SAMPLE PAPER

2019 JEE ADVANCED

MATHEMATICS

SET–2

Roll No. | | | | | | | |

Section 1 (Maximum Marks : 24)

(i) *This section contains **SIX** questions.*

(ii) *Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four options is correct.*

(iii) *For each question, darken the bubble corresponding to the correct option in the OMR Sheet.*

(iv) *For each question, marks will be awarded in one of the following categories:*

Full Marks : +4 If only the bubble corresponding to the correct option is darkened.

*Partial Marks : +1 For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.*

Zero Marks : 0 If none of the bubbles is darkened.

Negative Marks : –2 In all other cases.

(v) *For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks; and darkening (a) and (b) will result in –2 marks, as a wrong option in also darkened.*

1. If z_1 lies on $|z| = 1$ and z_2 lies on $|z| = 2$, then
   (a) $3 \le |z_1 - 2z_2| \le 5$
   (b) $1 \le |z_1 + z_2| \le 3$
   (c) $|z_1 - 3z_2| \ge 5$
   (d) $|z_1 - z_2| \ge 1$

2. The equation $x^3 + 1 = 2(2x - 1)^{1/3}$ has
   (a) one rational solution
   (b) two irrational solutions
   (c) sum of roots equal to zero
   (d) product of the roots as –1.

3. If roots of $x^3 + bx^2 + cx + d = 0$ are
   (a) In A.P. then $2b^3 - 9bc + 27d = 0$
   (b) In G.P. then $2b^3\ d = c^3$
   (c) In G.P. then $27d^3 = 9bcd^2 - 4c^3\ d$
   (d) equal then $c^3 = b^3 + 3bc$

4. Let x be the number of 5 digit numbers sum of whose digits is even and y be the number of 5 digit numbers sum of whose digits is odd, then
   (a) $x = y$
   (b) $x + y = 90{,}000$
   (c) $x = 45{,}000$
   (d) $x < y$

5. Let $S_n = \sum_{r=0}^{n} (-2)^r \left(\frac{{}^{n}C_r}{{}^{r+2}C_r} \right)$, then
   (a) $S_n = \frac{1}{n+1}$ if n is odd
   (b) $S_n = \frac{1}{n+2}$ if n is odd
   (c) $S_n = \frac{1}{n+1}$ if n is even
   (d) $S_n = \frac{1}{n+2}$ if n is even

6. Let A, B and C be 2×2 matrices with entires from the set of real numbers. Define * as follows:

$$A * B = \frac{1}{2}(AB' + BA')$$

   (a) $A * B = B * A$
   (b) $A * A = A^2$
   (c) $A * (B + C) = A * B + A * C$
   (d) $2A * I = A + A'$

Section-2 (Maximum Marks : 24)

(i) This section contains 8 questions.
*(ii) Each question has 4 options (a), (b), (c), (d). **Only One** of these options is correct.*
(iii) For each question, darken the bubble corresponding to the correct option in the OMR Sheet.
*(iv) For each question, marks will be awarded in **one of the following categories**:*
Full Marks : +3 If, only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative marks : –1 In all other cases.

7. The values of θ lying between θ = 0 and θ = π/2 and satisfying the equation

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 6\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 6\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 6\theta \end{vmatrix} = 0$$

are given by
(a) π/36, 5π/36
(b) 7π/36, 11π/36
(c) 5π/36, 7π/36
(d) 11π/36, π/36

8. A box contains 24 identical balls of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4th time on the 7th draw is
(a) 5/64
(b) 27/32
(c) 5/32
(d) 1/2

9. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$, then $f(\theta)$
(a) ≥ 0 only when θ ≥ 0
(b) ≤ 0 for all real θ
(c) ≥ 0 for all real θ
(d) ≤ 0 only when θ ≤ 0

10. In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is

(a) $\dfrac{3y}{2x(x+c)}$
(b) $\dfrac{3y}{2c(x+c)}$
(c) $\dfrac{3y}{4x(x+c)}$

(d) $\dfrac{3y}{4c\,(x+c)}$

11. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \pi/2$ is
    (a) 0
    (b) 1
    (c) 2
    (d) infinite

12. Let $0 < \alpha < \pi/2$ be a fixed angle. If $P = (\cos\theta, \sin\theta)$ and $Q = (\cos(\alpha-\theta), \sin(\alpha-\theta))$ then Q is obtained from P by
    (a) clockwise rotation around origin through an angle α.
    (b) anti clockwise rotation around origin through an angle α.
    (c) reflection in the line through origin with slope $\tan\alpha$.
    (d) reflection in the line through origin with slope $\tan(\alpha/2)$.

13. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals
    (a) $\sqrt{PQ.RS}$
    (b) $\dfrac{PQ+RS}{2}$
    (c) $\dfrac{2PQ+RS}{PQ+RS}$
    (d) $\sqrt{\dfrac{PQ^2+RS^2}{2}}$

14. The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of quadrilateral $PQRS$ is
    (a) 3
    (b) 6
    (c) 9
    (d) 15

Section-3 (Maximum Marks : 12)

(i) This section contains 4 question.
*(ii) Each question has 4 options (a), (b), (c), (d). **Only One** of these options is correct.*
(iii) For each question, darken the bubble corresponding to the correct option in the OMR Sheet.
*(iv) For each question, marks will be awarded in **one of the following categories**:*
Full Marks : +3 If, only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative marks : –1 In all other cases.

15. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x + y + 2z = 3$, $P_2 : 3x + 5y - 6z = 4$. Let $ax + by + cz = d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2. and perpendicular to planes P_1 and P_2.
Match Column - I with Column - II and select the correct answer using the code given below the lists :

Column - I	**Column - II**
P. a =	1. 13
Q. b =	2. –3
R. c =	3. 1
S. d =	4. –2

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	1	3	4	2
(c)	3	2	1	4
(d)	2	4	1	3

16. Match the columns

Column - I	**Column - II**
p. $\{x : [\sin^{-1} x] > [\cos^{-1} x]\}$ contains	1. $\{0, 1\}$
q. Range of $[\lvert\sin x\rvert + \lvert\cos x\rvert]$ contains	2. $\{1\}$
r. Range of $[x + 1/2] + [x - 1/2] + 2[-x]$	3. $[\sin 1, 1]$
s. Range of $\left[\frac{1}{[x-3]}\right]$ contains	4. $\{-1\}$

Codes:

	P	Q	R	S
(a)	2, 3	1, 2	4	1, 2, 4
(b)	2	1, 3	4	1, 2, 4
(c)	1	3	4	2
(d)	1, 3	2	4	3, 4

17. Let $f_1 : \mathbf{R} \to \mathbf{R}$, $f_2 : [0, \infty) \to \mathbf{R}$, $f_3 : \mathbf{R} \to \mathbf{R}$ and $f_4 : \mathbf{R} \to [0, \infty)$ be defined by

$$f_1(x) = \begin{cases} |x| & \text{if } x < 0, \\ e^x & \text{if } x \geq 0; \end{cases}$$

$f_2(x) = x^2$;

$$f_3(x) = \begin{cases} \sin x & \text{if } x < 0, \\ x & \text{if } x \geq 0; \end{cases}$$

and

$$f_4(x) = \begin{cases} f_2(f_1(x)) & \text{if } x < 0, \\ f_2(f_1(x)) - 1 & \text{if } x \geq 0; \end{cases}$$

Column - I	**Column - II**
P. f_4 is	1. onto but not one-one
Q. f_3 is	2. neither continuous nor one-one
R. f_2 *o* f_1 is	3. differentiable but not one-one
S. f_2 is	4. continuous and one-one

Codes:

	P	Q	R	S
(a)	3	1	4	2
(b)	1	3	4	2
(c)	3	1	2	4
(d)	1	3	2	4

18. Match the columns

	Column - I	**Column - II**
P.	The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)\,dx = 1$, is	1. 8
Q.	The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	2. 2
R.	$\int_{-2}^{2} \frac{3x^2}{(1+e^x)}\,dx$ equals	3. 4
S.	$\frac{\left(\int_{-1/2}^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}{\left(\int_{0}^{1/2} \cos 2x \log\left(\frac{1+x}{1-x}\right) dx\right)}$ equals	4. 0

Codes:

	P	Q	R	S
(a)	3	2	4	1
(b)	2	3	4	1
(c)	3	2	1	4
(d)	2	3	1	4