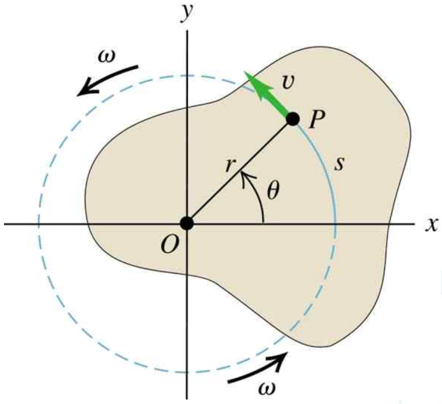
PHYSICS

Live eBook

01. Kinematics

Consider a rigid body rotating about a given fixed line.



The average angular velocity during the time interval Δt is $\omega = \frac{\triangle\theta}{\triangle t}$.

The instantaneous angular velocity at time t is $\omega = \frac{d\theta}{dt}$.

If the body rotates through equal angles in equal time intervals (irrespective of the smallness of the intervals), we say that it rotates with uniform angular velocity. In this case $\omega = d\theta/dt$ = constant and thus $\theta = \omega t$. If it is not the case, the body is said to be rotationally "accelerated". The angular acceleration is defined as

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2}.$$

If the angular acceleration α is constant, we have

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

and $$\omega^2 = \omega_0^2 + 2\alpha\theta$$

02. Relation between the Linear Motion of a Particle of a Right Body and its Rotation

The linear speed along the tangent is

$$v = \frac{ds}{dt} = r.\frac{d\theta}{dt}$$

$\therefore$ $$v = r\omega$$

and the linear acceleration along the tangent, i.e., the tangential acceleration, is

$$a = \frac{dv}{dt} = r.\frac{d\omega}{dt}$$

$\therefore$ $$a = r\alpha.$$

03. Moment of Inertia

Moment of inertia of a body about a given axis as the sum of the products of masses of all the particles of the body and squares of their respective perpendicular distances from the axis of rotation.

$$I = \sum_{i=1}^{i=n} m_i r_i^2$$

moment of inertia of a body about a given axis is numerically equal to torque acting on the body rotating with unit angular acceleration about it.

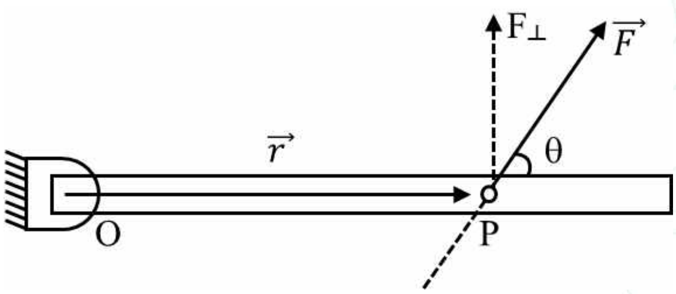
$$\vec{\tau} = I\vec{\alpha}$$

Moment of Inertia of Some bodies of regular shape

S.No.	Body	Axis	Moment of Inertia
i.	Thin circular ring, radius R	Perpendicular to plane, at centre	MR^2
ii.	Thin circular ring, radius R	Diameter	$M R^2/2$
iii.	Thin rod, length L	Perpendicular to rod, at mid point	$M L^2/12$
iv.	Circular disc, radius R	Perpendicular to disc at centre	$M R^2/2$
v.	Circular disc, radius R	Diameter	$M R^2/4$
vi.	Hollow cylinder, radius R	Axis of cylinder	$M R^2$
vii.	Solid cylinder, Radius R	Axis of cylinder	$M R^2/2$
viii.	Solid sphere, Radius R	Diameter	$2 M R^2/5$

04. Torque of a Force about a Point

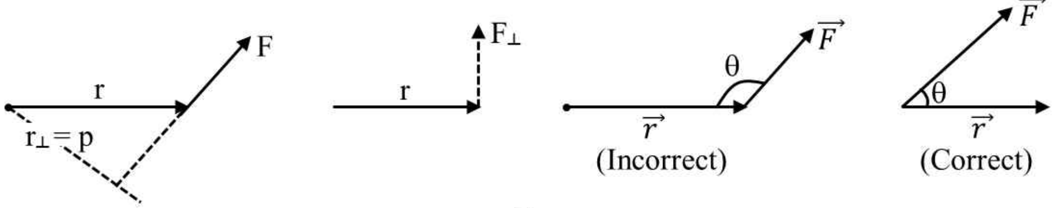
A pair of forces of equal magnitude but acting in opposite directions with different lines of action is known as a **couple** or **torque**. If you want to rotate the rod, you have to push any point *P* of the rod by a force *F*. The perpendicular component of the force, that is, $F_\perp$ (= $F \sin\theta$) is responsible for rotating rod. This turning or twisting or rotating effect of the force is called torque denoted by τ.



The rod rotates by the rotational effect (torque τ) of the force $\vec{F}$ which is given as $\vec{\tau} = \vec{r} \times \vec{F}$

i. $\tau = r\ F\ \sin\theta = pF = rF_{\perp}$, where θ = Angle between $\vec{r}$ and $\vec{F}$.

ii. Change in direction of $\vec{F}$ reverses the direction of $\vec{\tau}$.



Figure

05. Angular Momentum

Angular momentum $\vec{l}$ of a particle about an inertial reference point is defined as the vector product of the position vector $\vec{r}$ and linear momentum $\vec{p}$ of the particle relative to the reference point.

$$\vec{l} = \vec{r} \times \vec{p}\ (= \vec{r} \times m\vec{v})$$

where $\vec{p}$ is the linear momentum and $\vec{r}$ is the position vector of the particle from the given point. The angular momentum of a system of particles is the vector sum of the angular momenta of the particles of the system. Thus,

$$\vec{L} = \sum_i \vec{l_i} = \sum \left(\vec{r_i} \times \vec{p_i}\right)$$

06. Conservation of Angular Momentum

According to this principle, when no external torque acts on a system of particles, then the total angular momentum of the system remains always a constant.

For a system of n particles making up a rigid body, total torque acting on the body is due to external forces only. The internal forces between the particles of the body do not contribute to the torque.

As $$\vec{\tau}_{total} = \frac{d}{dt}\left(\vec{L}_{total}\right)$$

$\therefore$ When no external torque acts on the system, $\vec{\tau}_{total} = 0$

$\therefore$ $$\frac{d}{dt}\left(\vec{L}_{total}\right) = 0 \quad \text{or} \quad \vec{L}_{total} = constant$$

i.e., $$\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \ldots.\vec{L}_n = constant\ \ (vector)$$

This is the principle of conservation of angular momentum.