144} = 13$ units

2. (b)



Eq^n. of L_1 : $\dfrac{x-1}{1} = \dfrac{y+5}{1} = \dfrac{z-9}{1} = \lambda$

$\therefore (x,y,z) \equiv (\lambda + 1,\ \lambda - 5,\ \lambda + 9)$.

Let $(\lambda + 1,\ \lambda - 5,\ \lambda + 9)$ lie on the plane

$\therefore (\lambda + 1) - (\lambda - 5) + (\lambda + 9) = 5$

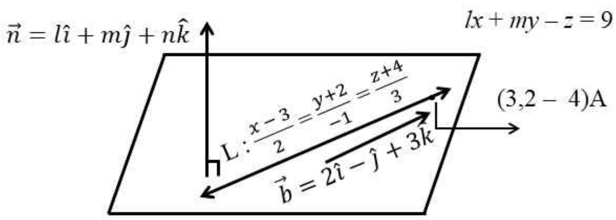
$\lambda = -10$

$\therefore D \equiv (-9, -15, -1)$

And, $AD = \sqrt{100+100+100}$

$= 10\sqrt{3}$ unit

3. (d)



D.R of plane are l, m – 1

Now, A(3, – 2,4) lies on the plane

$\therefore l(3) + m(-2) - (-4) = 9$

$3l - 2cm = 5$ (1)

$\& \ \vec{n} \perp \vec{b} \Rightarrow \vec{n}.\vec{b} = O$

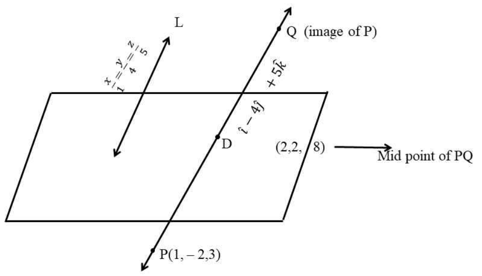
i.e. $2l - m = 3$ (2)

Solving (1) & (2) we get

$l = 1, m = -1$

$\therefore l^2 + m^2 = 2$

4. (d)



Eq[n]. of line passing through P harting

D. R 1,4,5 is : $\dfrac{x-1}{1} = \dfrac{y+2}{4} = \dfrac{z-3}{5}$

$\therefore (x,y,z) \equiv (\lambda + 1, 4\lambda - 2, 5\lambda + 3)$ lies on the plane

$\therefore 2(\lambda + 1) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\Rightarrow \lambda = 1$

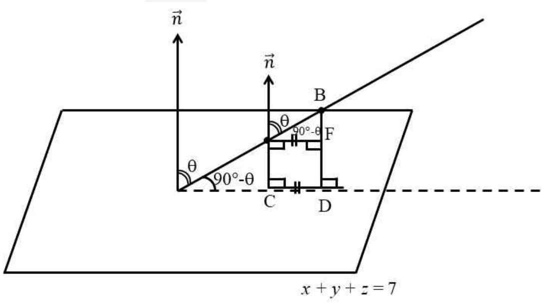
$\therefore D \equiv (2,2,8)$

$\therefore PQ = PD + DQ = 2PD$

$2\sqrt{1^2 + 4^2 + 5^2} = 2\sqrt{42}$

5. (i) → (d) (ii) → (c)

(i)



$A \equiv (5, -5, 4)$ $B \equiv (4, -1,3)$

$\overrightarrow{AB} = (4\hat{i} - \hat{j} + 3\hat{k}) - (5\hat{i} - \hat{j} + 4\hat{k}) = -\hat{i} + 0\hat{j} - \hat{k}.$

$\vec{n}.\overrightarrow{AB} = |\vec{n}||\overrightarrow{AB}|\cos\theta$

$(\hat{i} + \hat{j} + \hat{k}).(-\hat{j} + 0\hat{j} - \hat{k}) = \sqrt{3}\sqrt{2}\cos\theta$

$\Rightarrow \cos\theta = \dfrac{-\sqrt{2}}{\sqrt{3}}$

$\Rightarrow \sin\theta = 1/\sqrt{3}$

Projection = AF

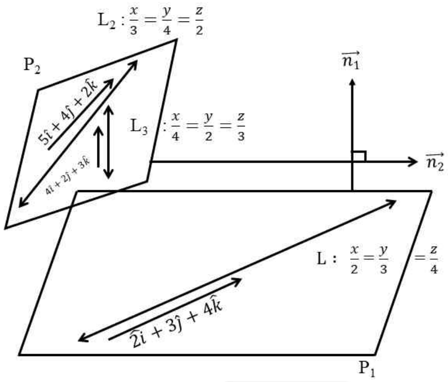
$\cos(90^\circ - \theta) = \dfrac{AF}{AB}$

$\Rightarrow$ AF= AB cos $(90^\circ - \theta)$

$= \left|\overrightarrow{AB}\right| \sin\theta$

$= \sqrt{2} \times \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{2}}{\sqrt{3}}$

(ii)



Now, $P_1 \perp P_2$

$\Rightarrow \overrightarrow{n_1} \perp \overrightarrow{n_2}$

& $\overrightarrow{n_1} \perp (2\hat{i} + 3\hat{j} + 4\hat{k})$ & $\overrightarrow{n_2} \perp (4\hat{i} + 2\hat{j} + 3\hat{k})$

& $\overrightarrow{n_2} \perp (3\hat{i} + 4\hat{j} + 2\hat{k})$ $\overrightarrow{n_2} \perp (4\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{n_2} = (3\hat{i} + 4\hat{j} + 2\hat{k}) \times (4\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{n_2} = 8\hat{i} - \hat{j} - 10\hat{k}$

& $\overrightarrow{n_1} = \overrightarrow{n_2} = 0 \Rightarrow 8a - b10c = 0$

& $\overrightarrow{n_1} . (2\hat{i} + 3\hat{j} + 4\hat{k}) = 0 \Rightarrow 2a + 3b + 4c = 0$

By (1) & (2),

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{1}$$

$\therefore$ Eq^n of plane is $x - 2y + z = 0$

6. (i) $\rightarrow$ 19/2 (ii) $\rightarrow$ 6

(i) $\dfrac{\vec{a}}{\vec{\ell}} = \dfrac{\hat{i} - \hat{j}}{\hat{i} + \hat{j} + \hat{k}}$

$(\vec{a} \times \vec{c}) + \vec{b} = \vec{O}$

$\Rightarrow \vec{a} \times \vec{c} = \overrightarrow{-b}$

$(\vec{a} \times \vec{c}) \times \vec{a} = (-\vec{b}) \times \vec{a}$

$(\vec{a}.\ \vec{b})\ \vec{c} - (\vec{c}.\ \vec{a})\ \vec{a} = -(\vec{b} \times \vec{a})$

$$\left|\vec{a}\right|^2 \vec{c} - 4\,\vec{a} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= -\hat{i} - \hat{j} + 2\hat{k}$$

$2\vec{c} - 4\vec{a} = (-\hat{i} - \hat{j} + 2\hat{k})$

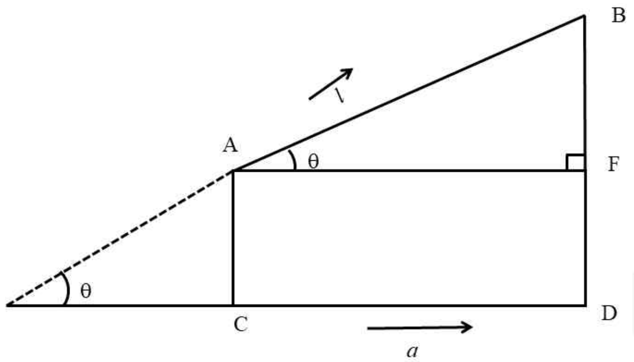
$2\vec{c} = (-\hat{i} - \hat{j} + 2\hat{k}) + (4\hat{i} - 4\hat{j} + 0\hat{k})$

$2\vec{c} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$$\vec{c} = \frac{3}{2}\hat{i} - \frac{5}{2}\hat{j} + \frac{2}{2}\hat{k}$$

$$|\vec{c}|^2 = (3/2)^2 + \left(\frac{-5}{2}\right)^2 + (1)^2$$
$$= 19/2$$

(ii)



$$\cos\theta = \frac{Af}{|\vec{\ell}|} \Rightarrow Af = |\vec{\ell}|\cos\theta = \frac{|\vec{a}|\vec{a}.\vec{a}}{|\vec{a}||\vec{b}|} = \frac{\vec{a}.\vec{b}}{|\vec{a}|}$$

$$\therefore \frac{\vec{a}.\vec{b}}{|\vec{a}|} = |\vec{a}|$$

So, $\vec{a}.\vec{b} = |\vec{a}|^2$

$\therefore$ $b_1 + b_2 + 2 = 4$

$b_1 + b_2 = 2$ (1) $(\because |\vec{a}| = 2)$

$\vec{a} + \vec{b} = (1 + b_1)\hat{i} + (1 + b_2)\hat{j} + (2\sqrt{2})\hat{k}\ (\vec{a} + \vec{b}).\vec{c} = 0$

$\Rightarrow 12 + 4b_1 = 0$

$\Rightarrow b_1 = -3$

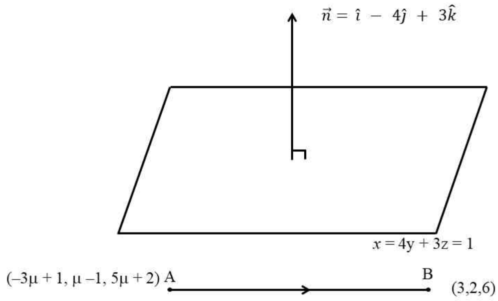
$b_2 = 5$

$\therefore$ $\vec{a} = -3\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$

$|\vec{a}| = \sqrt{9 + 25 + 2} = \sqrt{36} = 6$

7. (i) → $\mu = 1/4$. (ii) → $\lambda = -4$.

(i)



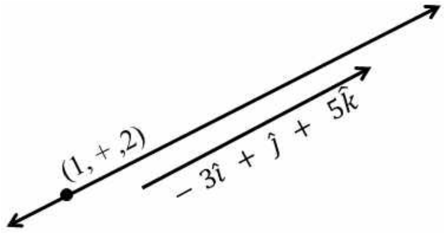
$\vec{n} \perp \overrightarrow{AB}$

$\Rightarrow$ $\vec{n}.\ \overrightarrow{AB} = O$ (1)

$\Rightarrow$ $(\hat{i} - 4\hat{j} + 3\hat{k}).\ \overrightarrow{AB} = O$

Let $\vec{r} = (1-4\mu)\hat{i} + (\mu-2)\hat{j} + (2+\mu)\hat{k}$

$\underbrace{(\hat{i} - \hat{j} + 2\hat{k})}_{\vec{a}} + \mu\underbrace{(-3\hat{i} + \hat{j}\ 5\hat{k})}_{\vec{b}}$



L : $\dfrac{x-1}{-3} = \dfrac{y+1}{1} = \dfrac{z-2}{5} = \mu$

$(x,y,z) = (-3\mu + 1,\ \mu - 1,\ 5\mu + 2)$

$\therefore\ \overrightarrow{AB} = (3\mu + 2)\ \hat{i} + (3 - \mu)\hat{j} + (4 - 5\mu)\ \hat{k}$

By(1)

$\mu = 1/4.$

(ii) $\exists$ constant x ;

$\alpha = x\beta$

$(\lambda - 2)\vec{a} + \vec{b} = x\ ((4\lambda - 2)x)\ \vec{a} + 3x\ \vec{b}$

$= ((4\lambda - 2)x)\ \vec{a} + 3x\ \vec{b}$

$\Rightarrow \underbrace{((\lambda-2) - (4\lambda - 2)x)}_{k_1}\vec{a} + \underbrace{(1-3x)}_{k_2}\vec{b} = 0$

$\Rightarrow 1 - 3x = 0 \Rightarrow 1 = x$ or $x = 1/3$

& $\lambda - 2 - (4\lambda - 2)x = 0$

$\Rightarrow \lambda = -4.$

8. (i) $\rightarrow$ $10\hat{i} + 5\hat{j}$ (ii) $\rightarrow$ $2\beta = -2$ i.e. $\beta = 1$. (iii) $\rightarrow$ $\therefore$ $|\alpha - \theta| = |\pi|2 - \pi|3| = 30^\circ$

(i) $[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 0$

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & \lambda^{2-}1 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 2\lambda^{2-}\ 9\lambda + 18 = 0$ (i)

$\lambda - 2$ is a factor of (i)

$\therefore\ \lambda^3 - 2\lambda^2 - 9\lambda + 18\ (\lambda - 2)\ (\lambda^2 - 9) = 0$

i.e. $(\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$

Values of λ are 2,3, – 3

$\vec{c} = 2\vec{i} + 4\hat{j} + 3\hat{k}$ or

$\vec{c} = 2\vec{i} + 4\hat{j} + 8\hat{k}$

$= 2\vec{a}$

$\therefore\ \vec{a}$ & $\vec{c}$ are colinear vectors

$\Rightarrow \vec{a} \times \vec{c} = \vec{0}$

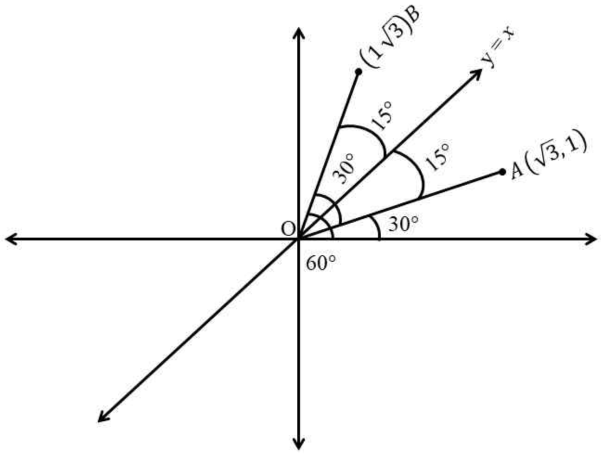
$\therefore\ \lambda \neq 3, -3.$

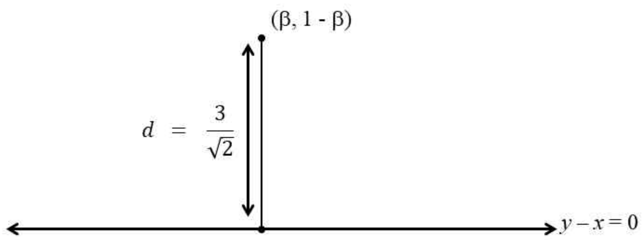
$\lambda = 2$

& $\vec{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore\ \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

(ii)





$$\frac{3}{\sqrt{2}} = d = \frac{|1-\beta-\beta|}{\sqrt{1^2+1^2}}$$

$\Rightarrow |1-2\beta| = 3$

$\Rightarrow 1 - 2\beta = \pm 3$

$\therefore 1 \pm 3 = 2\beta$

$\therefore 2\beta - 4 \Rightarrow \beta = 2$

or

$2\beta = -2$ i.e. $\beta = 1$.

(iii) $\vec{a} \times (\vec{b} \times \vec{c}) = 1/2\,\vec{b}$

$(\vec{a}.\ \vec{c})\vec{b} - (\vec{a}\ .\ \vec{b})\vec{c} = 1/2\,\vec{b}$

$\left((\vec{a}\ .\ \vec{c}) - \frac{1}{2}\right)\vec{b} - (\vec{a}\ .\ \vec{c})\ \vec{c} = 1/2\,\vec{b}$

As $\vec{a}$ & $\vec{c}$ are non collinear

$\Rightarrow \vec{a}.\vec{c}\ -1/2 = 0$ & $\vec{a}\ .\ \vec{b} = 0$

$\vec{a}\ .\ \vec{c} = = 1/2$ & $\vec{a} \perp \vec{b} = \alpha = 90^\circ = \pi/2$

$|\vec{a}||\vec{c}|\cos\theta = 1/2$ & $\alpha = \pi/2$

$\Rightarrow \theta = 60\circ = \pi/3$ & $\alpha = \pi/2$

$\therefore |\alpha - \theta| = |\pi|2 - \pi|3| = 30^\circ$

9. (a)

RHL : $\lim\limits_{h\to 0} \dfrac{\tan(\pi \sin^2(h)) + (h+0)^2}{h^2}$

i.e $\lim\limits_{h\to 0}\left(\dfrac{\tan(\pi/\sin^2(h)) \times \sin^2 h}{\pi \sin^2 h \times h^2}\right) + \lim\limits_{h\to 0}(1)$

i.e. $\left(\lim_{h\to 0}\frac{\tan\left(\pi\sin^2 h\right)}{\pi\sin^2 h}\right)\pi\lim\left(\frac{\sin h}{h}\right)^2+1$

ie $\pi + 1$

LHL : $\lim_{h\to 0}\frac{\tan\left(\sin^2(-h)\right)+\left(|-h|-\sin(-h[-h])\right)^2}{(-h)^2}$

(As $h \to 0$ & $h > 0$ & $-h < 0$ $\therefore$ $[-h] = -1$)

$= \lim_{h\to 0}\left(\frac{\tan\left(\pi\sin^2 h\right)}{h}+\left(\frac{h-\sin h}{h}\right)^2\right)$

$(1 \times \pi) + (1 + 1 - 2) = \pi$

$\therefore$ LHL $\neq$ RHL

10. (a)

(i) : $x \to 0$

$(1 + x)^n = 1\ nx$

$\sqrt{1+y^4} = (1+y^4)^{1/2}$

$= 1 + \frac{1}{2}y^4$

$\therefore \lim_{y\to 0}\frac{\sqrt{1+1+\frac{y^4}{2}}-\sqrt{2}}{y^4}$

$= \lim_{y\to 0}\frac{\sqrt{2+\frac{y^4}{2}}-\sqrt{2}}{y^4}$

$= \lim_{y\to 0}\frac{\sqrt{2}\left(\left(1+\frac{y^4}{4}\right)^{1/2}-1\right)}{y^4}$

$= \lim_{y\to 0}\frac{\sqrt{2}\left(1+\frac{1}{2}y\frac{4}{4}-1\right)}{y^4} = \frac{1}{4\sqrt{2}}$

CRASH COURSE

JEE ADVANCED 2021-22

PHYSICS

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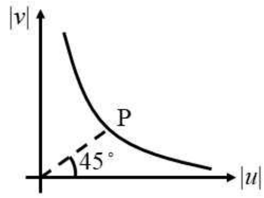
SET-1

(Q1 & 2) Only One Option Correct

1. The sum, difference and cross product of two vectors $\vec{A}$ and $\vec{B}$ are mutually perpendicular if
   (a) $\vec{A}$ and $\vec{B}$ are perpendicular to each other and $|\vec{A}| = |\vec{B}|$
   (b) $\vec{A}$ and $\vec{B}$ are perpendicular to each other
   (c) $\vec{A}$ and $\vec{B}$ are perpendicular but their magnitudes are arbitrary
   (d) $|\vec{A}| = |\vec{B}|$ and their directions are arbitrary

2. The $|u|$, $|v|$ graph for a concave mirror is as shown in figure. Here $|u| > |f|$. A line passing through origin of slope 1 cuts the graph at point P. Then co-ordinates of point P are



(a) (|2 f |, |2 f |)
(b) (|2 f |, | f |)
(c) (| f |, |2 f |)
(d) (| f |, | f |)

(Q3) More Than One Option Correct

3. Units off CR^2 is/are (C = capacitance and R = resistance).

(a) henry
(b) $\frac{\text{volt} - \text{second}}{\text{ampere}}$
(c) $\frac{\text{volt}}{\text{ampere}}$
(d) $\frac{\text{joule}}{\text{ampere}^2}$

(Q4) Matrix Match

4. For component of a vector $\vec{A} = (3\hat{i} + 4\hat{j} - 5\hat{k})$, match the following table

Table-1	Table-2
(a) Along y-axis	(p) 5 unit
(b) Along another vector $(2\hat{i} + \hat{j} + 2\hat{k})$	(q) 4 unit
(c) Along $(6\hat{i} + 8\hat{j} + 10\hat{k})$	(r) Zero
(d) Along another vector $(-3\hat{i} + 4\hat{j} + 5\hat{k})$	(s) None

(Q5 & 6) Only One Option Correct

5. A particle moves in space along the path $z = ax^3 + by^2$ in such a way that $\frac{dx}{dt} = c = \frac{dy}{dt}$ where a, b and c are constants. The acceleration of the particle is

(a) $(6ac^2x + 2bc^2)\hat{k}$
(b) $(2ax^2 + 6by^2)\hat{k}$
(c) $(4bc^2x + 3ac^2)\hat{k}$
(d) $(bc^2x + 2by)\hat{k}$

6. A particle is dropped from point A at a certain height from ground. It falls freely and passes through three points B, C and D with $BC = CD$. The time taken by the particle to move form B to C is 2 s and from C to D is 1 s. The time taken to move from A to B is

(a) 0.5 s
(b) 1.5 s
(c) 0.75 s
(d) 0.25 s

SET-2

(Q1 & 2) Only One Option Correct

1. The distance between two moving particles at any time is a. If v be their relative velocity and v_1 and v_2 be the components of v along and perpendicular to a. The time when they are closest to each other are

(a) $\frac{av_1}{v^2}$ (c) $\frac{av}{v_1^2}$

(b) $\frac{av_2}{v^2}$ (d) $\frac{av}{v_2^2}$

2. In the one-dimensional motion of a particle, the relation between position x and time t is given by $x^2 + 2x = t$ (here $x > 0$). Choose the correct statement

(a) the retardation of the particle $\frac{1}{4(x+1)^3}$

(b) the uniform velocity of the particle is $\frac{1}{(x+1)^3}$

(c) Both are correct

(d) Both are wrong

(Q3) More Than One Option Correct

3. Let $\vec{v}$ and $\vec{a}$ be the instantaneous velocity and acceleration of a particle moving in a plane. The, rate of change of speed $\frac{dv}{dt}$ of the particle is equal to

(a) $\left|\vec{a}\right|$

(b) $\frac{\vec{v}.\vec{a}}{v}$

(c) the component of $\vec{a}$ parallel to $\vec{v}$

(d) the component of $\vec{a}$ perpendicular to $\vec{v}$

(Q4 & 5) Comprehension Type

Passage

At time $t = 0$, particle A is at $(1m, 2m)$ and B is at $(5m, 5m)$. Velocity of B is $\left(2\hat{i} + 4\hat{j}\right)$ m/s velocity of particle A is $\sqrt{2}v$ at 45° with x-axis. A collides with B.

4. Value of v is...............m/s.

(a) 5 (c) 25

(b) 15 (d) 10

5. Time when A will collide with B is............... second.

(a) 0.5 s
(b) 1.5 s
(c) 4 s
(d) 3 s

Answer & Solutions

SET-1

(Q1 & 2) Only One Option Correct

1. (d)

Let $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$(\vec{A} + \vec{B}) \perp (\vec{A} - \vec{B})$ given

$(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$

$|\vec{A}| = |\vec{B}|$

$\vec{A} \times \vec{B} \perp$ to plane formed by $\vec{A}$ and $\vec{B}$ or $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$`

2. (a)

When object at centre of curvature, image coincides with object.

(Q3) More Than One Option Correct

3. (a, b, d)

Time constant in C-R and L-R circuits are CR and $\frac{L}{R}$

$CR = \frac{L}{R}$ or $CR^2 \equiv L$ units of CR^2 and L are same

$|E| = L\left(\frac{dI}{dt}\right)$ and $U = \frac{1}{2}Li^2$

$\Rightarrow$ Units of L or CR^2, $\frac{V - second}{A}$ and $\frac{J}{A^2}$

(Q4) Matrix Match

4. (a) $\rightarrow$ (q)
(b) $\rightarrow$ (r)
(c) $\rightarrow$ (s)
(d) $\rightarrow$ (s)

(Q5 & 6) Only One Option Correct

5. (a)

$$\frac{d\alpha}{dt} = \frac{dy}{dt} = c$$

$$\frac{d^2\alpha}{dt^2} = \frac{d^2y}{dt^2} = 0$$

$Z = ax^3 + by^2$

$= 3acx^2 + 2bcy$

$$\frac{d^2Z}{dt^2} = 6ac \times \left(\frac{dx}{dt}\right) + 2bc\left(\frac{dy}{dt}\right)$$

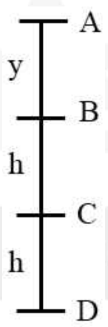
$= 6ac^2 + 2bc^2$

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$(6ac^2x + 2bc^2)\hat{k}$

6. (a)

$t_{AB} = t$



$y = \frac{1}{2}gt^2$

$y + h = \frac{1}{2}g(t + 2)^2$

$y + 2h = \frac{1}{2}g(t + 3)^2$ h

$\Rightarrow t = .5$ s

SET-2

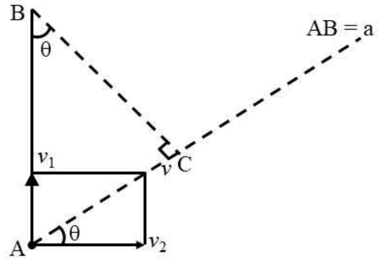
(Q1 & 2) Only One Option Correct

1. (a)

$$V^2 = V_1^2 + V_2^2 \Rightarrow \tan\theta = \frac{V_1}{V_2}$$

$$\cos\theta = \frac{V_2}{\sqrt{v_1^2 + v_2^2}}$$

$= \frac{V_2}{v}$



$\sin\theta = \frac{V_1}{\sqrt{V_1^2 + V_2^2}} = \frac{V_1}{V}$

Time $\frac{AC}{V} = \frac{AB\sin\theta}{V} = \frac{a v_1}{v^2}$

2. (a)

$\frac{dt}{dx} = 2(x+1) \Rightarrow v = \frac{dx}{dt}$

$\Rightarrow v = \frac{1}{2(x+1)}$ and $a = \frac{dv}{dt}$

$\Rightarrow -\frac{1}{2(x+1)^2} \cdot \frac{dx}{dt} = -\frac{1}{4(x+1)^3}$

(Q3) More Than One Option Correct

3. (b, c)

Speed $v^2 = v_x^2 + v_y^2$

$\Rightarrow 2V\frac{dv}{dt} = 2v_x\frac{dv_x}{dt} + 2v_y\frac{dv_y}{dt}$

$\Rightarrow \frac{dv}{dt} = \frac{v_x ax + v_y ay}{v} = \frac{\vec{v}.\vec{a}}{v}$

Compound of $\vec{a} \parallel$ to v

(Q4 & 5) Comprehension Type

4. (d)

$\vec{V}_A = V\hat{i} + V\hat{j}$

$\vec{V}_B = 2\hat{i} + 4\hat{j}$

$\vec{V}_{AB} = (V-2)\hat{i} + (V-4)\hat{j}$

$\overrightarrow{AB} = (4\hat{i} + 3\hat{j})$

$\vec{V}_{AB} \uparrow\uparrow \overrightarrow{AB} i + \frac{V-2}{4} = \frac{V-4}{3}$

$\Rightarrow$ V = 10

5.(d)

$\left|\overrightarrow{V}_{AB}\right| = 10$

$\left|\overrightarrow{AB}\right| = 5 \Rightarrow t = \dfrac{\left|\overrightarrow{AB}\right|}{\left|\overrightarrow{V}_{AB}\right|} = .5\,\text{sec}$

CHEMISTRY

JEE Advanced 2021-22 CRASH COURSE

JEE Advanced crash courses provides a preparation strategy & direction, a speedy revision and getting a high score for JEE Advanced. It is a focused course for the JEE Advanced aspirant's full preparation through a final mock test with important exam pattern, solving past questions and emphasize on the formulas to crack the JEE Advanced Exam.

▲Important problem-solving and revision of all important topics with the last 7 years JEE Advanced analysis. ▲Providing problem-solving tips and tricks for the exam. ▲100% JEE Advanced pattern questions with detailed solutions. ▲Those questions are the focus on chapters with a high weight. ▲ Misconceptions and repeated errors are cleared by the faculties. ▲The questions of compete syllabus designed by the experienced Misostudy faculty team. ▲Boosts confidence in students so that they can score well.

[One Option Correct]

1. 1.020g of metallic oxide contains 0.540g of the metal. Calculate the equivalent mass of the metal and hence its atomic mass with the help of Dulong and Petit's law. Taking the symbol for the metal as M. find the molecular formula of the oxide. The specific heat of the metal is 0.216 cal deg^{-1} g^{-1}.
   (a) M_2O_3
   (b) M_4O_3
   (c) M_2O_4
   (d) M_3O_5

2. A partially dried clay mineral contains 8% water. The original sample contained 12% water and 45% silica. The % of silica in the partially dried sample is nearly.
   (a) 50%
   (b) 49%
   (c) 55%
   (d) 47%

3. A mixture in which the mole ratio of H_2 and O_2 is 2:1 is used to prepare water by the reaction,
   $$2H_2(g) + O_2(g) \rightarrow 2H_2O(g)$$
   The total pressure in the container is 0.8 atm at 20°C before the reaction. Determine the final pressure at 120°C after reaction assuming 80% yield of water.
   (a) 0.8054 atm
   (b) 0.7864 atm
   (c) 0.9744 atm
   (d) 0.6964 atm

4. A mixture of HCOOH and $H_2C_2O_4$ is heated with concentrated H_2SO_4. The gas produced is collected and on treating with KOH solution, the volume of the gas decreases by 1/6th. Calculate the molar ratio of the two acids in the original mixture.
   (a) 2:3
   (b) 6:5
   (c) 4:1
   (d) 8:6

[Integer Type Questions]

5. A plant virus is found to consist of uniform cylindrical particles of 150Å in diameter and 5000Å long. The specific volume of the virus is 0.75 cm^3/g. If the virus is considered to be a single particle, find its molecular mass.
6. On dissolving 2.0g of metal in sulphuric acid, 4.51g of the metal sulphate was formed. The specific heat of the metal is 0.057 cal g^{-1}. What is the valency of the metal and exact atomic mass ?

[Matrix Matching]

7. Match the Column-X and Column-Y:

Column-X	**Column-Y**
(a) 1.6g CH_4	(i) 0.1 mol
(b) 1.7g NH_3	(ii) 6.023×10^{23} electrons
(c) HCHO	(iii) 40% carbon
(d) $C_6H_{12}O_6$	(iv) Vapour density = 15

[One Option Correct]

8. The ratio of the frequency corresponding to the third line in Lyman series of hydrogen atomic spectrum to that of the first line in Balmer series of Li^{2+} spectrum is

   (a) $\frac{4}{5}$ (c) $\frac{4}{3}$
   (b) $\frac{5}{4}$ (d) $\frac{3}{4}$

9. A parent nucleus X is decaying into daughter nucleus Y which in turn decays to Z. Half lives of X and Y are 40000 years and 20 years respectively. In certain sample, it is found that the number of Y nuclei hardly changes with time. If the number of X nuclei in the sample is 4×10^{20}, the number Y nuclei present in it is:

   (a) 2×10^{17} (c) 4×10^{23}
   (b) 2×10^{20} (d) 4×10^{20}

10. Three isotopes of an element have mass numbers *M*, (*M* + 1) and (*M* + 2). If the mean mass number is (*M* + 0.5), then which of the following ratios may be accepted for *M*, (*M* + 1), (*M* + 2) in that order?

    (a) 1 : 1 : 1
    (b) 4 : 1 : 1
    (c) 3 : 2 : 1
    (d) 2 : 1 : 1

Answer & Solutions

1. (a)

Mass of oxygen is the oxide = (1.020 − 0.540) = 0.480 gm

Equivalent mass of the metal = $\frac{0.540}{0.480} \times 8 = 9\,\text{gm}$

According to Dulong and Petit's law

Approx. atoms mass = $\frac{6.4}{\text{SP. heat}} = \frac{64}{0.216} = 29.63$

Valency of the metal = $\frac{\text{At. mass}}{\text{Eq. heat}} = \frac{29.63}{9} \approx 3$

Hence,

the formula of the oxide = M_2O_3

2. (d)

	Clay	Silica	Water
Initial stage	43%	45%	12%
Final stage	$(92-x)$	x	8%

Ratio of silica and clay will remain constant, before and after drying.

$$\frac{45}{43} = \frac{x}{92-x}$$

∴ x = 47%

(a)

3. (b)

$$pH_2 = \frac{2}{3} \times 0.8 = 0.533\ atm$$

$$pO_2 = \frac{1}{3} \times 0.8 = 0.266\ atm$$

$$2H_2 + O_2 \rightarrow 2H_2O$$

t = 0 0.533 0.266 0

After the reaction = $\frac{0.533 \times 20}{100} = 0.1066, \frac{0.266 \times 20}{100} = 0.0533, \frac{0.533 \times 80}{100}\ 0.4264.$

Total pressure = 0.1066 + 0.0533 + 0.4264 = 0.5863

Using Gay-Lussac's law

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{0.5863}{293} = \frac{P_2}{393}$$

⇒ P_2 = 0.7864 atm

4. (c)

$$\underset{1\ \text{mol}}{HCOOH} \xrightarrow[\text{Heat}]{\text{conc. } H_2SO_4} \underset{1\ \text{mol}}{CO} + H_2O$$

$$\underset{1\ \text{mol}}{H_2C_2O_4} \xrightarrow[\text{Heat}]{\text{conc. } H_2SO_4} \underset{1}{CO} + \underset{1}{CO_2} + H_2O$$

Let "a" moles of HCOOH and "b" moles of $H_2C_2O_4$ be present in the original mixture

moles of CO formed = $a + b$

moles of CO_2 formed = b

Total moles of gases = $a + 2b$

Now

$$\frac{a+2b}{6} = b$$

$\Rightarrow$ $a = 4b \Rightarrow \frac{a}{b} = 4$

$\Rightarrow$ $a : b = 4 : 1$

5. 114.72

Equivalent mass of SO_4^{2-} radical = $\frac{\text{Ionic mass}}{\text{Valency}}$

$= \frac{96}{2} = 48$

Mass of metal sulphate = 4.51 gm

Mass of metal = 2.0 gm

Mass of sulphate radical = 4.51 − 2 = 2.51 gm

2.51 gm of sulphate combine with 2 gm of metal.

So, 48 gm of sulphate will combine with = $\frac{2}{2.51} \times 48 = 38.24\,\text{gm metal}$

$\therefore$ Equivalent mass of metal = 38.24 gm

According to Dulong and Petit's law

Approximate atomic mass = $\frac{6.4}{\text{Specific heat}} = \frac{6.4}{0.057} = 112.5$

Valency = $\frac{\text{Approximate atomic mass}}{\text{Equivalent mass}} = \frac{112.5}{38.24} \approx 3$

Exact atomic mass = 38.24 × 3 = **114.72**

6. (a) → (i), (ii),
(b) → (i), (ii),
(c) → (iii), (iv),
(d) → (iii)

(a) 1.6g CH_4 = $\frac{1.6}{16} = 0.1\,\text{mole}$

$= 0.1 \times 6.022 \times 10^{23} \times 10$

$= 6.022 \times 10^{23}$ electron.

(b) 1.7g NH_3 = $\frac{1.7}{17} = 0.1$ mole

$= 0.1 \times 6.022 \times 10^{23} \times 10$

$= 6.022 \times 10^{23}$ electron.

(c) % of "c" = $\frac{12}{30} \times 100 = 40\%$

$MM = 2 \times VD$

$VD = \frac{MM}{2} = \frac{30}{2} = 15$

(d) % of "c" = $\frac{6 \times 12}{180} \times 100 = \frac{72}{180} \times 100 = 40\%$

7. (d)

For third line in Lyman series.

$n_1 = 1$; $n_2 = 4$

$$V_H = \frac{C}{\lambda} = C.R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= C.\ (R_H)\ (1)^2 \left[\frac{1}{1^2} - \frac{1}{4^2}\right]$$

$$V_H = \frac{15}{16} R_H C$$

For first line in Balmer series for Li^{2+}

$n_1 = 2$; $n_2 = 3$

$$V_{Li^{2+}} = (R_H)(Z^2)\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= (R_H)(3^2)\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= C\ R_H \times 9 \times \frac{5}{36} = \frac{5}{4} CR_H$$

$$\therefore \quad \frac{V_H}{V_{Li^{2+}}} = \frac{15}{6} \times \frac{4}{5} = \frac{3}{4}$$

8. (a)

$$X \xrightarrow{\lambda_x} Y \xrightarrow{\lambda_y} Z$$

At equilibrium

$\lambda_x N_x = \lambda_y N_y$

$$N_y = \frac{\lambda_x}{\lambda_y} \times N_x$$

$$= \frac{(t_{1/2})_y}{(t_{1/2})_x} \times N_x$$

$$= \frac{20}{40000} \times 4 \times 10^{20}$$

$= 2 \times 10^{17}$

9. (b)

Let the ratio is, M : (M + 1) : (M + 2) = x : y : z

Mean atomic mass = $\frac{M\times x+(M+1)\times y+(M+2)\times z}{(x+y+z)}$

$$M + 0.5 = \frac{xM+y(M+1)+z(M+2)}{(4+1+1)}$$

$$= \frac{4M+1(M+1)+1(M+2)}{6}$$

$$= \frac{6M+3}{6} = \frac{3(2M+1)}{6}$$

$$= M+\frac{1}{2} = M+0.05 = RHS$$

Hence, "b" is the correct option

10. (a) (c)

$$mvr = \frac{nh}{2\pi}$$

$$E_n = E_1 \times \frac{z^2}{n^2}$$

MATHEMATICS

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1. If z, z_2 are non-zero complex numbers such that $|z_1| = |z_2| = |z_1 + z_2|$ then z_1/z_2 can be
   (a) 1
   (b) ω
   (c) ω^2
   (d) − 1

2. Modulus of complex number whose reciprocal is
   Match the entries in Column I with entries in Column II

Column-I	Column-II
(a) $\frac{1}{a}+\frac{1}{b+ic}$	(p) $\frac{\sqrt{a^2+b^2}\,\sqrt{a^2+c^2}}{\lvert b-c\rvert}$
(b) $\frac{1}{a-ib}-\frac{1}{a-ic}$	(q) $\sqrt{a^2+(b+c)^2}$
(c) $\frac{b}{a+ib}+\frac{c}{a-ic}$	(r) $\frac{\lvert a\rvert\,\sqrt{b^2+c^2}}{\sqrt{(a+b)^2+c^2}}$
(d) $\frac{1}{a+ib+ic}$	(s) $\frac{\sqrt{a^2+b^2}\,\sqrt{a^2+c^2}}{\lvert a\rvert\,\lvert b+c\rvert}$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	q	r	s

3. Let α, β be roots of the equation $ax^2 + bx + c = 0$, then equation whose roots are
Match the entries in Column-I with entries in Column-II

Column-I	Column-II
(a) $-1/\alpha, -1/\beta$	(p) $ax^2 + 2bx + 4c = 0$
(b) $-\alpha, -\beta$	(q) $a^2x^2 + (2ac - b^2) + c^2 = 0$
(c) α^2, β^2	(r) $cx^2 - bx + a = 0$
(d) $2\alpha, 2\beta$	(s) $ax^2 - bx + c = 0$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	q	r	s

4. **Statement-I :** If all the four roots of $x^4 - 4x^3 + ax^2 - bx + 1 = 0$ are positive, than $a = 6$ and $b = 4$.
Statement-II : If polynomial equation $P(x) = 0$ has four positive roots, then the polynomial equation $P'(x) = 0$ has atleast 3 positive roots.
 (a) Statement-I is True, Statement-II is True; Statement-II is correct explanation for Statement-I.
 (b) Statement-I is True, Statement-II is true; Statement-II is not a correct explanation for Statement-I.
 (c) Statement-I is True, Statement-II is False.
 (d) Statement-I is False, Statement-II is True.

5. Let $a, b, c \in \mathbf{C}$ such that $a + b + c = 0$.
If $|a| = |b| = |c| = 1$, then $|a - b|^3 + |b - c|^3 + |c - a|^3 - 3\,|a - b|\,|b - c|\,|c - a|$ is equal to

6. $a, b, c \in \mathbf{R}$ and a, b, c are in A.P. Match the expression in Column-I with the conditions/properties in Column-II.

Column-I	Column-II
(a) a^2, b^2, c^2 are in A.P.	(p) $a = b = c$
(b) a^2, b^2, c^2 are in G.P.	(q) $-\frac{1}{2}a, b, c$ are in G.P.
(c) a^2, b^2, c^2 are in H.P.	(r) $a, b, -\frac{1}{2}c$ are in G.P.
(d) $a + b + c = \frac{3}{2}$	(s) $b = \frac{1}{2}$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	q	r	s

7. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Statement-I : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

Statement-II : The number b_1, b_2, b_3, b_4 are in H.P.

(a) Statement-I is false and Statement-II is true.
(b) Statement-I is true and Statement-II is false
(c) Statement-I and Statement-II both are true
(d) Statement-I and Statement-II both are false

8. **Statement-I :** $\frac{1^2}{(1)(3)} + \frac{2^2}{(3)(5)} + \dots + \frac{n^2}{(2n-1)(2n+1)} = \frac{n(n+1)}{2(2n+1)}$

Statement-II : $\frac{1}{(1)(3)} + \frac{2}{(3)(5)} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{1}{2n+1}$

(a) Statement-I is false and Statement-II is true.
(b) Statement-I is true and Statement-II is false
(c) Statement-I and Statement-II both are true
(d) Statement-I and Statement-II both are false

Paragraph Question

9. Given a sequence t_1, t_2, ... if its possible to find a function $f(r)$ such that

$$t_r = f(r+1) - f(r)$$

then

$$\sum_{r=1}^{n} t_r = f(n+1) - f(1)$$

(i) Sum of the series $\sum_{r=1}^{\infty} \frac{1}{4r^2-1}$ is

(a) 2
(b) 1
(c) 1/2
(d) 1/4

(ii) If $u_1 = 1$, $u_{n+1} = 2u_n + 1$, then u_{n+1} equals

(a) $2^n + 1$
(b) $2^{n+1} - 1$
(c) $2^n - 2$
(d) $2^{n+1} - 2$

(iii) If $x_n = 1^2 + (2)(2^2) + 3^2 + (2)(4^2) + \dots$

$= n(n+1)^2/2$ if n is even, then $\frac{x_{51}}{(13)(51^2)}$ is

10. Let m and n be two positive integers such that $m \geq n$. The number of ways of Match the entries in Column I with entries in Column II

	Column-I	Column-II
(a)	distributing m distinct books among n children	(p) 0
(b)	arranging n distinct books at m places	(q) m^n
(c)	selecting m persons out of n persons so that two particular persons are not selected	(r) n^m
(d)	number of functions from $\{1, 2, 3, \ldots n\}$ to $\{1, 2, 3, \ldots m\}$	(s) $({}^mC_n)\ (n!)$

	p	q	r	s
(a)	p	q	r	s
(b)	p	q	r	s
(c)	p	q	r	s
(d)	p	p	r	s

Answer & Solutions

1. (b),(c)

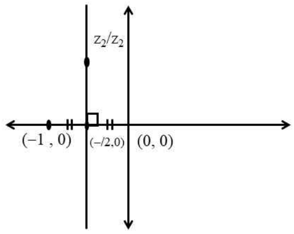
$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}. \quad \text{But } |z_1| = |z_2| = 1$$

$\therefore$ $$\left|\frac{z_1}{z_2}\right| = 1$$

Also, $$\frac{|z_1 + z_2|}{|z_2|} = \left|\frac{z_1}{z_2} + 1\right| = 1$$

$\therefore$ $$\left|\frac{z_1}{z_2} - (0+0i)\right| = \left|\frac{z_1}{z_2} - (-1-0i)\right| = 1$$

$\therefore$ z_1/z_2 lies on $\perp$ bisector of line segment joining $0 + 0i$ & $(-1 + 0i)$



$\therefore$ $\text{Re }(z_1/z_2) = -1/2$

$\therefore$ $z_1/z_2 = -1/2 + ai$

But , $$\left|\frac{z_1}{z_2}\right| = 1$$

$\therefore$ $|-1/2 + ai| = 1$

$\Rightarrow$ $$\frac{1}{4} + a^2 = 1 \Rightarrow a^2 = 3/4 \Rightarrow a = \pm\sqrt{3}/2$$

$\therefore$ $$\frac{z_1}{z_2} = -1/2 \pm \frac{\sqrt{3}}{2}1$$

$= \acute{\omega}, \acute{\omega}^2$

2. (a) $\rightarrow$ (r),(b) $\rightarrow$ (p),(c) $\rightarrow$ (s),(d) $\rightarrow$ (q)

(a)

$$\left|\frac{a + b + i\,c}{a\,(b + i\,c)}\right| = |1/z|$$

$\Rightarrow$ $$\frac{1}{|z|} = \frac{\sqrt{(a+b)^2 + c^2}}{|a|\sqrt{b^2 + c^2}}$$

$\therefore$ $$|z| = \frac{|a|\sqrt{b^2 + c^2}}{\sqrt{(a+b)^2 + c^2}}$$

$a \rightarrow r$

(b) $$\left|\frac{1}{z}\right| = \left|\frac{a - ic - a + ib}{(a - ib)(a - ic)}\right|$$

$$\frac{1}{|z|} = \frac{|i(b - c)|}{|a - ib|\ |a - ic|} = \frac{|b - c|}{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}$$

$$\therefore \quad |z| = \frac{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}{|b - c|}$$

$$b \rightarrow p$$

(c) $$\left|\frac{1}{z}\right| = \left|\frac{ab - ib + ac + ibc}{(a + ib)(a - ib)}\right|$$

$$= \frac{|a(b + c)|}{|(a + ib)(a - ic)|}$$

$$\frac{1}{|z|} = \frac{|a|\ |b + c|}{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}$$

$$|z| = \frac{\sqrt{a^2 + b^2}\ \sqrt{a^2 + c^2}}{|a|\ |b + c|}$$

$$c \rightarrow s$$

(d) $$\left|\frac{1}{z}\right| = \left|\frac{1}{a + i(b + c)}\right|$$

$$\frac{1}{|z|} = \frac{1}{\sqrt{a^2 + i(b + c)^2}}$$

$$\therefore \quad |z| = \sqrt{a^2 + (b + c)^2}$$

$$d \rightarrow q$$

3. (a) $\rightarrow$ (r),(b) $\rightarrow$ (s),(c) $\rightarrow$ (q),(d) $\rightarrow$ (p)

(a) Replace x by $-1/x$ in $ax^2 + bx + c = o$ to get $a - bx + cx^2 = 0$.

$$a \rightarrow r$$

(b) Replace x by $-x$ in $ax^2 + bx + c = o$ to get $ax^2 - bx + c^2 = 0$

$$b \rightarrow s$$

(c) Replace x by $\sqrt{x}$ in $ax^2 + bx + c = o$ to get $a^2\ x^2 + (2ac - b^2)\ \ x + c^2 = 0$

$$c \rightarrow q$$

(d) Replace x by $x/2$ in $ax^2 + bx + c = o$ to get $ax^2 + 2bx + 4c = 0$

$$d \rightarrow q$$

4. (b)

let x_1, x_2, x_3, x_4 be the 4 roots of $x^4 - 4x^3 + ax^2 - bx + 1 = O$

$$\therefore \quad x_1 + x_2 + x_3 + x_4 = 4$$

$$x_1\ x_2\ x_3\ x_4 = 1$$

$$\therefore \quad \underbrace{\tfrac{1}{4}(x_1 + x_2 + x_3 + x_4)}_{\text{A.M. of } x_1, x_2, x_3, x_4 = 1} \qquad \underbrace{(x_1\ x_2\ x_3\ x_4)^{1/4}}_{\text{G.M. of } x_1, x_2, x_3, x_4 = 1}$$

$$\Rightarrow \quad x_1 = x_2 = x_3 = x_4 = 1.$$

∴ $x^4 - 4x^3 + bx^2 - bx + 1 = (x - 1)^4$

⇒ a = 6 & b = 4

Also, Between any 2 roots of P(x) lies one root of P'(x) where P(x) is a polynomial

∴ Statement 1 & 2 both are true (b)

5. The integer 0

$$|b-c|^2 + |b+c|^2 = 2\left(|b|^2 + |c|^2\right)$$

$$|b-c|^2 + |-a|^2 = 2(1+1) = 1$$

$$\therefore \quad |b-c|^2 = 3$$

$$|b-c| = \sqrt{3} = |a-b| - |a-c|$$

$$\therefore \quad |a-b|^3 + |b-c|^3 + |c-a|^3 - 3|a-b|\,|b-c|\,|c-a|$$

$$3\sqrt{3} + 3\sqrt{3} + 3\sqrt{3} - 3\sqrt{3}\,\sqrt{3}\,\sqrt{3} = 0.$$

6. (a) → (p),(b) → (p),(c) → (p,q,r),(d) → (s)

2b = a + c

& $2b^2 = a^2 + c^2$

(a) $(a + c)^2 = (2b)^2$

$= 4b^2$

$= 2(2b^2)$

$a^2 + c^2 + 2ac = 2a^2 + 2c^2 \quad (a - c)^2 = O$

⇒ a = c but 2b = a + c

⇒ a = b = c

a → p

(b) $(b^2)^2 = a^2c^2$

$b^2 = \pm\, ac$

∴ $b^2 = ac$

⇒ a,b,c are in GP

Already, a,b,c are in ap

∴ a = b = c

∴ b → p.

(c) $b^2 = \dfrac{2a^2c^2}{a^2 + c^2}$

but, $b^2 = \left(\dfrac{a+c}{2}\right)^2$

$$\left(\frac{a+c}{4}\right)^2 = \frac{2a^2c^2}{a^2+c^2}$$

$$\underbrace{\left((a+c)^2+2ac\right)}_{\downarrow} \qquad \left((a-c)^2\right)=0 \longrightarrow a=b=c$$

$\frac{-a}{2}$, b,c are in GP

or

a,b, − c/2 are in GP

c → p, q, r.

(d) $a + b + c = 3/2$

$b + 2b = 3/2$

$3b = 3/2$

∴ $b = 1/2$

∴ d → s

7. (b)

Let a_1, a_2, a_3, a_4, be in GP

∴ $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a^4 = ar^3$ with r = common ratio.

∴ $b_1 = a$

$b_2 = a + ar = a(1 + r)$

$b_3 = a + ar + ar^2$

$= a(1 + \lambda + \lambda^2)$

$b4 = a(1 + \lambda + \lambda^2 + \lambda^3)$

Now,

$b_2 - b_1 \neq b_3 - b_2$

∴ b_1, b_2, b_3, b_4 are not in AP

Also, $\frac{b_2}{b_1} \neq \frac{b_3}{b_2}$

∴ b_1, b_2, b_3, b_4 are not in GP.

Also, $1/b_2 - 1/b_1 \neq 1/b_3 - 1/b_2$

∴ $\frac{1}{b_1}, \frac{1}{b_2}, \frac{1}{b_3}, \frac{1}{b_4}$ are in AP

∴ b_1, b_2, b_3, b_4 are not in H.P

∴ Statement (1) is true & (2) statement is false

8. (b)

$$tr = \frac{r^2}{(2r-1)(2r+1)}$$

$$4tr = \frac{4r^2-1+1}{(2r-1)(2r+1)}$$

$$4tr = 1 + \frac{1}{2}\left(\frac{1}{(2r-1)(2r+1)}\right)$$

$$4\sum_{r=1}^{n} tr = \sum_{r=1}^{n} 1 + \frac{1}{2}\sum_{r=1}^{n}\frac{1}{(2r-1)(2r+1)}$$

$$4\sum_{r=1}^{n} tr = n + \frac{1}{2}\left(\frac{1}{1} - \frac{2}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \ldots \frac{-1}{2n+1}\right)$$

$$= n + \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$

$$= n + \frac{2}{2n+1} = \frac{n(2n+1)+n}{2n+1}$$

$$\therefore \quad \sum_{r=1}^{n} tr = \frac{1}{4}\left(\frac{2n(n+1)}{2n+1}\right)$$

$$= \frac{2n(n+1)}{2(2n+1)}$$

And, $tr = \dfrac{1}{(2r-1)(2r+1)}$

$$= \frac{1}{2}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$$

$$\sum_{r=1}^{n} tr = \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$

$$= \frac{n}{2n+1}$$

$\therefore$ Statement (1) is true , Statement (2) is false.

9. (1) $\rightarrow$ (c),(2) $\rightarrow$ (b),(3) $\rightarrow$ (2)

(1)

$$\sum_{r=1}^{\infty} \frac{1}{4r^2-1} = \sum_{r=1}^{\infty} \frac{1}{(2r-1)(2r+1)}$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(\sum_{r=1}^{n}\left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)\right)$$

$$= \lim_{n\to\infty} \frac{1}{2}\left(1 - \frac{1}{2n+1}\right) = 1/2 \quad \text{(C)}$$

(2) $U_{n+1} - U_n = 2U_n + 1 - 2U_{n-1} - 1$

$= 2U_n - 2U_{n-1}$

$= 2\ (U_n - U_{n-1})$

$= 2.2(U_n - 1 - U_n - 2)$

$= 2.2\ (U_{n-1} - U_{n-2})....$

$= 2^{n-1}\ (U_2 - U_1) = 2^{n-1}\ (3-1)$

$= 2^n$

$\therefore\ U_{n+1} = 2^n + U_n = 2^n + 2^{n-1} + U_{n-1}$

$= 2^n + 2^{n-1} + 2^{n-2} + U_{n-2}$

$$= 2^n + 2^{n-1} + ... + 2^1 + U_1 = 2\left(\frac{2^n-1}{2-1}\right) + 1 = 2^{n+1} - 1$$

(3) $x_{51} = x_{50} + 51^2$

$$= \frac{25(51)^2}{2} + 51^2$$

$= 25\ (51)^2 + 51$

$= 51^2 \times 26 = 51^2 \times 13 \times 2$

$$\frac{x_{51}}{13 \times 51^2} = 2$$

10. (a) → (r),(b) → (s),(c) → (p),(d) → (q)

(1) m disticut books can be distributed among n children = n^m ways

(2) ${}^mC_n \times n!$

(3) Out of n persons, 2 are not selected

∴ m persons are to be selected from n–2 persons. But, $m \geq n > n - 2 \Rightarrow m > n - 2$

∴ Not possible

(4) Each members of domain can be mapped in m ways & domain has n members

∴ No of maps = m^n