SET–1

Roll No. | | | | | | | |

CBSE 2019 Sample Question Paper

MATHEMATICS

Time allowed: 3 hours ***Maximum marks: 100***

General Instructions:

(i) *All question are compulsory.*

(ii) *This question paper contains 29 questions.*

(iii) *Question number **1** to **4** in Section A very short-answer type questions carrying **1** mark each.*

(iv) *Questions number **5** to **11** in Section B very short-answer type questions carrying **2** marks each.*

(v) *Questions number **12** to **22** in Section C are long-answer I type questions carrying **4** marks each.*

(vi) *Questions number **23** to **29** in Section D are long-answer II type questions carrying **6** marks each.*

Section A

1. Find the minor of element 6 in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

2. Differentiate $\sin(\cos(x^2))$ with respect to x.

3. Find the order and degree, if defined, of each of the differential equation.

$$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

4. Find the direction cosines of x, y and z-axis.

OR

Find the angle between the pair of lines given by
$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$
and $\vec{r} = 5\hat{i} + 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Section – B

5. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that A + aA + bI = O.

6. Let * be the binary operation on **N** defined by a * b = H.C.F. of a and b. Is * commutative? Is * associative? Does there exist identity for this binary operation on **N**?

7. Find $\int \frac{(x^2+1)e^x}{(x+1)^2} dx$

OR

Evaluate the intergral : $\int_4^9 \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx$

8. Form the differential equation representing the family of curves $y = a \sin(x + b)$, where a, b are arbitrary constants.

9. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 8$, then find $|\vec{x}|$.

OR

Show that the points $A(-2\hat{i} + 3\hat{j} + 5\hat{k})$, $B(\hat{i} + 2\hat{j} + 3\hat{k})$ and $C(7\hat{i} - \hat{k})$ are collinear.

10. If A and B are two independent events, then show that the probability of occurrence of at least one of A and B is given by 1–P(A′) P(B′).

11. Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly and then one card is drawn randomly. If it is known that the number on the drawn card is more than 3, what is the probability that it is an even number?

OR

If P(A) = 0.8, P(B) = 0.5 and P (B/A) = 0.4. find P(A ∪ B)

Section – C

12. Show that the relation R in the set **R** of real numbers, defined as R = $\{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

OR

Consider $f : R_+ \to (-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{(\sqrt{y+6})-1}{3}\right)$.

13. Show that $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{13} = \pi$.

14. If a, b, c are positive and unequal, show that value of the determinant $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

15. Prove that the greatest integer function defined by $f(x) = [x]$, $0 < x < 3$ is not differentiable at $x = 1$ and $x = 2$.

OR

If $y = \sin^{-1}x$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

16. If $y = \cos^{-1}x$, Find $\frac{d^2y}{dx^2}$ in terms of y alone.

17. Find the slope of the normal to the curve $x = a\ cos^3\theta$, $y = a\ sin^3\theta$ at $\theta = \frac{\pi}{4}$.

18. Find $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$.

19. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$.

20. Find the particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$ given that $y = 1$, when $x = 0$.

OR

Find the equation of a curve passing through the point (−2, 2), given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

21. If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ are the position vectors of points A, B, C and D respectively, then find the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Deduce that $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

22. Find the shortest distance between the lines l_1 and l_2 whose vector equations are

$$\vec{r} = \hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}-\hat{k}) \quad \text{..... (1)}$$

and $$\vec{r} = 2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}-2\hat{k}) \quad \text{..... (2)}$$

Section – D

23. Solve the system of equations

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$
$$\frac{4}{x}+\frac{6}{y}+\frac{5}{z}=1$$
$$\frac{6}{x}+\frac{9}{y}+\frac{20}{z}=2$$

OR

If $A=\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$, then verify that A(*adj* A) = |A| I. Also find A^{-1}.

24. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

25. Find the area of the region enclosed between the two circles : $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

OR

Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.

26. Find the vector equation of the plane passing through the intersection of the planes $\bar{r}.(\hat{i}+\hat{j}+\hat{k}) = 6$ and $\bar{r}.(2\hat{i}+3\hat{j}+4\hat{k}) = -5$, and the point (1,1,1)

OR

Find the coordinates of the point where the line through the points A(3, 4, 1) and B(5, 1, 6) crosses the XY plane.

27. A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron. 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin A in the diet ? What is the minimum amount of vitamin A?

28. Find the mean of the Binomial distribution $B\left(4, \frac{1}{3}\right)$.

29. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

OR

Find the shortest distance between the lines whose vector equations are
$\vec{r} = (1-t)\,\hat{i} + (t-2)\,\hat{j} + (3-2t)\,\hat{k}$ and
$\vec{r} = (s+1)\,\hat{i} + (2s-1)\,\hat{j} - (2s+1)\,\hat{k}$