PHYSICS

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01. Radioactive Decay Law

The number of atoms disintegrating per second of a radioactive sample at any tie is directly proportional to the number of atoms present at that time. The rate of disintegration of the sample cannot be altered by changing the external factors, such as pressure, temperature, etc. It is known as **radioactive decay law**.

Consider that a radioactive sample contains N_0 atoms initially ($t = 0$). As the time elapses, the atoms of the sample decrease due to disintegration. Suppose that after time t, the number of the atoms reduce to N and after time $t + dt$, the number of atoms further decrease to $N - dN$. Obviously, in the time interval between t and $t + dt$ *i.e.* equal to dt, the number of atoms decrease by $N - (N - dN)$ *i.e.* dN.

Therefore, rate of disintegration of atoms at time t** = $\frac{dN}{dt}$

According to radioactive decay law, the rate of disintegration at any time t is directly proportional to the number of atoms present at time t *i.e.*

$$\frac{dN}{dt} \propto N$$

or $$\frac{dN}{dt} = -\lambda N, \qquad \text{...(i)}$$

where the constant of proportionally λ is called ***decay constant*** of the radioactive sample. It is also known as ***disintegration constant*** or ***transformation constant***. Its value depends upon the nature of the radioactive sample. Further, the negative sign indicates that the number of the atoms of the sample decreases with the passage of time.

From the equation (i), we have

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating, we have

$$\int \frac{dN}{N} = -\lambda \int dt$$

or $$\log_e N = -\lambda t + k, \qquad \text{...(ii)}$$

where k is constant of integration and it can be evaluated from the following boundary condition :

When $t = 0, N = N_0$

On setting $t = 0$ and $N = N_0$, the equation (ii), becomes

$$\log_e N_0 = -\lambda \times 0 + k$$

or $$k = \log_e N_0$$

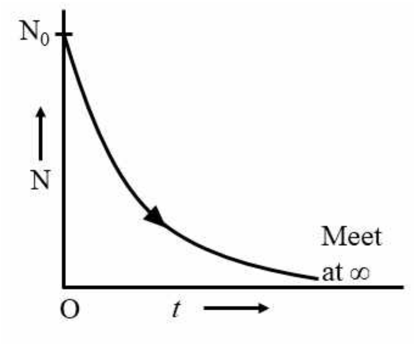
Substituting for k in the equation (ii), we get

$$\log_e N = -\lambda t + \log_e N_0$$

or $$\log_e \frac{N}{N_0} = -\lambda t$$

or $$\frac{N}{N_0} = e^{-\lambda t}$$

or $$N = N_0\, e^{-\lambda t} \qquad \text{...(iii)}$$



02. Radioactive Decay Constant

The decay constant of a radioactive sample is a characteristic of the radioactive substance. The radioactive substances having large value of the decay constant, decay rapidly in comparison to those having a small value of decay constant.

According to radioactive decay law,

$$\frac{dN}{dt} = -\lambda N$$

or $$\lambda = \frac{-dN/dt}{N}$$

Hence, *radioactive decay constant of a substance (radioactive) may be defined as the ratio of its instantaneous rate of disintegration to the number of atoms present at that time.*

Again, $$N = N_0 e^{-\lambda t}$$

If $$t = \frac{1}{\lambda},$$

then $$N = N_0 e^{-\lambda \frac{1}{\lambda}} = \frac{1}{e} N_0 = \frac{N_0}{2.718} = 0.368 N_0$$

Hence, *radioactive decay constant of a substance may also be defined as the reciprocal of the time, after which the number of atoms of a radioactive substance decreases to 0.368 (or 36.8%) of their number present initially.*

03. Half−Life

The half−life of a radioactive substance is defined as the time during which the nuclei of half of the atoms of the radioactive substance will disintegrate. It is denoted by T.

Consider that radioactive sample contains N_0 atoms at time $t = 0$. Then, the number of atoms left behind after time t is given by

$$N = N_0 e^{-\lambda t} \qquad \text{...(iv)}$$

From the definition of half-life, it follows that when t = T (half-life of the sample), $N = \frac{N_0}{2}$

Setting the above condition in equation (iv), we have

$$\frac{N_0}{2} = N_0\, e^{-\lambda T}$$

or $$e^{-\lambda T} = \frac{1}{2}$$

or $$e^{\lambda T} = 2$$

or $$\lambda T = \log_e 2 = 2.303 \log_{10} 2 = 2.303 \times 0.3010 = 0.693$$

or $$T = \frac{0.693}{\lambda} \qquad ...(v)$$

Thus, half-life of a radioactive substance is inversely proportional to its decay constant and is a characteristic property of its nucleus.

04. Mean-Life or Average-Life

The average-life of a radioactive substance is defined as the average time for which the, nuclei of the atoms of the radioactive substance exist. It is denoted by T_a.

Obviously, the average-life of a radioactive sample may be calculated by finding the total life of all the atoms of the substance in a sample and then dividing it by the total number of atoms present in that sample.

Consider that initially (t = 0), the number of atoms present in a sample of radio-active substance is N_0. Suppose that at time t, the number of atoms reduces to N and at time $t + dt$, the number of atoms becomes N − dN. Obviously, each of dN atoms disintegrating during the time interval between t and $t + dt$ has lived for a time t.

Therefore, the total life of dN atoms = $t\ d$N

and the total life of all the N_0 atoms = $\int_0^{N_0} t\ dN$

hence, the average-life of the radioactive sample*,

$$T_a = \frac{\int_0^{N_0} t\ dN}{N_0} = \frac{N_0/\lambda}{N_0}$$

or $$T_a = \frac{1}{\lambda} \qquad ...(vi)$$

Thus, the average-life of a radioactive substance is equal to the reciprocal of its dec ay constant.

From the equations (v) and (vi), we have

$$T = 0.693\ T_a \qquad ...(vii)$$

i.e. half-life of a radioactive substance is equal to 0.693 times its average-life.

05. Activity of Radioactive Substance

The activity of a radioactive substance may be defined as the rate at which the nuclei of its atoms in the sample disintegrate.

If a radioactive sample contains N atoms at any time t, then its activity at time t is defined as

$$R = -\frac{dN}{dt} \qquad ...(viii)$$

The negative sign shows that with the passage of time, the activity of the radio-active substance decreases.

Since according to the radioactive decay law,

$$\frac{dN}{dt} = -\lambda N,$$

the equation (viii) may be expressed as

$$R = \lambda N \qquad ...(ix)$$

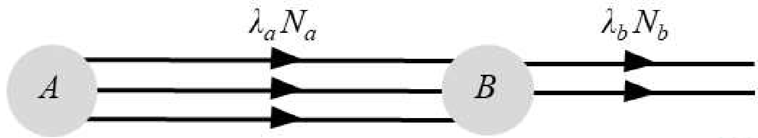
Since $N = N_0 e^{-\lambda t}$, we have

$$R = \lambda N_0 e^{-\lambda t} \qquad ...(x)$$

Here, $\lambda N_0 = R_0$ is activityn of the radioactive sample at time $t = 0$.

06. Successive Disintegration

Suppose a parent radioactive nucleus A (decay constant = λ_a) has number of atoms N_0 at time t = 0. After disintegration it converts into a nucleus B (decay constant = λ_b) which is further radioactive. Initially (t = 0), number of atoms of B are zero. We are interested in finding N_b, the number of atoms of B at time t.



	A	B
At t = 0	N_0	0
At t = t	$N_a = N_0 e^{-\lambda_a t}$	$N_b = ?$

At time t, net rate of formation of B = rate of disintegration of A − rate of disintegration of B

$\therefore \qquad \dfrac{dN_b}{dt} = \lambda_a N_a - \lambda_b N_b$

or $\qquad \dfrac{dN_b}{dt} = \lambda_a N_0 e^{-\lambda_a t} - \lambda_b N_b \qquad$ (as $N_a = N_0 e^{-\lambda_a t}$)

or $\qquad dN_b + \lambda_b N_b \, dt = \lambda_a N_0 e^{-\lambda_a t}$

Multiplying this equation by $e^{\lambda_b t}$, we have

$$e^{\lambda_b t} dN_b + e^{\lambda_b t} \lambda_b N_b \, dt = \lambda_a N_0 e^{(\lambda_b - \lambda_a) t}$$

$$\therefore \quad d N_b e^{\lambda_b t} = \lambda_a N_0 e^{(\lambda_b - \lambda_a) t} \, dt$$

Integrating both sides, we get

$$N_b e^{\lambda_b t} = \left(\frac{\lambda_a}{\lambda_b - \lambda_a}\right) N_0 e^{(\lambda_b - \lambda_a) t} + C \qquad \text{...(xi)}$$

where, C is the constant of integration, which can be found as under.

At time, $t = 0, \quad N_b = 0$

$$\therefore \quad C = \left(\frac{\lambda_a}{\lambda_b - \lambda_a}\right) N_0$$

Substituting this value in equation (xi), we have

$$N_b = \frac{N_0 \lambda_a}{\lambda_b - \lambda_a} (e^{-\lambda_a t} - e^{-\lambda_b t})$$