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Complete MATH

IIT-JEE · CBSE eBOOKS

CLASS 11&12th

CLASS 11th

Permutations & Combinations

01. Factorial

Factorial *The continued product of first n natural numbers is called the "n factorial" and is denoted by* $\lfloor n$ *or* $n!$ *i.e.*
$n! = 1 \times 2 \times 3 \times 4 \times \ldots \times (n-1) \times n$

02. Exponent of Prime *p* In Factorial *n*

Let p be a prime number and n be a positive integer.
Let $E_p(n)$ denote the exponent of the prime p in the positive inter n. Then,

$$E_p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots \left[\frac{n}{p^s}\right],$$

where s is the largest positive integer such that $p^s \le n < p^{s+1}$

03. Some Useful Symbols

If n is a natural number and r is a positive integer satisfying $0 \le r \le n$, then the natural number $\frac{n!}{(n-r)!}$ is denoted by the symbol ${}^nP_r$ or, $P(n, r)$.

i.e., ${}^nP_r = P(n,r) = \frac{n!}{(n-r)!}$

If n is *a* natural number and r is a positive integer satisfying $0 \le r \le n$, then the natural number $\frac{n!}{(n-r)!\,r!}$ is denoted by the symbol ${}^nC_r$, or, $C(n, r)$. Thus,

$${}^nC_r = C(n, r) = \frac{n!}{(n-r)!\,r!}$$

Property I ${}^nC_r = {}^nC_{n-r}$, *for* $0 \le r \le n$.

Remark The above property can be restated as follow:
If x and y are non-negative integers such that ${}^nC_x = {}^nC_y$, then $x = y$ or, $x + y = n$.

Property II *Let n and r be non-negative integers such that* $1 \le r \le n$.

Then, ${}^nC_r = \frac{n}{r} \cdot {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ ${}^nC_{r-1} = {}^{n+1}C_r$

Property III *Let n and r be non-negative integers such that* $1 \le r \le n$.

Then, ${}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1}$

Property IV *If* $1 \le r \le n$, *then* $n.\ {}^{n-1}C_{r-1} = (n - r + 1)\,{}^nC_{r-1}$

Property V *If n is even, then the greatest value of* ${}^nC_r$ $(0 \le r \le n)$ *is* ${}^nC_{n/2}$.

Property VI *If n is odd, then the greatest value of* ^nC_r $(0 \le r \le n)$ is $\dfrac{^nC_{n+1}}{2}$ or, $\dfrac{^nC_{n-1}}{2}$

04. Fundamental Principles of Counting

Fundamental Principle of Multiplication *If there are two jobs such that one of them can be completed in m ways, and when it has been completed in any one of these m ways, second job can be completed in n ways; then the jobs in succession can be completed in* $m \times n$ *ways.*

Remark *The above principle can be extended for any finite number of jobs as stated below: If there are n jobs* $J_1, J_2, \ldots J_n$ *such that job* J_i *can be performed independently in* m_i *ways in which all the jobs can be performed is* $m_1 \times m_2 \times m_3 \times \ldots \times m_n$.

Fundamental Principle of Addition *If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in* $(m + n)$ *ways.*

05. Permutations and Combinations

Combination *Each of the different selections made by takin some or all of a number of distinct objects or items, irrespective of their arrangements or order in which they are placed, is called a combination.*

Permutations *Each of the different arrangements which can be made by taking some or all of a number of distinct objects is called a permutation.*

Results I *Let r and n be positive integers such that* $1 \le r \le n$. *Then, prove that the number of all permutations of n distinct items or objects taken r at a time is*

$$n(n-1)(n-2)(n-3)\ldots(n-(r-1))$$

Remark 1 *We have,*

$n(n-1)(n-2)\ldots(n-(r-1))$

$$= \frac{n(n-1)(n-2)\ldots(n-(r-1))(n-r)!}{(n-r)!} = \frac{n!}{(n-r)!} = {}^nP_r$$

So, the total number of arrangements (permutations) of n-distinct items, taking r at a time is ^nP_r *or* $P(n, r)$.

Results II *The number of all permutations (arrangements) of n distinct objects taken all at a time is* $n!$.