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CLASS 11&12th

CLASS 11th

Gravitation

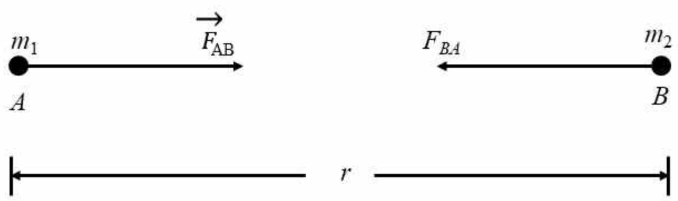
01. Newton's Law of Universal Gravitation

All physical bodies are subject to the action of the forces of mutual gravitational attraction. The basic law describing the gravitational forces was stated by Sir Issac Newton and it is called Newton's Law of Universal gravitation.

The law is stated as : "*Between any two particles of masses m_1 and m_2 at separation r from each other there exist attractive forces* $\vec{F}_{AB}$ and $\vec{F}_{BA}$ *(as shown in figure) directed from one body to the other and equal in magnitude which is directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between the two*". Thus we can write

$$F_{AB} = F_{BA} = G\frac{m_1 m_2}{r^2} \quad ...(i)$$

Where G is called universal gravitational constant. The law of gravitation can be applied to the bodies whose dimensions are small as compared to the separation between the two or when bodies can be treated as point particles.



02. Gravitational Field

We can state by Newton's universal law of gravitation that every mass M produces, in the region around it, a physical situation in which, whenever any other mass is placed, force acts on it, is called gravitational field. This filed is recognized by the force that the mass M exerts another mass, such as m, brought into the region.

03. Strength of Gravitational Field

We define gravitational filed strength at any point in space to be the gravitational force per unit mass on a test mass (mass brought into the filed for experimental observation). Thus for a point in space if a test mass m_0, experiences a force $\vec{F}$, then at that point in space gravitational field strength which is denoted by $\vec{g}$, is given as

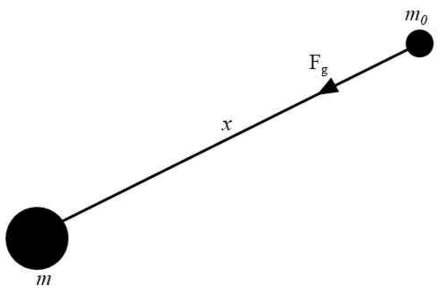
$$\vec{g} = \frac{\vec{F}}{m_0} \quad ...(i)$$

Gravitational field strength $\vec{g}$ is a vector quantity and has same direction as that of the force on the test mass in field.

04. Gravitational Field Strength of a Point Mass

As per our pervious discussion we can state that every point mass also produces a gravitational field in its surrounding. To find the gravitational field strength due to a point mass, we put a test mass m_0 at a point P at distance x from a point mass m then force on m_0 is given as

$$F_g = \frac{Gmm_0}{x^2} \quad \text{...(i)}$$



Now if at point P, gravitational field strength due to m is g_p then it is given as

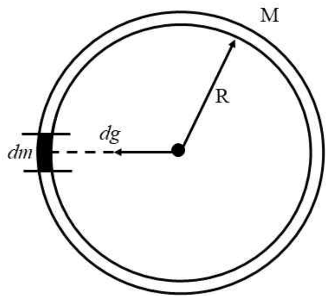
$$g_p = \frac{F_g}{m_0} = \frac{Gm}{x^2} \quad \text{...(ii)}$$

The expression in equation- (ii) gives the gravitational field strength at a point due to a point mass.

05. Gravitational Field Strength due to a Ring

Case-I : At the centre of ring

To find gravitational field strength at the centre of a ring of mass M and radius R, we consider an elemental mass dm on it as shown in figure. Let dg be the gravitational field at the centre of ring C due to the element dm.



Here we can simply state that another element of same mass exactly opposite to dm on other half of ring will produce an equal gravitational field at C in opposite direction. Thus due to all the elements on ring, the net gravitational field at centre C will be vectorially nullified and hence net gravitational field strength at C will be 0.

Case-II: At a point on the axis of ring

Figure shows a ring of mass M and radius R placed in YZ plane with centre at origin. Here we wish to find the gravitational filed strength at a point P on its axis at a distance x from its centre.

To find this we consider an element of length dl on ring as shown in figure. the mass dm of this element can be given as

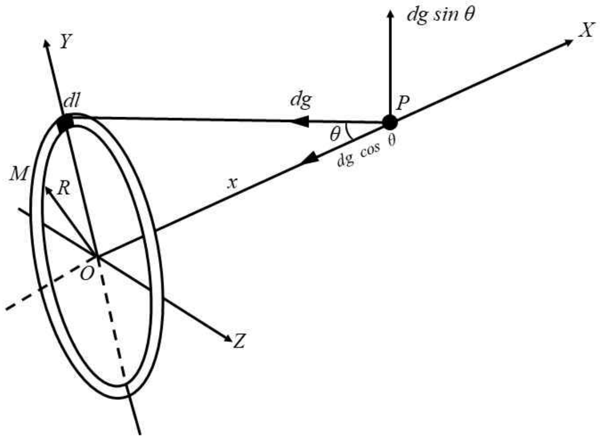
$$dm = \frac{M}{2\pi R}dl \qquad ...(i)$$

Let the gravitational field strength at point P due to the element dm is dg then it is given as

$$dg = \frac{Gdm}{(x^2 + R^2)}$$

or
$$= \frac{GM\,dx}{2\pi R(x^2 + R^2)}$$

This elemental gravitational field strength dg has two rectangular components, one along the axis of ring dg cosθ and other perpendicular to the axis of ring, dg sinθ. Here when we integrate the result for the complete ring, dg sinθ component will be cancelled out by symmetry and dg cosθ will be summed up to give the net gravitational filed strength at P.



Thus here net gravitational field strength at P is given as

$$g = \int dg\ \cos\theta = \int_0^{2\pi R} \frac{GM\,dx}{2\pi R(x^2 + R^2)} \times \frac{x}{\sqrt{x^2 + R^2}}$$

or
$$= \frac{GMx}{2\pi R(x^2 + R^2)^{3/2}} \int_0^{2\pi R} dl$$

$$= \frac{GMx}{2\pi R(x^2 + R^2)^{3/2}}[2\pi R]$$

$$= \frac{GMx}{(x^2 + R^2)^{3/2}}$$