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Complete MATH

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CLASS 11&12th

CLASS 12th

Inverse Trigonometric Functions

01. Inverse of a Function

We know that corresponding to every bijection (one-one onto function) $f: A \to B$ there exists a bijection $g: B \to A$ defined by

$$g(y) = x \text{ if and only } f(x) = y$$

The function $g: B \to A$ is called the inverse of function $g: A \to B$ and is denoted by f^{-1}. Thus, we have

$$f(x) = y \Leftrightarrow f^{-1}(y) = x.$$

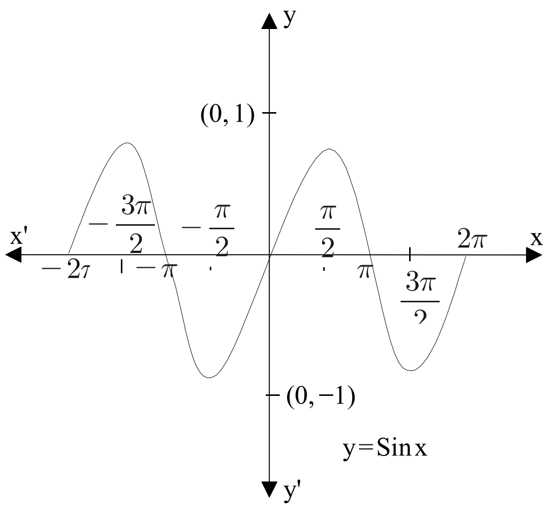
We have also learnt that

$$(f^{-1} of)(x) = f^{-1}(f(x)) = f^{-1}(y) = x, \text{ for all } x \in A.$$

and $$(of\, f^{-1})(y) = f(f^{-1}(y)) = f(x) = y, \text{ for all } x \in B.$$

02. Inverse of Sine Function

Consider the function $f: R \to R$ given by $f(x) = \text{Sin}\, x$. It is a many-one into function as it attains same value at infinitely many points and its range $[-1, 1]$ is not same as its co-domain. We known that any function can be made an onto function, if we replace its co-domain by its range. Therefore, $f: R \to [-1, 1]$ is a many-one onto functions.



Journey to obtain inverse of Sine function-

In order to make f a one-one function, we will have to restrict its domain in such a way that in that domain there is no turn in the graph of the function and the function takes every value between -1 and 1. It is evident from the graph of $f(x) = \text{Sin}\, x$ that if we take the domain as $[-\pi/2, \pi/2]$, then $f(x)$ becomes one-one. Thus,

$f: [-\pi/2, \pi/2] \to [-1, 1]$ given

$f(\theta) = \text{Sin}\, \theta$

is a bijection and hence invertible.

The inverse of the sine function is denoted by Sin^{-1}. Thus, Sin^{-1} is a function with domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$ such that

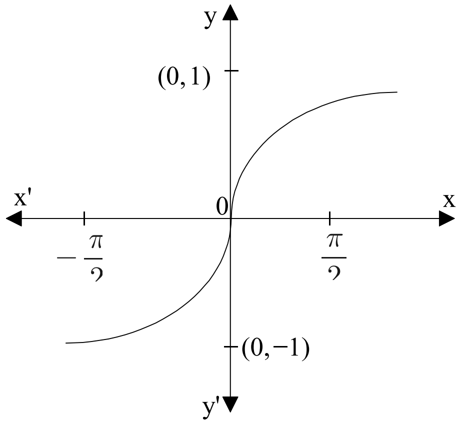
$$\text{Sin}^{-1}x = \theta \Leftrightarrow \text{Sin}\,\theta = x.$$

Also,

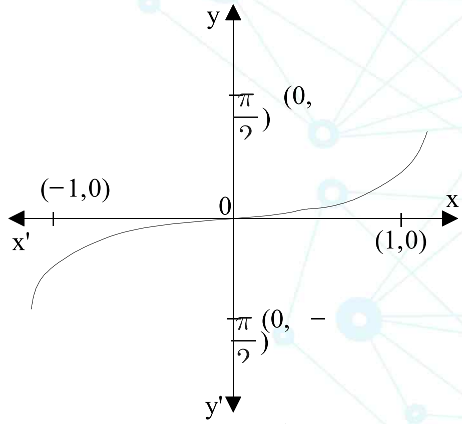
$$\text{Sin}^{-1}(\text{Sin}\,\theta) = \theta \quad \text{for all } \theta \in [-\pi/2, \pi/2] \qquad \left[\begin{array}{l} \because f^{-1}of(x) = f^{-1}(f(x)) = x \\ \text{and } fof^{-1}(y) = f\left(f^{-1}(y)\right) = y \end{array}\right]$$

and, $\text{Sin}\left(\text{Sin}^{-1}x\right) = x$ for all $x \in [-1, 1]$

The graph of the function $f : [-\pi/2, \pi/2] \rightarrow [-1, 1]$ given by $f(x) = \text{Sin}\,x$ is shown in Figure. In order to obtain the graph of $\text{Sin}^{-1} : [-1, 1] \rightarrow [-\pi/2, \pi/2]$ we interchange x and y axes as shown in Figure.



y=Sinx



y=Sin^{-1}x

03. Principal Value Branches of Inverse Trigonometric Functions

(i) $y = \text{Sin}^{-1}x \Rightarrow x = \text{Sin}\,y$

In $x = \text{Sin}\,y$, for one value of x, y can take infinite values.

But if $y = \text{Sin}^{-1}x$ is a function, then y should possess only one value of y for every value of x. This means we should restrict the values which y can possess. The restricted set of values which y can possess is its Principal Value Branch.

Here $-1 \leq \text{Sin}\,y \leq 1 \Rightarrow \dfrac{-\pi}{2} \leq y \leq \dfrac{\pi}{2}$

$\Rightarrow$ Domain: $x \in [-1, 1]$

Range: $y \in \left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$

Principal Value Branch of $\text{Sin}^{-1}x \equiv \dfrac{-\pi}{2} \leq y \leq \dfrac{\pi}{2}$