PHYSICS

CLASS NOTES FOR CBSE

Chapter 15. Oscillations

01. Wave Motion

Periodic Motion
Defined as that motion which repeats itself after equal intervals of time.
Oscillatory Motion
Oscillatory or vibratory motion is defined as a periodic and bounded motion of a body about a fixed point.
Difference between Periodic and Oscillatory Motion
Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. e.g., circular motion (or the motion of planets around the sun) is a periodic motion, but it is not oscillatory because, the basic concept of to and fro motion about the mean position for oscillatory motion is not present here.

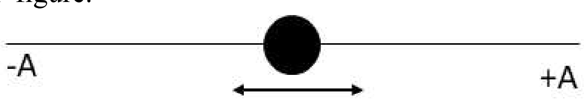
02. Periodic and Oscillatory Motions

Period is the smallest interval of time after which the motion is repeated.
Frequency is defined as the number of oscillations per unit time. It is the reciprocal of time period *T*. It is represented by the symbol *v*.
The relation between *v* and T is

$$v = \frac{1}{T} \qquad \text{...(i)}$$

03. Simple Harmonic Motion

Consider a particle oscillating to and fro, about the origin of an x-axis, between the limits +A and –A as shown in figure.



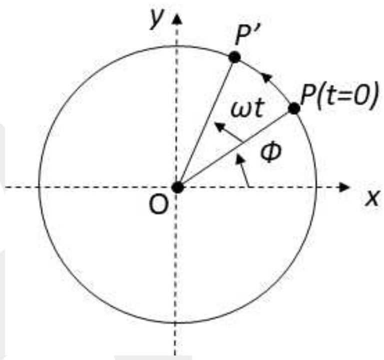
This is considered simple harmonic motion if displacement x, of the particle from the origin, varies with time as

$$x_{(t)} = A\cos(\omega t + \phi) \qquad \text{(i)}$$

Where A, ω and ϕ are constants.
SHM is not only periodic motion, but one in which displacement is a sinusoidal function of time.

04. Simple Harmonic Motion and Uniform Circular Motion

Given figure describes the same situation mathematically. Let a particle P move uniformly in a circle of radius A with angular speed ω in anti-clockwise direction.



The initial position vector of the particle makes an angle ϕ with positive direction of x-axis at t = 0. The position of P' on the x-axis as the particle moves on the circle is given by

$$x(t) = A\cos(\omega t + \phi)$$

Which is the defining equation of S.H.M.

Example Find the time taken by the particle in going form x = 0 to x = $\frac{A}{2}$ where A is the amplitude.

Solution

$$x = A\sin\omega t$$

$$\frac{A}{2} = A\sin\omega t$$

$$\sin\frac{\pi}{6} = \sin\frac{2\pi}{T}t$$

$$\frac{\pi}{6} = \frac{2\pi}{T}t$$

$$\Rightarrow \quad t = \frac{T}{12}$$

05. Velocity and Acceleration in Simple Harmonic Motion

The speed V of a particle in uniform circular motion is, its angular speed ω times the radius of the circle A.

$$v = \omega A$$

The direction of velocity $\bar{v}$ at a time t is along the tangent to the circle at the point where the particle is located at that instant.

$$v(t) = -\omega A sin(\omega t + \phi)$$

The instantaneous acceleration is than

$$a(t) = -\omega^2 A cos(\omega t + \phi)$$

$$a(t) = -\omega^2 x(t) \qquad ...(i)$$

$$x = A sin\omega t$$

Velocity in S.H.M. at instant t is

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

$$v = A\omega\cos\omega t$$

$$= A\omega\sqrt{1-\sin^2\omega t}$$

$$= A\omega\sqrt{1-\frac{x^2}{A^2}}$$

or $\boxed{v = \omega\sqrt{A^2 - x^2}}$ ...(ii)

Thus, the velocity of a particle in S.H.M. changes with displacement x of the particle, when the displacement is zero (x = 0), i.e., when it passes through its equilibrium position, velocity is maximum $(v_{\max} = A\omega)$ and when the displacement is maximum (y = A), velocity is zero (v = 0).

$$a = \frac{dv}{dt} = -A\omega^2\sin\omega t$$

$$a = -\omega^2 x$$

$$a \propto -x$$

Thus, acceleration of SHM is proportional to the displacement x and its direction is opposite to the direction of displacement from the equilibrium.

The period is defined as the time taken by the particle executing SHM to complete one vibration.

06. Energy in Simple Harmonic Motion

KE and PE of a particle in SHM vary between zero and their maximum values. The velocity of a particle executing SHM is zero at the extreme positions (it is a periodic function of time). So, the kinetic energy (K) of such a particle is

$$K = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi)$$

$$= \frac{1}{2}kA^2\sin^2(\omega t + \phi) \qquad ...(i)$$

This is also a periodic function of time, being zero when the displacement is maximum and minimum when the particle is at the mean position.

The concept of potential energy is possible only for conservative forces. The spring force, F = -kx is a conservative force, with associated potential energy.